Cross-border M&As, greenfield investment, and the gains from openness

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Abstract

This paper develops a general equilibrium model of the cross-border M&As and greenfield foreign direct investment. Firms engaging in the cross-border M&As enhance the pair-wise efficiency of target and acquirer firms, whereas firms engaging in the greenfield foreign direct investment use their original technology. This paper shows how these two types of firms affect the gains from cross-border M&As and greenfield foreign direct investment.

Keywords: cross-border M&A; greenfield foreign direct investment; gains

JEL classification: F11; F23; O33

1 Introduction

There are three general ways through which multinational enterprises (MNEs) access foreign markets: exports, greenfield investment (GFI), and cross-border merger and acquisition (CBMA). Among these three options, the importance of CBMAs has been increasing over time: more and more MNEs choosing to acquire existing local firms, rather than exporting or building their own establishment from scratch.

To shed light on implications of this trend, we extend the benchmark Ricardian model of trade and multinational production (Eaton and Kortum, 2002, Ramondo and Rodriguez-Clare, 2013, and Alviarez, 2019) by incorporating two different modes of foreign market access: GFI and CBMAs. We assume that each firm owns knowledge-capitals, which are tradable across firms in different countries. As in the standard model in the literature, GFI firms receive a productivity draw to produce abroad based on their nationality. Additionally, we allow firms to have another option of CBMAs: firms receive another draw that determines an efficiency of using knowledge capital of foreign firms in supplying their goods and services in their markets. In the CBMA market, some firms become an acquirer if they turn out to be the most efficient firm in utilizing local knowledge capital in a targeted country. Some firms end up with a low level of productivity and a low level of efficiency to utilize knowledge capital of firms in other countries, and become a target in the CBMA market. Overall, local firms, and exporter firms, GFI firms, and CBMA firms all compete in the output market to offer the lowest output price.

Using this model, we investigate the contribution of the CBMA channel to the total gains from openness. Ramondo and Rodriguez-Clare (2013) study gains from trade and multinational production. Alviarez (2019) adds sectoral heterogeneity of multinational production, and finds that the gains from openness can be even larger. However, in these models, GFI and CBMAs are not explicitly distinguished. We show that the increase in CBMA firms can lower the gains from GFI because CBMAs tend to be a GFI substitute in serving foreign markets. However, it does not imply that CBMAs lower the gains from foreign direct investment because CBMAs are GFI independent when we measure the total gains from foreign direct investment.

The remainder of the paper is organized as follows. Section 2 describes our theoretical model with CBMA and GFI. Section 3 describes the gains from CBMA, GFI, foreign direct investment, and openness. Section 4 concludes the paper.

2 Model

We build on and extend the work of Eaton and Kortum (2002) and Ramondo and Rodríguez-Clare (2010, 2013) introducing knowledge capital owned by firms, which is tradable among firms in the CBMA market. Locations $i, l, n \in \{1, \cdots, I\}$ denote the origin, production, and destination countries, respectively. We index a continuum of final goods and intermediate goods by $u \in [0,1]$ and $v \in [0,1]$, respectively.

2.1 Productivity distributions

Firms from country i have their own knowledge capital m_i, which represents their accumulated know-how. They draw productivity z_{li} to utilize their knowledge capital in producing their goods in country l. The productivity vectors for final and intermediate goods, $\mathbf{z_i^f}(u) \equiv \{z_{1i}^f(u), \cdots, z_{Ii}^f(u)\}$ and $\mathbf{z_i^g}(v) \equiv \{z_{1i}^g(v), \cdots, z_{Ii}^g(v)\}$, respectively, are independently drawn across countries from the following multivariate Fréchet distribution with the same location parameter $T_i(> 0)$ and dispersion parameter $\theta(> 1)$:

$$F_i(z) = \exp\left[- T_i \sum_l (z_{li}^s)^{-\theta} \right], \tag{1}$$

for $s = f, g$. f and g denote final and intermediate good sectors, respectively.

Furthermore, we allow another option for firms to set up operations in foreign countries. Firms from country i draw pair-specific productivity z_{ni}^{sm} to use knowledge capital of local firms m_n in country n and can participate in the CBMA market to purchase it. We assume that the efficiency of CBMA firms from bidder country i is randomly drawn from the multivariate Fréchet distribution:

$$F_i^m(z) = \exp\left[- \sum_l T_{li}^m (z_{li}^{sm})^{-\vartheta} \right], \tag{2}$$

where $T_{li}^m (> 0)$ governs pair-specific efficiency of CBMA firms. z_{li}^{sm} captures the pair-wise synergy effect originating from the CBMAs between bidder firms in country i and target firms in country n. $\vartheta (> 1)$ controls the dispersion of idiosyncratic efficiency in CBMA knowhow.

2.2 CBMA versus GFI

The production function of final good u in country n is

$$q_n^f(u) = \mathfrak{z}_n^f(u) L_n^f(u)^{\alpha} Q_n^f(u)^{1-\alpha}, \tag{3}$$

where $\mathfrak{z}_n^f(u)$, $L_n^f(u)$, and $Q_n^f(u)$ denote productivity, the quantity of labor, and the composite intermediate good, respectively.[1] $\mathfrak{z}_n^f(u)$ depends on two types of technology:

$$\mathfrak{z}_n^f = \begin{cases} z_{ni}^{fm}(u) \text{ for CBMA firms} \\ z_{ni}^f(u) \text{ for other firms} \end{cases}. \tag{4}$$

As in the existing literature, if firms from country i draw a high productivity draw from the distribution (1), they tend to be GFI firms using their own knowledge capital $m_i^f(u)$ operating in country n. Furthermore, if firms from country i draw a relatively high pair-wise productivity from the distribution (2) in utilizing local specific capital $m_n^f(u)$, they can purchase it from local firms in country n to become an acquirer in the CBMA market.

If local firms draw high productivity from the distribution (1), they do not sell their knowledge capital even when they draw low productivity from the distribution (2). If local firms are not competitive drawing low productivity from both of the distributions

[1] Similarly, intermediate good v is assumed to be produced according to $q_n^g(v) = \mathfrak{z}_n^g(v) L_n^g(v)^{\beta} Q_n^g(v)^{1-\beta}$.

(2) and (1), they may become a target to sell their knowledge capital to other firms in the CBMA market.

There are four possible channels that firms from country i sell their goods and services to consumers in country n. First, firms located in $l = i$ can export goods to consumers in country $n \neq l$, which incurs in iceberg-type trade costs: d_{nl}. Second, firms from i can become GFI firms setting up an affiliate in $l = n$, which incurs in GFI costs: h^f_{ni}. Third, firms from i can perform CBMAs in $l = n$ incurring in CBMA costs: τ^f_{ni}. The fourth case is to pay h^g_{li} and τ^g_{ni} as well as d_{nl} and where firms from i produce in $l \neq i$ and sell good to $n \neq i, l$ as in Ramondo and Rodríguez-Clare (2013).

2.3 Equilibrium

We drop the index u and v and label final goods and intermediate goods by $Z^f \equiv (z^f_1, z^{fm}_1, \cdots, z^f_I, z^{fm}_I)$ and $Z^g \equiv (z^g_1, z^{gm}_1, \cdots, z^g_I, z^{gm}_I)$, respectively.

Letting $c^f_n \equiv w^\alpha_n (P^g_n)^{1-\alpha}$ and $c^g_n \equiv w^\beta_n (P^g_n)^{1-\beta}$ denote the unit cost of the input in country n, where w_n and P^g_n are the wage and the price of the composite intermediate good, respectively, the price of good is given by $p^s_n = \frac{c^s_n}{\mathfrak{z}^s_n}$ for $s = f, g$. If multi-regional firms choose the option of GFI, they have to pay additional costs h^s_{ni}, which implies $c^s_{ni} = c^s_n h^s_{ni}$. If they succeed in cross-border M&As, $c^s_{ni} = c^s_n \tau^s_{ni}$.

In a competitive equilibrium the prices of final and intermediate goods in country n, respectively, become

$$p^f_n(Z^f) = \min_i \left\{ \frac{c^f_n \tau^f_{ni}}{z^{fm}_{ni}}, \frac{c^f_n h^f_{ni}}{z^f_{ni}} \right\} \tag{5}$$

$$p^g_n(Z^g) = \min_{i,l} \left\{ \frac{c^g_l \tau^g_{li} d_{nl}}{z^{gm}_{li}}, \frac{c^g_l h^g_{li} d_{nl}}{z^g_{li}} \right\}. \tag{6}$$

Letting $\Phi^{fm}_{ni} \equiv T^m_{ni} [c^f_n (\tau^f_{ni})^{\vartheta/\theta}]^{-\theta}$, $\Phi^{fh}_{ni} \equiv T_i (c^f_n h^f_{ni})^{-\theta}$, and $\Phi^f_n \equiv \sum_i \Phi^{fm}_{ni} + \sum_i \Phi^{fh}_{ni}$, the shares of CBMA and GFI expenditures by country n on final goods produced with pair-specific technology and country i's own technology, respectively, are

$$\phi_{ni}^{fm} = \frac{\Phi_{ni}^{fm}}{\Phi_n^f} \quad \text{and} \quad \phi_{ni}^{fh} = \frac{\Phi_{ni}^{fh}}{\Phi_n^f}. \tag{7}$$

The value of CBMA and GFI firms in final goods by country i operating in country n and the price index in n, respectively, can be expressed as

$$Y_{ni}^{fm} = \phi_{ni}^{fm} w_n L_n \tag{8}$$

$$Y_{ni}^{fh} = \phi_{ni}^{fh} w_n L_n \tag{9}$$

$$P_n^f = \gamma(\Phi_n^f)^{-1/\theta}, \tag{10}$$

where γ is positive constant.

Similarly, letting $\Phi_{ni}^{gm} \equiv \sum_l T_{li}^m [c_l^g (\tau_{li}^g)^{\vartheta/\theta} d_{nl}]^{-\theta}$, $\Phi_{ni}^{gh} \equiv T_i \sum_l (c_l^g h_{li}^g d_{nl})^{-\theta}$, and $\Phi_n^g \equiv \sum_i \Phi_{ni}^{gm} + \sum_i \Phi_{ni}^{gh}$, the expenditure shares of CBMAs and GFIs by country n on intermediate goods produced with pair-specific technology and country i's own technology, respectively, are

$$\phi_{ni}^{gm} = \frac{\Phi_{ni}^{gm}}{\Phi_n^g} \quad \text{and} \quad \phi_{ni}^{gh} = \frac{\Phi_{ni}^{gh}}{\Phi_n^g}. \tag{11}$$

The price index in country n for intermediate goods is given by

$$P_n^g = \gamma(\Phi_n^g)^{-1/\theta}. \tag{12}$$

Total imports by country n from l are given by the sum of intermediate goods produced in country $l \neq n$ with technologies from any other country:

$$X_{nl} = \frac{\sum_i T_{li}^m (\tau_{li}^g)^{-\vartheta} (c_l^g d_{nl})^{-\theta} + \sum_i T_i (h_{li}^g)^{-\theta} (c_l^g d_{nl})^{-\theta}}{\Phi_n^g} \eta w_n L_n \tag{13}$$

$$= \frac{\widetilde{T}_l^m (c_l^g d_{nl})^{-\theta} + \widetilde{T}_l (c_l^g d_{nl})^{-\theta}}{\Phi_n^g} \eta w_n L_n \tag{14}$$

$$= \frac{\widetilde{\mathbf{T}}_l (c_l^g d_{nl})^{-\theta}}{(P_n^g/\gamma)^{-\theta}} \eta w_n L_n, \tag{15}$$

where $\eta \equiv (1-\alpha)/\beta$, $\widetilde{T}_l^m \equiv \sum_i T_{li}^m(\tau_{li}^g)^{-\vartheta}$, $\widetilde{T}_l \equiv \sum_i T_i(h_{li}^g)^{-\theta}$, and $\widetilde{\mathbf{T}}_l \equiv \widetilde{T}_l^m + \widetilde{T}_l$, respectively. $\widetilde{T}_l^m$ represents the set of technologies of country l that equals the local productivity plus the productivity of CBMA firms operating in country l. CBMA barrier τ_{li}^g limits CBMAs discounting the technology of CBMAs in the host country l. $\widetilde{T}_l$ indicates the set of available technologies in country l with local or foreign GFI firms, which is discounted by the GFI barriers h_{li}^g. Following Alviarez (2019), we call $\widetilde{T}_l^m$ and $\widetilde{T}_l$ effective technology to distinguish them from the local technology T_l.

3 Gains

In this section, we compute the gains from trade (GT), the gains from GFI ($GGFI$), the gains from CBMA ($GCBMA$), the gains from foreign direct investment ($GFDI$), and gains from openness (GO), for each country. In autarky, which attains when trade, CBMA, and GFI costs are infinite ($d_{nl}, \tau_{li}^s, h_{li}^s \to \infty$ for all $n \neq l, l \neq i$, and $s = f, g$), the equilibrium real wage becomes

$$\lim_{h,\tau,d\to\infty} \frac{w_n}{P_n^f} = \widetilde{\gamma}\mathbf{T}_n^{\frac{(1+\eta)}{\theta}}. \tag{16}$$

Gains are measured by the proportional change in country n's real wage, w_n/P_n^f, as we move from the above-mentioned counterfactual equilibrium characterized by above-mentioned isolation to the actual equilibrium:[2] For example, $GCBMA$ for country n can be expressed by the proportional change in country n's real wage as we move from the counterfactual equilibrium with trade and GFI but no CBMA to the actual equilibrium. Similarly, $GGFI$ is given by the proportional change in country n's real wage as we move from the counterfactual equilibrium with trade and CBMA but no GFI to the actual equilibrium.

Lemma 1. *The gains from CBMAs and GFI, respectively, can be expressed as CBMA*

[2] $GT_n \equiv \frac{\frac{w_n}{P_n^f}}{\lim_{d\to\infty}\frac{w_n}{P_n^f}} = \left(\frac{X_{nn}}{\sum_j X_{nj}}\right)^{-\frac{\eta}{\theta}}$ as in the literature.

and GFI shares as follows:

$$GCBMA_n \equiv \frac{\frac{w_n}{P_n^f}}{\lim_{\tau\to\infty}\frac{w_n}{P_n^f}} = \left(\frac{Y_{nn}^{fm}+\sum_j Y_{nj}^{fh}}{\sum_j Y_{nj}^f}\right)^{-\frac{1}{\theta}} \times \left(\frac{Y_{nn}^{gm}+\sum_j Y_{nj}^{gh}}{\sum_j Y_{nj}^g}\right)^{-\frac{\eta}{\theta}} \tag{17}$$

$$GGFI_n \equiv \frac{\frac{w_n}{P_n^f}}{\lim_{h\to\infty}\frac{w_n}{P_n^f}} = \left(\frac{\sum_j Y_{nj}^{fm}+Y_{nn}^{fh}}{\sum_j Y_{nj}^f}\right)^{-\frac{1}{\theta}} \times \left(\frac{\sum_j Y_{nj}^{gm}+Y_{nn}^{gh}}{\sum_j Y_{nj}^g}\right)^{-\frac{\eta}{\theta}}. \tag{18}$$

Therefore, it shows that CBMAs tend to be a GFI substitute because CBMA and GFI are alternative ways to do business in a particular market. If GFI share, $\frac{\sum_j Y_{nj}^{fh}}{\sum_j Y_{nj}^f}$ and $\frac{\sum_j Y_{nj}^{gh}}{\sum_j Y_{nj}^g}$, increases, the gains from CBMA decreases. If CBMA share, $\frac{\sum_j Y_{nj}^{fm}}{\sum_j Y_{nj}^f}$ and $\frac{\sum_j Y_{nj}^{gm}}{\sum_j Y_{nj}^g}$, increases, the gains from GFI decreases.

Proposition 1. *CBMA is GFI independent when we measure the total gains from FDI and openness. $GFDI$ and GO become, respectively,*

$$GFDI_n \equiv \frac{\frac{w_n}{P_n^f}}{\lim_{\tau,h\to\infty}\frac{w_n}{P_n^f}} = \left(\frac{Y_{nn}^{fm}+Y_{nn}^{fh}}{\sum_j Y_{nj}^f}\right)^{-\frac{1}{\theta}} \times \left(\frac{Y_{nn}^{gm}+Y_{nn}^{gh}}{\sum_j Y_{nj}^g}\right)^{-\frac{\eta}{\theta}} \tag{19}$$

$$GO_n \equiv \frac{\frac{w_n}{P_n^f}}{\lim_{h,\tau,d\to\infty}\frac{w_n}{P_n^f}} = GT_n \times GFDI_n. \tag{20}$$

Therefore, CBMA share does not necessarily lower the gains from foreign direct investment because CBMAs are one component of foreign direct investment. Rather, CBMAs can have a favorable effect on the gains from foreign direct investment and the gains from openness because they offer an option for firms to serve in the foreign market even in the case that trade and GFI costs are too deterrent.

l, the relative sales are more likely to increase, the smaller GFI firms' productivity dispersion parameter θ, and the higher CBMA firms' productivity dispersion parameter ϑ.

4 Conclusion

There are many possible channels that increase the gains from foreign direct investment and openness. In recent decades, the cross-border M&A play an important role for countries benefiting from their interaction with the rest of the world. In this paper, we theoretically examine how productive firms in the cross-border M&A market acquire local knowledge and start local business to offer the lowest output price. We show that the cross-border M&As are a promising tool to increase the gains from foreign direct investment and openness.

Appendix A: Cross-border M&A market

We introduce a CBMA market where local firms producing final good $u \in [0,1]$ in country n may sell their local knowledge capital $m_{nn}(u)$ to other firms from country i. We assume these cross-border CBMA firms have the following efficiency: $m_{ni}(u)' = \psi_{ni}(u)m_{nn}(u)$. $\psi_{ni}(u)$ captures the pair-wise synergy effect originating from the CBMAs between bidder and target firms.

If bidder firms offer higher bid price $p_n^m(u)$ than $\psi_{nn}(u)$, local firms want to be the target and host them. Hence, the profit function of M&A bidder firms from country i to operate in country n becomes

$$\Pi_{ni}(u) = \psi_{ni}(u)m_{nn}(u) - p_n^m(u)m_{nn}(u),$$

where $p_n^m(u)$ denotes the acquisition price of local knowledge capital for good u. Hence, as far as $\psi_{ni}(u) > p_n^m(u)$, firms from country i want to participate in the CBMA market, local firms choose to become a target if their efficiency $\psi_{nn}(u)$ is lower than $p_n^m(u)$.

The efficiency in utilizing local knowledge capital m_{nn} depends on two terms: pair-specific efficiency z_{ni}^m and pair-wise cost $\tau_{ni}(u)$. $\tau_{ni}(u)$ captures bilateral M&A costs that bidder firms from country i face: $\tau_{ni} > \tau_{nn} = 1$.

We assume perfect competition in the CBMA market. The maximum bidding price of the CBMAs in country n from country i can be expressed as follows:

$$p_{ni}^m(u) = \psi_{ni}(u) = \frac{z_{ni}^m(u)}{\tau_{ni}}.$$

The realized CBMA price p_n^m in country n that CBMA firms would pay becomes the highest one across countries i:

$$p_n^m(u) = \max\{p_{ni}^m(u); i = 1, \cdots, I\}.$$

The probability that country n hosts firms from country i becomes

$$\pi_{ni}^m = \frac{T_{ni}^m \tau_{ni}^{-\vartheta}}{\sum_j T_{nj}^m \tau_{nj}^{-\vartheta}}.$$

Appendix B: Gravity equation in the M&A market

Letting $G_{ni}^m(\psi) \equiv \Pr[\psi_{ni} \geq \psi]$, we obtain

$$\begin{aligned} G_{ni}^m(\psi) &= \Pr\left[z_{ni}^m \geq \tau_{ni}\psi \text{ for all } i\right] = 1 - F_{ni}^m\left(\tau_{ni}\psi\right) \\ &= 1 - \exp^{-[T_{ni}^m \tau_{ni}^{-\vartheta}]\psi^{-\vartheta}}. \end{aligned}$$

If we denote the maximum price as $\psi_n \equiv \max\{\psi_{ni}, \cdots, \psi_{n_I}\}$ and let $G_n^m(\psi) \equiv \Pr[\psi_n \geq \psi]$ be the distribution of M&A deal values in country n, we obtain

$$\begin{aligned} G_n^m(\psi) &= \Pr\left[\psi_n \geq \psi\right] = 1 - \Pi_i^m \Pr\left[\psi_{ni} \leq \psi\right] \\ &= 1 - \Pi_i F_{ni}^m \\ &= 1 - \Pi_i \exp^{-[T_{ni}^m \tau_{ni}^{-\vartheta}]\psi^{-\vartheta}} \\ &= 1 - \exp^{\Phi_n^m \psi^{-\vartheta}} \end{aligned}$$

where $\Phi_n^m \equiv \sum_i T_{ni}^m \tau_{ni}^{-\vartheta}$.

For a certain value of $\psi_{ni} = \psi$, the probability that country i is the highest bidder to country n becomes

$$\begin{aligned} \Pr[\psi_{ni} \geq \max_{r \neq i} \psi_{nr}] &= \Pi_{r \neq i} \Pr[\psi_{nr} \leq \psi] = \Pi_{r \neq i} F_{nr}^m \\ &= \exp^{-[(\Phi_n^m)^{-i} \psi^{-\vartheta}]}, \end{aligned}$$

where $(\Phi_n^m)^{-i} \equiv \sum_{r \neq i} T_{nr}^m \tau_{nr}^{-\vartheta}$.

Hence, if we integrate over this probability for all values of ψ multiplied by the density

$dG_{ni}^m(\psi)$, we obtain the probability that country n hosts firms from country i becomes

$$\begin{aligned}
\pi_{ni}^m &\equiv \int_0^\infty e^{-(\Phi_n^m)^{-i}\psi^{-\vartheta}} T_{ni}^m \tau_{ni}^{-\vartheta}(-\vartheta)\psi^{-\vartheta-1} e^{-[T_{ni}^m \tau_{ni}^{-\vartheta}]\psi^{-\vartheta}} d\psi \\
&= \left(\frac{T_{ni}^m \tau_{ni}^{-\vartheta}}{\Phi_n^m}\right) \int_0^\infty (-\vartheta)\Phi_n^m \psi^{-\vartheta-1} e^{-\Phi_n^m \psi^{-\vartheta}} d\psi \\
&= \left(\frac{T_{ni}^m \tau_{ni}^{-\vartheta}}{\Phi_n^m}\right) \int_0^\infty dG_n^m(\psi) d\psi \\
&= \frac{T_{ni}^m \tau_{ni}^{-\vartheta}}{\Phi_n^m}
\end{aligned}$$

Therefore, we obtain the following gravity equation

$$m_{ni} = \frac{T_{ni}^m \tau_{ni}^{-\vartheta}}{\Phi_n^m} m_n.$$

Appendix C: Gains from openness

The normalized import share in country n becomes

$$\frac{X_{nl}/\eta w_n L_n}{X_{ll}/\eta w_l L_l} = \left(\frac{P_l^g d_{nl}}{P_n^g}\right)^{-\theta}.$$

Summing up, we obtain

$$\eta w_l L_l \sum_n \frac{X_{nl}}{X_{ll}} = \sum_n \left(\frac{P_l^g d_{nl}}{P_n^g}\right)^{-\theta} \eta w_n L_n \equiv \Psi_l.$$

The sum of the values of CBMA and GFI firms in intermediates by country i in country l to serve country n is $(\phi_{ni}^{gm}\pi_{nli}^{gm} + \phi_{ni}^{gh}\pi_{nli}^{gh})\eta w_n L_n$. This implies

$$\begin{aligned}
Y_{li}^g &= \sum_n [\phi_{ni}^{gm}\pi_{nli}^{gm} + \phi_{ni}^{gh}\pi_{nli}^{gh}]\eta w_n L_n \\
&= \frac{T_{li}^m(\tau_{li}^g)^{-\vartheta}(c_l^g)^{-\theta} + T_i(h_{li}^g)^{-\theta}(c_l^g)^{-\theta}}{\Phi_n^g}\Psi_l \\
&= \widetilde{\mathbf{T}}_{li}\left(\frac{c_l^g}{P_l^g/\gamma}\right)^{-\theta}\Psi_l,
\end{aligned}$$

where $\widetilde{\mathbf{T}}_{li} \equiv T^m_{li}(\tau^g_{li})^{-\vartheta} + T_i(h^g_{li})^{-\theta}$. Hence, we obtain

$$Y^g_{ll} = \gamma^{-\theta}\mathbf{T}_l\left(\frac{c^g_l}{P^g_l}\right)^{-\theta}\Psi_l.$$

where $\mathbf{T}_l \equiv T^m_{ll} + T_l$. Using $c^g_l = Bw^\beta_l(P^g_l)^{1-\beta}$, the real wage can be expressed as

$$\frac{w_l}{P^g_l} = (\gamma B)^{-1/\beta}\mathbf{T}_l^{1/\beta\theta}\left(\frac{Y^g_{ll}}{\Psi_l}\right)^{-1/\beta\theta}.$$

The domestic share by country n on final goods becomes

$$\frac{Y^f_{nn}}{\sum_j Y^f_{nj}} = \frac{\mathbf{T}_n(c^f_n)^{-\theta}}{\sum_i \mathbf{T}_i(c^f_{ni})^{-\theta}} = \frac{\mathbf{T}_n}{\widetilde{\mathbf{T}}^f_n},$$

where $\widetilde{\mathbf{T}}^f_n \equiv \sum_i T^m_{ni}(\tau^f_{ni})^{-\vartheta} + \sum_i T_i(h^f_{ni})^{-\theta}$. Using $c^f_n = Aw^\alpha_n(P^g_n)^{1-\alpha}$, the price index for final goods becomes

$$\begin{aligned} P^f_n &= \gamma(\widetilde{\mathbf{T}}^f_n)^{-\frac{1}{\theta}}c^f_n \\ &= \gamma(\widetilde{\mathbf{T}}^f_n)^{-\frac{1}{\theta}}Aw^\alpha_n(P^g_n)^{1-\alpha}. \end{aligned}$$

Rearranging this equation with respect to the real wage, we obtain

$$\begin{aligned} \frac{w_n}{P^f_n} &= (\gamma A)^{-1}(\widetilde{\mathbf{T}}^f_n)^{\frac{1}{\theta}}\left(\frac{w_n}{P^g_n}\right)^{1-\alpha} \\ &= \widetilde{\gamma}\mathbf{T}_n^{\frac{\eta}{\theta}}(\widetilde{\mathbf{T}}^f_n)^{\frac{1}{\theta}}\left(\frac{Y^g_{nn}}{\Psi_n}\right)^{-\frac{\eta}{\theta}} \\ &= \widetilde{\gamma}\mathbf{T}_n^{\frac{(1+\eta)}{\theta}}\left(\frac{\mathbf{T}_n}{\widetilde{\mathbf{T}}^f_n}\right)^{-\frac{1}{\theta}}\left(\frac{Y^g_{nn}}{\eta w_n L_n}\frac{X_{nn}}{\sum_j X_{nj}}\right)^{-\frac{\eta}{\theta}}, \end{aligned}$$

where $\widetilde{\gamma} \equiv (\gamma A)^{-1}(\gamma B)^{-\eta}$. When $\eta w_n L_n = \sum_i Y^g_{ni}$, the real wage is then

$$\frac{w_n}{P^f_n} = \widetilde{\gamma}\mathbf{T}_n^{\frac{1+\eta}{\theta}}\left(\frac{Y^f_{nn}}{\sum_j Y^f_{nj}}\right)^{-\frac{1}{\theta}} \times \left(\frac{Y^g_{nn}}{\sum_j Y^g_{nj}}\right)^{-\frac{\eta}{\theta}} \times \left(\frac{X_{nn}}{\sum_j X_{nj}}\right)^{-\frac{\eta}{\theta}}.$$

In autarky, the equilibrium real wage becomes

$$\lim_{h,\tau,d\to\infty} \frac{w_n}{P_n^f} = \widetilde{\gamma}\mathbf{T}_n^{\frac{(1+\eta)}{\theta}}.$$

In each counter-factual equilibrium, the real wage becomes

$$\lim_{h\to\infty} \frac{w_n}{P_n^f} = \widetilde{\gamma}\mathbf{T}_n^{\frac{(1+\eta)}{\theta}} \left(\frac{Y_{nn}^f}{\sum_j Y_{nj}^{fm} + Y_{nn}^{fh}}\right)^{-\frac{1}{\theta}} \times \left(\frac{Y_{nn}^g}{\sum_j Y_{nj}^{gm} + Y_{nn}^{gh}}\right)^{-\frac{\eta}{\theta}} \times \left(\frac{X_{nn}}{\sum_j X_{nj}}\right)^{-\frac{\eta}{\theta}}$$
$$\lim_{\tau\to\infty} \frac{w_n}{P_n^f} = \widetilde{\gamma}\mathbf{T}_n^{\frac{(1+\eta)}{\theta}} \left(\frac{Y_{nn}^f}{\sum_j Y_{nj}^{fh} + Y_{nn}^{fm}}\right)^{-\frac{1}{\theta}} \times \left(\frac{Y_{nn}^g}{\sum_j Y_{nj}^{gh} + Y_{nn}^{gm}}\right)^{-\frac{\eta}{\theta}} \times \left(\frac{X_{nn}}{\sum_j X_{nj}}\right)^{-\frac{\eta}{\theta}}.$$

The gains from CBMA, GGFI, GFDI, and GO, respectively, become

$$GT_n \equiv \frac{\frac{w_n}{P_n^f}}{\lim_{d\to\infty} \frac{w_n}{P_n^f}} = \left(\frac{X_{nn}^h}{\sum_j X_{nj}}\right)^{-\frac{\eta}{\theta}}$$
$$GCBMA_n \equiv \frac{\frac{w_n}{P_n^f}}{\lim_{\tau\to\infty} \frac{w_n}{P_n^f}} = \left(\frac{Y_{nn}^{fm} + \sum_j Y_{nj}^{fh}}{\sum_j Y_{nj}^f}\right)^{-\frac{1}{\theta}} \times \left(\frac{Y_{nn}^{gm} + \sum_j Y_{nj}^{gh}}{\sum_j Y_{nj}^g}\right)^{-\frac{\eta}{\theta}}$$
$$GGFI_n \equiv \frac{\frac{w_n}{P_n^f}}{\lim_{h\to\infty} \frac{w_n}{P_n^f}} = \left(\frac{\sum_j Y_{nj}^{fm} + Y_{nn}^{fh}}{\sum_j Y_{nj}^f}\right)^{-\frac{1}{\theta}} \times \left(\frac{\sum_j Y_{nj}^{gm} + Y_{nn}^{gh}}{\sum_j Y_{nj}^g}\right)^{-\frac{\eta}{\theta}}$$
$$GFDI_n \equiv \frac{\frac{w_n}{P_n^f}}{\lim_{\tau,h\to\infty} \frac{w_n}{P_n^f}} = \left(\frac{Y_{nn}^{fm} + Y_{nn}^{fh}}{\sum_j Y_{nj}^f}\right)^{-\frac{1}{\theta}} \times \left(\frac{Y_{nn}^{gm} + Y_{nn}^{gh}}{\sum_j Y_{nj}^g}\right)^{-\frac{\eta}{\theta}}$$
$$GO_n \equiv \frac{\frac{w_n}{P_n^f}}{\lim_{h,\tau,d\to\infty} \frac{w_n}{P_n^f}} = GT_n \times GFDI_n.$$

Assuming $h_{li}^f = h_{li}^g$ and $\tau_{li}^f = \tau_{li}^g$ for all l and i, $\frac{Y_{nn}^f}{\sum_j Y_{nj}^f} = \frac{Y_{nn}^g}{\sum_j Y_{nj}^g}$. Hence,

$$GCBMA_n = \left(\frac{Y_{nn}^m + \sum_j Y_{nj}^h}{\sum_j Y_{nj}}\right)^{-\frac{1+\eta}{\theta}}$$
$$GGFI_n = \left(\frac{\sum_j Y_{nj}^m + Y_{nn}^h}{\sum_j Y_{nj}}\right)^{-\frac{1+\eta}{\theta}}$$
$$GFDI_n = \left(\frac{Y_{nn}^m + Y_{nn}^h}{\sum_j Y_{nj}}\right)^{-\frac{1+\eta}{\theta}}$$
$$GO_n = GT_n \times GFDI_n.$$

References

Alviarez, V., 2019. “Multinational production and comparative advantage,” *Journal of International Economics*, 119, 1–54.

Eaton, J., Kortum, S. 2002. “Technology, geography, and trade.” *Econometrica*, 70, 1741-1779.

Ramondo, N., Rodríguez-Clare, A., 2010, “ Growth, size, and openness: A quantitative approach,” *American Economic Review, Papers and Proceedings*, 100, 62-67.

Ramondo, N., Rodríguez-Clare, A., 2013, “Trade, multinational production, and the gains from openness,” *Journal of Political Economy*, 121, 273-322.