# Announcement Returns, Volume, and 

# Bidder/Target Relative Size in Takeovers 

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#### Abstract

We develop a competitive model of takeovers in which announcement returns (ARs), volume, and bidder/target relative size are endogenously determined. Because the signs of ARs identify the public information about firms, empirical observations of positive target ARs and much lower and more dispersed bidder ARs indicate: (i) firms’ tradeable is not public information, (ii) firms' non-tradeable was public information for deals with negative bidder ARs. A parameterized model shows that the relative bidder size increases (decreases) in the productivity of non-tradeable (tradeable). Transaction costs reduce volume, but increase the relative bidder size if non-tradeable is sufficiently important in production. (JEL L1, G3, D8)


Key words: Announcement Return, Takeovers, Volume.

[^0]
## 1 Introduction

A large body of empirical literature studies announcement returns in takeovers. One welldocumented fact is that target returns are robustly positive, while bidder returns exhibit more dispersion and are negative on average (Andrade et al.(2001), Fuller et al. (2002)). While researchers have investigated a long list of firm- and deal-characteristics, there still appears to be no consensus on the economic forces that drive these patterns. This is a difficult task because, quoting from Fuller et al. (2002, p.1763), "Researchers have been unable to successfully explain much of this variation, partially because the announcement of a takeover reveals information about numerous things." We present a simple analytical framework to understand the nature of the information revealed around a takeover announcement.

We follow Holmes and Schmitz (1990, 1995) and Jovanovic and Braguinsky (2004) in modeling a competitive model of takeovers in which good projects (tradeable) and good organizations (non-tradeable) are complements. Because some firms with a good organization set out with a bad project, and vice versa, takeovers serve to reallocate good tradeable from bad organizations to good organizations. Along with announcement returns for bidders and targets, the model endogenously determines which firms become targets and bidders, overall takeover volume, and the relative size of bidders and targets.

We first show that, under general conditions, target and bidder announcement returns for takeovers generated by the complementarity of assets identify the public knowledge about firms before the deal announcements. More precisely, we show that announcement returns are negative (positive) for target firms and positive (negative) for bidder firms if firms' tradeable (non-tradeable) is public knowledge. To see why, suppose that firms' tradeable is public knowledge before the takeover announcement. From investors' perspective, conditional on a known tradeable quality, firms with heterogeneous non-tradeable are pooled. Hence, announcement returns reflect the information about bidder and target firms' nontradeable. Because of the complementarity between tradeable and non-tradeable, having the same quality of tradeable and facing the same market price for it, firms with worse non-tradeable should have higher incentive to be a target (a seller of tradeable). Accordingly, the announcement of being a target firm reveals a bad information about the firm's non-tradeable, while that of being a bidder firm reveals a good information. Relative to the pre-announcement expected firm value conditional on the known quality of tradeable, a target market value drops and a bidder market value goes up, i.e., target discounts and bidder premia. A symmetric reasoning explains target premia and bidder discount if firms' non-tradeable is public knowledge.

We also show that, if firms' status quo value - a value that would obtain if firms do not
participate in takeovers - is public knowledge but tradeable and non-tradeable are not separately observed, then announcement returns are positive for both targets and bidders, given that the likelihood of takeovers is sufficiently small. When firms' status quo value is public information, firms with different combinations of tradeable and non-tradeable that generate the same status quo value are pooled. If a measure of firms that participate in takeovers is sufficiently small, then the separation from non-participating firms drives announcement returns. These announcement returns must be positive because firms participate in takeovers only if doing so increases their values relative to their status quo values.

What does this imply for the empirical observations of announcement returns? In principle, a sample of takeover deals may include all the three cases of public knowledge about firms described above, and announcement returns can be positive or negative depending on what investors know about firms before the announcements. However, we know that negative target returns are uncommon - indicating that firms' tradeable is not public knowledge. Through the lens of our model, empirically documented (i) robust target premia, (ii) much smaller bidder returns, and (iii) a large dispersion in bidder returns, jointly indicate that it is difficult to know firms' tradeable a priori, while non-tradeable may be public for some firms, in which case we should expect their large value losses if they turn out to be bidders.

In terms of understanding the dispersion of bidder returns, our model suggests that controlling for investors' prior information is critically important. At the end of Section 3, we discuss another set of evidence that supports our hypothesis - large loss deals are concentrated among serial or frequent bidders (Moeller et al. (2005)) and their returns decline from deal to deal (Renneboog and Vansteenkiste (2019)). ${ }^{1}$ In the literature, there are two approaches to this phenomenon. First, Fuller et al. (2002) argue that "Since the same bidder chooses different types of targets and methods of payment, any variation in returns must be due to the characteristics of the target and the bid". Second, bidder firms might change as they experience multiple deals over a short period of time. In particular, they might grow overconfidence (Billet and Qian (2008), Jaffe et al. (2013), El-Khativ et al. (2015)) or might be learning (Aktas et al. (2009, 2011, 2013)).

Our model offers an alternative hypothesis: declining returns might be due to investors' gradual learning about serial bidders' non-tradeable. To be clear, our message is not to dispute that changes in bidder firms can affect announcement returns. However, our model shows that changes in investors' information can be a significant - in that it can change the sign of announcement returns - contributing factor. When examining announcement returns

[^1]in takeover markets, controlling for investors' prior information about firms is critical as it can change their sign even without changes in firm- or deal-characteristics.

In Section 4, we solve a parameterized model in closed form to derive other predictions. We show that (i) takeover volume decreases in transaction costs, and (ii) a technological change that favors non-tradeable (tradeable) increases (decreases) the size of bidders relative to their matched targets. We also show that (iii) transaction costs that distort prices (e.g. M\&A advisory fees that depend on target prices) increase the relative bidder size if a production technology does not exhibit decreasing returns to non-tradeable, and (iv) costs that directly reduce post-takeover firm values (e.g. integration costs) increase the relative bidder (target) size if a production technology features higher returns to non-tradeable (tradeable) than tradeable (non-tradeable). To our knowledge, these comparative statics results are new to the literature. Taken together, the model predicts the following crossindustry patterns of takeovers: takeover volume should be smaller in industries with higher transaction costs, while the relative size of bidders should be larger in industries where nontradeable is more important in production than tradeable. Moreover, transaction costs and technology interact: the positive effect of transaction costs on the relative bidder size should be more noticeable in industries with technologies that rely more on non-tradeable.

The rest of the paper is organized as follows. In Section 2, we develop a model of takeovers. In Section 3, we derive implications for announcement returns. In Section 4, we solve a parameterized model in closed-form to investigate other testable implications. Section 5 concludes.

## 2 Model

Each firm is endowed with a tradeable factor $A$, and a non-tradeable factor $X$. We interpret $A$ as tangible asset that can be traded and $X$ as intangible asset that is difficult to trade, such as management quality and organizational capital. A tradeable factor is indivisible, and at most one tradeable factor can be managed by each firm. This assumption is meant to capture the indivisible nature of takeovers. ${ }^{2}$ Finally, we assume that being a target and a bidder simultaneously is too costly. This is also consistent with observations - firms are typically active on only one side of takeover markets. There is a continuum of firms with different $A$ and $X$. We make the following assumption for the distribution of $A$ and $X$.

## Assumption 1 (Distribution)

[^2](a) Tradeable $A$ is distributed over $\left[0, A_{\max }\right]$ with a cumulative distribution function $\Phi_{A}$ and a continuous and strictly positive density $\phi_{A}$.
(b) Non-tradeable $X$ is distributed over $\left[0, X_{\max }\right.$ ] with a cumulative distribution function $\Phi_{X}$ and a continuous and strictly positive density $\phi_{X}$.
(c) $A$ and $X$ are independent.

Assumption 1(a,b) are technical assumptions. Assumption 1(c) implies that some firms have high $A$ and low $X$, and vice versa. ${ }^{3}$ Takeovers reallocate high $A$ from firms with low $X$ to firms with high $X$. Using $(A, X) \in\left[0, A_{\max }\right] \times\left[0, X_{\max }\right]$, firms can produce $F(A, X)$, or participate in takeovers. We make the following assumption for the production function $F$. We let $F_{A}$ and $F_{X}$ denote partial derivatives.

## Assumption 2 (Technology)

For any $(A, X) \in\left[0, A_{\max }\right] \times\left[0, X_{\max }\right], F(A, X)$ satisfies the following properties:
(a) $F_{A}>0, F_{X}>0$, and $F_{A X}>0$.
(b) $F_{A}(A, 0)=0$ and $F_{A}(A, \infty)=\infty$.
(c) $F(0, X)=0, F(A, 0)=0$, and $F(A, \infty)=\infty$.

Assumption 2(a) states that both factors are useful and they are complementary. Combined with Assumption 1(c), this is the source of gains from takeovers in our model. Assumption 2(b) ensures the interiority of takeover choice. Assumption 2(c) ensures the existence of marginal firms on both sides of the market.

A firm with $(A, X)$ faces three options. First, it can produce with endowed $(A, X)$, which results in firm value $F(A, X)$. Second, it can sell its tradeable $A$ at a market price $P(A)$. If $A$ is private information to the firm, it must disclose it before selling it. ${ }^{4}$ Third, it can abandon its initial tradeable $A$ to buy a new tradeable $a \in\left[0, A_{\max }\right]$ at a market price $P(a)$. A market price $P(A)$ is an equilibrium object, which firms take as given. In this setup, a decision problem facing a firm with $(A, X)$ is

$$
\begin{equation*}
\max \{F(A, X), \underbrace{P(A)}_{\text {target }}, \underbrace{\max _{a \in\left[0, A_{\max }\right]}\{F(a, X)-P(a)\}}_{\text {bidder }}\} . \tag{1}
\end{equation*}
$$

[^3]In formulating (1), we implicitly assumed that the firm cannot sell an endowed tradeable $A$ and buy a new tradeable (i.e., cannot act as both a bidder and a target). If allowed, this would achieve the payoff $P(A)+\max _{a \in\left[0, A_{\max }\right]}\{F(a, X)-P(a)\}$. Such a buy-and-sell strategy might be reasonable if $A$ is a specific asset. Yet, in a context of takeovers, we rarely observe a firm simultaneously selling itself and buying another firm. Accordingly, we infer that a cost of doing both must be very high. We make this intuition formal in Lemma 5, and show that if a cost of doing both is at least $F(A, X)$, then no firm does both.

Transaction costs. While our results on announcement returns can be derived in a model without transaction costs, we incorporate them because they are important in takeover markets and yield testable implications. We assume that target firms selling their $A$ receive $\left(1-\tau_{T}\right) P(A)$, while bidder firms buying $A$ pay $\left(1+\tau_{B}\right) P(A)$, where $\tau_{T} \in[0,1]$ and $\tau_{B} \in[0, \infty)$ are constants. For concreteness, we interpret $\left(\tau_{T}, \tau_{B}\right)$ as fees charged by M\&A advisers (e.g. investment banks). Bidder firms buying $A$ may additionally incur integration costs $c F(A, X)$, where $c \in[0,1]$ is a constant. With these transaction costs $\left(\tau_{T}, \tau_{B}, c\right)$, a decision problem facing a firm with $(A, X)$ becomes

$$
\begin{gather*}
\max \left\{F(A, X), \quad \Pi_{T}(A, X), \quad \Pi_{B}(A, X)\right\}, \text { where }  \tag{2}\\
\Pi_{T}(A) \equiv\left(1-\tau_{T}\right) P(A), \\
\Pi_{B}(X) \equiv \max _{a \in\left[0, A_{\max }\right]}\left\{(1-c) F(a, X)-\left(1+\tau_{B}\right) P(a)\right\} . \tag{3}
\end{gather*}
$$

In the following, we work with (2), which subsumes (1) as a special case $\tau_{T}=\tau_{B}=c=0$.

### 2.1 Bidder firms

We first study bidder firms' action and participation decision. Given an increasing and twice-differentiabile $P(A)$, the first and second order conditions for (3) are

$$
\begin{align*}
& (1-c) F_{A}(A, X)=\left(1+\tau_{B}\right) P^{\prime}(A)>0  \tag{4}\\
& (1-c) F_{A A}(A, X)<\left(1+\tau_{B}\right) P^{\prime \prime}(A) \tag{5}
\end{align*}
$$

Assumption 2(b) ensures that (4) has a unique solution for $X$. We denote this solution by $x(A)$ and call it a matching function. By the implicit function theorem applied to (4),
the matching function $x(A)$ satisfies

$$
\begin{aligned}
x^{\prime}(A) & =\frac{F_{A}(A, x(A))}{F_{A X}(A, x(A))}\left(\frac{P^{\prime \prime}(A)}{P^{\prime}(A)}-\frac{F_{A A}(A, x(A))}{F_{A}(A, x(A))}\right) \\
& =\frac{1}{F_{A X}(A, x(A))}\left(\frac{1+\tau_{B}}{1-c} P^{\prime \prime}(A)-F_{A A}(A, x(A))\right)>0,
\end{aligned}
$$

where the second equality is by (4) and the last inequality is by (5). Hence, the inverse matching function $x^{-1}(X) \equiv a(X)$ is well defined and increasing. Using $a(X)$, a value of bidder firms endowed with $(A, X)$ is

$$
\begin{equation*}
\Pi_{B}(X)=(1-c) F(a(X), X)-\left(1+\tau_{B}\right) P(a(X)) . \tag{6}
\end{equation*}
$$

By the optimality of $a(X), \Pi_{B}^{\prime}(X)=(1-c) F_{X}(a(X), X)>0$. Also, $\Pi_{B}(X)>0$ if and only if $\frac{P(a(X))}{F(a(X), X)}<\frac{1-c}{1+\tau_{B}}$. By using (4) to replace $\frac{1-c}{1+\tau_{B}}$ with $\frac{P^{\prime}(a(X))}{F_{A}(a(X), X)}$ and rearranging, $\Pi_{B}(X)>0$ if and only if $\frac{F_{A}(a(X), X)}{F(a(X), X)}<\frac{P^{\prime}(a(X))}{P(a(X))}$. Lemma 1 summarizes the results so far.

## Lemma 1 (Matching)

The interior solution of bidders' problem (3) characterized by (4) and (5) implies:
(a) A matching function $x(A) \in(0, \infty)$ uniquely defined by (4) satisfies

$$
\begin{equation*}
x^{\prime}(A)=\frac{F_{A}(A, x(A))}{F_{A X}(A, x(A))}\left(\frac{P^{\prime \prime}(A)}{P^{\prime}(A)}-\frac{F_{A A}(A, x(A))}{F_{A}(A, x(A))}\right)>0 . \tag{7}
\end{equation*}
$$

(b) $\Pi_{B}(X)$ given in (6) satisfies $\Pi_{B}^{\prime}(X)>0$. Also, $\Pi_{B}(X)>0$ if and only if

$$
\begin{equation*}
\frac{F_{A}(a(X), X)}{F(a(X), X)}<\frac{P^{\prime}(a(X))}{P(a(X))} \tag{8}
\end{equation*}
$$

Lemma 1(a) shows that at the interior optimum of bidders' problem, the matching is positive assortative (i.e., $x^{\prime}(A)>0$ ). Lemma $\mathbf{1}(\mathbf{b})$ shows that in any equilibrium with a positive bidder payoff $\Pi_{B}(X)$, the price elasticity must be greater than the production elasticity with respect to a tradeable.

Participation. A participation constraint for bidder firms is

$$
F(A, X) \leq \Pi_{B}(X)=(1-c) F(a(X), X)-\left(1+\tau_{B}\right) P(a(X))
$$

Assumption 2(c) implies that there is a unique $A_{B}(X) \in(0, a(X))$ defined by

$$
\begin{equation*}
F\left(A_{B}(X), X\right)=(1-c) F(a(X), X)-\left(1+\tau_{B}\right) P(a(X)) \tag{9}
\end{equation*}
$$

such that $F(A, X) \leq \Pi_{B}(X)$ if and only if $A \leq A_{B}(X)$. Firms with a non-tradeable $X$ are indifferent between $F\left(A_{B}(X), X\right)$ and $\Pi_{B}(X)$. We later verify that $A \leq A_{B}(X)$ implies another participation constraint $\Pi_{T}(A)<\Pi_{B}(X)$. To characterize $A_{B}(X)$, we apply the implicit function theorem to (9) and use (4):

$$
A_{B}^{\prime}(X)=-\frac{F_{X}\left(A_{B}(X), X\right)-(1-c) F_{X}(a(X), X)}{F_{A}\left(A_{B}(X), X\right)}
$$

This is positive if and only if $F_{X}\left(A_{B}(X), X\right)<(1-c) F_{X}(a(X), X)$. Using (9), an algebra shows that $A_{B}^{\prime}(X)>0$ if and only if

$$
\begin{equation*}
\frac{\frac{F_{X}\left(A_{B}(X), X\right)}{F\left(A_{B}(X), X\right)}}{\frac{F_{X}(a(X), X)}{F(a(X), X)}}<1+\left(1+\tau_{B}\right) \frac{P(a(X))}{F\left(A_{B}(X), X\right)} . \tag{10}
\end{equation*}
$$

Because $A_{B}(X)<a(X),(10)$ is satisfied if $\frac{F_{X}(A, X)}{F(A, X)}$ is weakly increasing in $A$. These results are summarized in Lemma 2.

## Lemma 2 (Bidder firms)

(a) $F(A, X) \leq \Pi_{B}(X)$ if and only if $A \leq A_{B}(X)$, where $A_{B}(X) \in(0, a(X))$ is uniquely defined by (9).
(b) $A_{B}^{\prime}(X)>0$ if $\frac{F_{X}(A, X)}{F(A, X)}$ is weakly increasing in $A$.

The condition on $F$ in Lemma 2(b) states that an increase in $A$ weakly increases the elasticity of $F$ with respect to $X$. This is stronger than $F_{A X}>0$. A simple example that satisfies this condition is a multiplicably separable production function $F(A, X)=g(A) h(X)$. For $A_{B}(X)$ to be increasing, however, this condition is not necessary. A necessary and sufficient condition (10) allows $\frac{F_{X}(A, X)}{F(A, X)}$ to be decreasing in $A$, but not too much. For our characterization of announcement returns, what matters is that $A_{B}(X)$ is increasing. So, the sufficient condition being not necessary is a good thing; our characterization of announcement returns is valid for more general $F$, beyond the multiplicably separable case.

### 2.2 Target firms

Next, we characterize target firms. A participation constraint for target firms with $(A, X)$ is

$$
F(A, X) \leq \Pi_{T}(A)=\left(1-\tau_{T}\right) P(A)
$$

By Assumption 2(c), there exists a unique threshold $X_{T}(A)>0$ defined by

$$
\begin{equation*}
F\left(A, X_{T}(A)\right)=\left(1-\tau_{T}\right) P(A) \tag{11}
\end{equation*}
$$

such that $F(A, X) \leq \Pi_{T}(A)$ if and only if $X \leq X_{T}(A)$. Firms with a tradeable $A$ are indifferent between $F\left(A, X_{T}(A)\right)$ and $\Pi_{T}(A)$. We later verify that $X \leq X_{T}(A)$ implies another participation constraint $\Pi_{B}(X)<\Pi_{T}(A)$. To characterize $X_{T}(A)$, we rewrite (9) using $X=x(A) \Leftrightarrow A=a(X)$ :

$$
P(A)=\frac{1-c}{1+\tau_{B}} F(A, x(A))-\frac{1}{1+\tau_{B}} F\left(A_{B}(x(A)), x(A)\right) .
$$

Substituting this into (11),

$$
\begin{aligned}
F\left(A, X_{T}(A)\right) & =\left(1-\tau_{T}\right)\left\{\frac{1-c}{1+\tau_{B}} F(A, x(A))-\frac{1}{1+\tau_{B}} F\left(A_{B}(x(A)), x(A)\right)\right\} \\
& <F(A, x(A))
\end{aligned}
$$

This implies $X_{T}(A)<x(A)$. By applying the implicit function theorem to (11),

$$
X_{T}^{\prime}(A)=-\frac{F_{A}\left(A, X_{T}(A)\right)-\frac{F\left(A, X_{T}(A)\right)}{P(A)} P^{\prime}(A)}{F_{X}\left(A, X_{T}(A)\right)}
$$

Therefore, $X_{T}^{\prime}(A)>0$ if and only if

$$
\begin{equation*}
\frac{F_{A}\left(A, X_{T}(A)\right)}{F\left(A, X_{T}(A)\right)}<\frac{P^{\prime}(A)}{P(A)} \tag{12}
\end{equation*}
$$

Recall from (8) that for a tradeable $A$ to attract bidder firms, $\frac{F_{A}(A, x(A))}{F(A, x(A))}<\frac{P^{\prime}(A)}{P(A)}$ is necessary. Because $X_{T}(A)<x(A)$, if $\frac{F_{A}(A, X)}{F(A, X)}$ is weakly increasing in $X$, then (8) implies (12). The results on target firms' participation are summarized in Lemma 3.

## Lemma 3 (Target firms)

(a) $F(A, X) \leq \Pi_{T}(A)$ if and only if $X \leq X_{T}(A)$, where $X_{T}(A) \in(0, x(A))$ is uniquely defined by (11).
(b) $X_{T}^{\prime}(A)>0$ if $\frac{F_{A}(A, X)}{F(A, X)}$ is weakly increasing in $X$.

Lemma 2(b) and Lemma 3(b) are the key to our analysis of announcement returns.

### 2.3 Participation decision

We establish that firms with $A \leq A_{B}(X)$ choose to be bidders, not targets, and that firms with $X \leq X_{T}(A)$ choose to be targets, not bidders.

Lemma $4 \quad A \leq A_{B}(X)$ implies $\Pi_{T}(A)<\Pi_{B}(X)$, while $X \leq X_{T}(A)$ implies $\Pi_{B}(X)<\Pi_{T}(A)$.

Proof. On the ( $X, A$ )-plane (taking $X$ on the horizontal axis), $X_{T}(A)<x(A)$ implies that the graph $X=X_{T}(A)$ lies left to the graph $X=x(A)$. Similarly, $A_{B}(X)<a(X)$ implies that the graph $A=A_{B}(X)$ lies below the graph $A=a(X)$. Because $X=$ $x(A)$ and $A=a(X)$ are an identical and increasing function, any point on or left to the graph $X=X_{T}(A)$ (i.e., $\left.X \leq X_{T}(A)\right)$ lies strictly above the graph $A=A_{B}(X)$ (i.e., $\left.A>A_{B}(X)\right)$. Therefore at any such point, $\Pi_{B}(X)<F(A, X) \leq \Pi_{T}(A)$ holds. Similar reasoning applied to any point on or below the graph $A=A_{B}(X)$ (i.e., $A \leq A_{B}(X)$ ) implies $\Pi_{T}(A)<F(A, X) \leq \Pi_{B}(X)$.

## Proposition 1 summarizes Lemmas 1-4.

## Proposition 1 (Firms' actions)

Assume $\frac{\partial}{\partial A}\left(\frac{F_{X}}{F}\right) \geq 0$ and $\frac{\partial}{\partial X}\left(\frac{F_{A}}{F}\right) \geq 0$. Given an increasing and twice-differentiable $P(A)$, firms' optimal actions imply:
(a) Bidders and targets are matched according to $X=x(A)$ defined by (4), and $x^{\prime}(A)>0$.
(b) Firms become targets if and only if $X \leq X_{T}(A)$, and become bidders if and only if $A \leq A_{B}(X) . X_{T}(A)$ defined by (11) satisfies $X_{T}^{\prime}(A)>0$ and $0<X_{T}(A)<x(A)$, while $A_{B}(X)$ defined by (9) satisfies $A_{B}^{\prime}(X)>0$ and $0<A_{B}(X)<a(X) \equiv x^{-1}(X)$.
(c) $\Pi_{T}^{\prime}(A)>0$ and $\Pi_{B}^{\prime}(X)>0$. Also, $\Pi_{B}(X)>0$ if and only if (8) holds.

Figure 1 illustrates firms' matching and selection stated in Proposition 1(a,b).


Figure 1. Firms' actions.

Note. A black solid line is a matching function $x(A)=X \Leftrightarrow a(X)=A$. A blue dashed line represents marginal target firms with $\left(A, X_{T}(A)\right)$. A red double dashed line represents marginal bidder firms with $\left(A_{B}(X), X\right)$.

In the area left to the blue dashed line $X=X_{T}(A), \Pi_{B}(X)<F(A, X)<\Pi_{T}(A)$ holds. Firms in this region have low organization quality $X$ relative to their tradeable $A$. Their best option is to sell their endowed $A$. In the area below the red double dashed line $A=A_{B}(X)$, $\Pi_{T}(A)<F(A, X)<\Pi_{B}(X)$ holds. Firms in this region have low tradeable quality $A$ relative to their organization $X$. Their best option is to buy and integrate another firm with a better tradeable. Finally, in the neighborhood of the black solid line (a matching function $x(A)=X), \max \left\{\Pi_{T}(A), \Pi_{B}(X)\right\}<F(A, X)$ holds. Firms in this region find it optimal not to participate in takeovers. Even if firms with high $A$ in this region received a bid $P(A)$, they would reject it because their status quo value $F(A, X)$ is higher than $\Pi_{T}(A)$.

To close this subsection, we show that if simultaneously being a target and a bidder (i.e., a buy-and-sell strategy) costs at least $F(A, X)$, no firm does this.

Lemma 5 If a buy-and-sell strategy costs $F(A, X)$ or more, a firm with $(A, X)$ does not choose this strategy.

Proof. For target firms with $X \leq X_{T}(A)$, an additional benefit from being bidder is $\Pi_{B}(X)$. Therefore, if a cost of doing both $\phi$ satisfies $\phi \geq \Pi_{B}(X)$, no target firm has such an incentive. Recall that $\Pi_{B}(X)<F(A, X)$ holds for these firms. Similarly, for bidder firms with $A \leq A_{B}(X), \phi \geq \Pi_{T}(A)$ prevents them from doing both, and $\Pi_{T}(A)<F(A, X)$ holds for these firms. For non-participating firms, $\phi \geq \Pi_{T}(A)+\Pi_{B}(X)-F(A, X)$ prevents them from doing both. Because max $\left\{\Pi_{T}(A), \Pi_{B}(X)\right\}<F(A, X)$ holds for these firms, $\Pi_{T}(A)+\Pi_{B}(X)-F(A, X)=\min \left\{\Pi_{T}(A), \Pi_{B}(X)\right\}+\max \left\{\Pi_{T}(A), \Pi_{B}(X)\right\}-F(A, X)<$ $\min \left\{\Pi_{T}(A), \Pi_{B}(X)\right\}<F(A, X)$. Therefore, a sufficient (but not necessary) condition to deter a buy-and-sell strategy for any firm is $\phi \geq F(A, X)$.

In equilibrium, $\min \left\{\Pi_{T}(A), \Pi_{B}(X)\right\}<F(A, X)$ holds for any firm. For target and bidder firms, the incremental benefit from switching to a buy-and-sell strategy (from their current value max $\left.\left\{\Pi_{T}(A), \Pi_{B}(X)\right\}\right)$ is smaller than $F(A, X)$. For non-participating firms, a buy-and-sell strategy means combining their second and third best options (after their current value $F(A, X)$ ). Its net benefit (net of $F(A, X)$, as firms cannot "keep it and sell it") is smaller than the payoff of their third best option $\min \left\{\Pi_{T}(A), \Pi_{B}(X)\right\} .{ }^{5}$

### 2.4 Market-clearing prices, volume, and relative size

So far we assumed the existence of increasing and twice-differentiable $P(A)$, and it was left implicit in $x(A), X_{T}(A)$, and $A_{B}(X)$. We close the model by writing down demand and supply as a function of $P(A)$. First, consider supply. There is a measure $\phi_{A}(A)$ of firms endowed with a tradeable $A$. Among them, those with a non-tradeable $X \leq X_{T}(A)$ become targets. Hence, a measure of target firms with a tradeable $A$ is $\phi_{A}(A) \Phi_{X}\left(X_{T}(A)\right)$. By integrating this supply density over an interval $\left[a, A_{\max }\right.$ ], we obtain the corresponding supply $\int_{a}^{A_{\max }} \phi_{A}(A) \Phi_{X}\left(X_{T}(A)\right) d A$.

Next, consider firms endowed with a non-tradeable $X$. There is a measure $\phi_{X}(X)$ of such firms, and among them those with a tradeable $A \leq A_{B}(X)$ become bidders. Therefore, a demand for a tradable $a(X)$ from bidder firms endowed with a non-tradeable $X$ is $\phi_{X}(X) \Phi_{A}\left(A_{B}(X)\right)$. Because a tradeable $A$ is matched with a non-tradeable $x(A)$ and $x^{\prime}(A)>0$ by Lemma 1, the relevant demand to be equated with supply $\int_{a}^{A_{\max }} \phi_{A}(A) \Phi_{X}\left(X_{T}(A)\right) d A$ is $\int_{x(a)}^{x\left(A_{\max }\right)} \phi_{X}(X) \Phi_{A}\left(A_{B}(X)\right) d X$. A market-clearing condition is then:

$$
\int_{a}^{A_{\max }} \phi_{A}(A) \Phi_{X}\left(X_{T}(A)\right) d A=\int_{x(a)}^{x\left(A_{\max }\right)} \phi_{X}(X) \Phi_{A}\left(A_{B}(X)\right) d X \quad \forall a \in\left[0, A_{\max }\right]
$$

[^4]Taking a derivative with respect to $a$ yields

$$
\begin{equation*}
\phi_{A}(A) \Phi_{X}\left(X_{T}(A)\right)=\phi_{X}(x(A)) \Phi_{A}\left(A_{B}(x(A))\right) x^{\prime}(A) \quad \forall A \in\left[0, A_{\max }\right] . \tag{13}
\end{equation*}
$$

Finally, we impose a boundary condition $x\left(A_{\max }\right)=X_{\max }$. In the market-clearing condition (13), $A_{B}(X)$ defined by (9) and $X_{T}(A)$ defined by (11) depend on $P(A)$, while $x(A)$ defined by (4) depends on $P^{\prime}(A)$. Therefore, (13) defines a second-order differential equation in $P(A)$. We also define a total volume of takeovers by

$$
\begin{equation*}
V \equiv \int_{0}^{A_{\max }} \phi_{A}(A) \Phi_{X}\left(X_{T}(A)\right) d A \tag{14}
\end{equation*}
$$

Because $V$ is an equilibrium measure of target firms, the maximum value of $V$ is $\frac{1}{2}$. We define the relative bidder size by

$$
\begin{equation*}
R B(A) \equiv \frac{\Pi_{B}(x(A))}{\Pi_{T}(A)} \tag{15}
\end{equation*}
$$

In Section 4 we solve (13) and characterize (14) and (15) for a parameterized model.

## 3 Announcement returns

To define announcement returns by changes in the post-announcement firm value relative to the pre-announcement firm value, we assume the following time line of events:

Stage 1. A continuum of firms forms. Based on its characteristics $I$ which is public knowledge, a firm's market value is $q(I)$.

Stage 2. The firm may enter the takeover market as a bidder or a target. It can stay out of the takeover market.

Stage 3. The takeover market clears at the prices $P(A)$ for $A \in\left[0, A_{\max }\right]$. A bidder's market value is $\Pi_{B}(X)$, while a target's firm value is $\Pi_{T}(A)$.

At Stage 2, firms know their own $(A, X)$ and act as we analyzed in the previous section. At Stage 3, the takeover market clears and target firms' $A$ and bidder firms' $X$ become public. Using Stage-1 price $q(I)$ and Stage- 3 price $\left(\Pi_{B}(X), \Pi_{T}(A)\right)$, announcement returns are $\frac{\Pi_{T}(A)-q(I)}{q(I)}$ for targets that satisfy $X \leq X_{T}(A)$, and $\frac{\Pi_{B}(X)-q(I)}{q(I)}$ for bidders that satisfy $X \leq X_{T}(A)$, where $I$ is public knowledge about the firm at Stage 1.

Clearly, the behavior of announcement returns crucially depends on $I$. For example, if $I=(A, X)$, i.e., both factors are public knowledge at Stage 1, then

$$
q(A, X)= \begin{cases}\Pi_{T}(A) & \text { for firms with } X \leq X_{T}(A) \\ \Pi_{B}(X) & \text { for firms with } A \leq A_{B}(X) \\ F(A, X) & \text { for other firms }\end{cases}
$$

implies zero announcement returns for all targets and bidders. Thus, non-zero announcement returns require some information revelation about $(A, X)$. We study the following three cases for $I$ :

Case A. $I=A$. Only a tradeable factor $A=a$ is public knowledge at Stage 1 .
Case X. $I=X$. Only a non-tradeable factor $X=x$ is public knowledge at Stage 1.
Case F. $I=F(A, X)$. Only a status-quo firm value $F(A, X)=f \in \mathbb{R}_{+}$is public knowledge (neither $A$ nor $X$ is public knowledge) at Stage 1.

In Case A, suppose that investors observe a firm's tradeable is $A=a \in\left(0, A_{B}\left(X_{\max }\right)\right)$ at Stage $1 .{ }^{6}$ They rationally anticipate that the firm becomes a target if $X \leq X_{T}(a)$ and it becomes a bidder if $X \in\left[A_{B}^{-1}(a), X_{\max }\right]$. At Stage 1, investors value this firm according to

$$
q(a)=\left(\int_{0}^{X_{T}(a)} \phi_{X}(X) d X\right) \Pi_{T}(a)+\int_{X_{T}(a)}^{A_{B}^{-1}(a)} F(a, X) \phi_{X}(X) d X+\left(\int_{A_{B}^{-1}(a)}^{X_{\max }} \phi_{X}(X) d X\right) \widehat{\Pi}_{B}(a)
$$

where $\widehat{\Pi}_{B}(a) \equiv \frac{\int_{A_{B}^{-1}(a)}^{X_{\max }} \Pi_{B}(X) \phi_{X}(X) d X}{\int_{A_{B}^{-1}(a)}^{X_{\max }} \phi_{X}(X) d X}$ is the average bidder value conditional on $A=a$. With this Stage-1 price $q(a)$, the target announcement return is $\frac{\Pi_{T}(a)-q(a)}{q(a)} \equiv R_{T}(a)$, while the average bidder announcement return is $\frac{\widehat{\Pi}_{B}(a)-q(a)}{q(a)} \equiv R_{B}(a)$.

In Case $\mathbf{X}$, suppose that investors observe a firm's non-tradeable is $X=x \in\left(0, X_{T}\left(A_{\max }\right)\right)$ at Stage $1 .{ }^{7}$ They rationally anticipate that the firm becomes a bidder if $A \leq A_{B}(x)$, while it becomes a target if $A \in\left[X_{T}^{-1}(x), A_{\max }\right]$. Investors value this firm according to

$$
q(x)=\left(\int_{0}^{A_{B}(x)} \phi_{A}(A) d A\right) \Pi_{B}(x)+\int_{A_{B}(x)}^{X_{T}^{-1}(x)} F(A, x) \phi_{A}(A) d A+\left(\int_{X_{T}^{-1}(x)}^{A_{\max }} \phi_{A}(A) d A\right) \widehat{\Pi}_{T}(x)
$$

[^5]where $\widehat{\Pi}_{T}(x) \equiv \frac{\int_{X_{T}(x)}^{A_{\max }^{1}(x)} \Pi_{T}(A) \phi_{A}(A) d A}{\int_{X_{T}^{-1}(x)}^{A_{\max }} \phi_{A}(A) d A}$ is the average target value conditional on $X=x$. With this Stage-1 price $q(x)$, the bidder announcement return is $\frac{\Pi_{B}(x)-q(x)}{q(x)} \equiv R_{B}(x)$, while the average target announcement return is $\frac{\widehat{\Pi}_{T}(x)-q(x)}{q(x)} \equiv R_{T}(x)$.

In Case $\mathbf{F}$, suppose that investors observe a firm's status quo value $F(A, X)=f \in(0, \bar{f})$ at Stage 1 , where $\bar{f} \equiv \min \left\{\bar{f}_{A}, \bar{f}_{X}\right\}, \bar{f}_{A} \equiv F\left(A_{\max }, X_{T}\left(A_{\max }\right)\right)$, and $\bar{f}_{X} \equiv F\left(A_{B}\left(X_{\max }\right), X_{\max }\right) .^{8}$ The average target and bidder values conditional on $F(A, X)=f$ are given by ${ }^{9}$

$$
\begin{aligned}
\widehat{\Pi}_{T}(f) & \equiv E\left[\Pi_{T}(A) \mid X \leq X_{T}(A), F(A, X)=f\right] \\
\widehat{\Pi}_{B}(f) & \equiv E\left[\Pi_{B}(X) \mid A \leq A_{B}(X), F(A, X)=f\right]
\end{aligned}
$$

The Stage-1 price $q(f)$ is the weighted average of $\left\{f, \widehat{\Pi}_{T}(f), \widehat{\Pi}_{B}(f)\right\}$. The average bidder return is $\frac{\widehat{\Pi}_{B}(f)-q(f)}{q(f)} \equiv R_{B}(f)$, while the average target announcement return is $\frac{\widehat{\Pi}_{T}(f)-q(f)}{q(f)} \equiv$ $R_{T}(f)$. Figure 2 illustrates what takeover announcements reveal for each case.


Figure 2. Information revealed relative to pre-announcement information.
Note. In each panel, a dotted line represents investors' information set at Stage 1. In Case A, investors know $A=a<A_{B}\left(X_{\max }\right)$. In Case X, investors know $X=x<X_{T}\left(A_{\max }\right)$. In Case F , investors know $F(A, X)=f<\bar{f}$.

## Proposition 2 (Announcement returns)

[^6](a) In Case $A, R_{T}(A)<0<R_{B}(A)$ holds for any $A \in\left(0, A_{B}\left(X_{\max }\right)\right)$.
(b) In Case $X, R_{B}(X)<0<R_{T}(X)$ holds for any $X \in\left(0, X_{T}\left(A_{\max }\right)\right)$.
(c) In Case $F$, when the number of takeovers is sufficiently small, $0<\min \left\{R_{T}(f), R_{B}(f)\right\}$ holds for any $f \in(0, \bar{f})$.

Proof. In Case A, conditional on $A=a \in\left(0, A_{B}\left(X_{\max }\right)\right)$, investors rationally anticipate that firms choose actions according to their unobserved $X: \Pi_{T}(a) \leq F(a, X) \leq$ $\Pi_{B}(a)$. Both $F(a, X)$ (non-participating firm value) and $\Pi_{B}(X)$ (bidder firm value) are strictly increasing in $X$. Because rational investors form $q(a)$ as the weighted average of $\left\{\Pi_{T}(a), F(a, X), \Pi_{B}(X)\right\}, \Pi_{T}(a)$ is always smaller than $q(a)$, while $\widehat{\Pi}_{B}(a)$ is always greater than $q(a)$. A symmetric reasoning in Case $\mathbf{X}$ shows that conditional on $X=x \in$ $\left(0, X_{T}\left(A_{\max }\right)\right), \Pi_{B}(x)$ is smaller than $q(x)$, while $\widehat{\Pi}_{T}(x)$ is greater than $q(x)$. In Case $\mathbf{F}$, a firm with $F(A, X)=f \in(0, \bar{f})$ can be a target or a bidder or a non-participant. Because $F_{A X}>0$, conditional on $F(A, X)=f$, investors know that if the firm has a high $A$ it should have low $X$ and vice versa. Note that $f<\min \left\{\widehat{\Pi}_{T}(f), \widehat{\Pi}_{B}(f)\right\}$ holds, because $f \leq \Pi_{T}(A)$ for targets and $f \leq \Pi_{B}(X)$ for bidders on the locus $\{F(A, X)=f\}$. Because $q(f)$ is the weighted average of $\left\{f, \widehat{\Pi}_{T}(f), \widehat{\Pi}_{B}(f)\right\}$, for a sufficiently large weight on $f$ (i.e., when the number of takeovers is sufficiently small), $f<q(f)<\min \left\{\widehat{\Pi}_{T}(f), \widehat{\Pi}_{B}(f)\right\}$ holds.

Proposition 2 shows that when only a tradeable $A$ is public knowledge at Stage 1, target discounts and bidder premia occur, while the opposite pattern arises when only a nontradeable $X$ is public knowledge at Stage 1. This result generalizes Jovanovic and Braguinsky (2004), which assumed Case $\mathbf{X}$ to rationalize target premia and bidder discounts.

The key takeaway of Proposition 2 is that investors' information set at Stage 1 determines which firms are pooled, which in turn determines information revealed at Stage 3. In Case A (see Figure 2(a)), investors observe $A=a$ at Stage 1 and firms with heterogeneous $X$ are pooled. Therefore, takeover announcements reveal high $X$ for bidder firms and low $X$ for target firms, generating $R_{T}(a)<0<R_{B}(a)$ (bidder premia and target discounts). A symmetric reasoning in Case $\mathbf{X}$ (Figure 2(b)) implies that takeover announcements reveal high $A$ for target firms and low $A$ for bidder firms, generating $R_{B}(x)<0<R_{T}(x)$ (target premia and bidder discounts). In Case F (Figure 2(c)), investors observe the status quo value $F(A, X)=f$ at stage 1 and firms with different $(A, X)$ are pooled. Therefore, takeover announcements reveal higher $X$ for bidder firms and higher $A$ for target firms relative to other firms with the same status quo value $f$. If the likelihood of takeovers is sufficiently
small, this generates $0<\min \left\{R_{T}(f), R_{B}(f)\right\}$ (target and bidder premia). ${ }^{10}$

Discussion. A large body of empirical literature found that target returns are robustly positive while bidder returns exhibit more dispersion and are negative on average (Andrade et al.(2001), Fuller et al. (2002)). While researchers examined a long list of variables to control for firm- and deal-characteristics, not much effort has been done to control for the public knowledge about firms before announcements. Our model shows that it can be a key driver of announcement returns. A given takeover sample may include all the three cases we considered, Cases A, X, F. Then, the fact that target firms' market value almost always increases upon the announcement of takeovers indicates that deals to which Case $A$ applies must be rare, i.e., firms' tradeable factors are typically not public knowledge. This is consistent with the idea that a project development is subject to a high level of uncertainty (consider R\&D), and a firm has an incentive to keep its outcome secret. If a takeover sample mostly consists of deals to which Case $\mathbf{X}$ or Case $\mathbf{F}$ applies, then bidder returns can be positive or negative, while target returns are always positive. Thus, through the lens of our model, (i) robust target premia, (ii) much smaller bidder returns, and (iii) a large dispersion in bidder returns, can be jointly explained by the nature of public knowledge about firms investors do not know firms' $A$.

While Case $\mathbf{F}$ (a status quo firm value is public knowledge) seems natural, one may wonder what Case $\mathbf{X}$ means empirically. Another well known empirical fact may shed light on this: firms that conduct many takeovers - serial bidders - are common. ${ }^{11}$ Importantly, the large loss deals are concentrated among serial bidders (Moeller et al. (2005)), and their announcement returns decline from deal to deal (Renneboog and Vansteenkiste (2019)). In the empirical literature, the serial bidders' underperformance is usually attributed to fundamental changes in bidder firms. The literature mostly focused on CEO overconfidence (Billet and Qian (2008), Jaffe et al. (2013), El-Khativ et al. (2015)) or CEO learning (Aktas et al. $(2009,2011,2013)$ ) as a driving force of serial bidders' persistent underperformance.

Our model offers an alternative hypothesis: investors may be learning about serial bidders' non-tradeable. Bidder firms' skill with which they successfully complete takeovers and implement subsequent integration is likely to be non-tradeable. For its first takeover, a firm's non-tradeable - an organizational skill to manage projects - may be uncertain from investors' point of view. However, if the firm successfully implements multiple takeovers in a

[^7]short period of time, investors can learn about this skill. Therefore, we view a sample with serial bidders as a good example of Case X. Also, the fact that bidder returns decline from deal to deal is consistent with investors' gradual learning about the firm's non-tradeable.

El-Khativ et al. (2015) offers a further supporting evidence for our hypothesis. They examined a network position of CEOs, and find that high-centrality CEOs carry greater value losses. They also find that internal governance mechanisms are unrelated to the magnitude of value losses. These findings are consistent with our hypothesis: high-centrality CEOs' nontradeable skill is more likely to be public knowledge, and negative announcement returns are driven by investors' disappointment about their currently managed projects. When the negative returns are not driven by CEOs' self-interest but by investors' learning, governance mechanisms intended to correct agency frictions should not matter much.

To close our discussion, we want to stress that our message is not to dispute the importance of CEOs' overconfidence and learning, or more generally, fundamental changes in bidder firms as determinants of announcement returns. However, our model shows that differences in investors' prior information about firms can be a significant - in that they can flip a sign of announcement returns - contributing factor. To interpret observed announcement returns in takeover markets, controlling for public knowledge about firms is critical as it can change their sign even if firm- or deal-characteristics are identical. ${ }^{12}$

## 4 Other Testable Implications

The model admits a closed form solution when $\Phi_{A}(A)=\frac{A}{A_{\max }}, \Phi_{X}(X)=\frac{X}{X_{\max }}$, and $F(A, X)=A^{\alpha} X^{\beta}$. We use this parameterized model to generate testable implications. All the proofs for this section are gathered in the Appendix. First, the market-clearing condition (13) becomes

$$
\begin{equation*}
X_{T}(A)=A_{B}(x(A)) x^{\prime}(A) \tag{16}
\end{equation*}
$$

Using $\eta_{f}(A) \equiv \frac{f^{\prime}(A)}{f(A)} A$ to denote the elasticity of a function $f(A), x^{\prime}(A)$ given in (7) is

$$
x^{\prime}(A)=\frac{F_{A}(A, x(A))}{F_{A X}(A, x(A)) A}\left(\eta_{P^{\prime}}(A)-\frac{F_{A A}(A, x(A))}{F_{A}(A, x(A))} A\right) .
$$

[^8]Using $F(A, X)=A^{\alpha} X^{\beta}$ to evaluate this expression,

$$
\begin{equation*}
x^{\prime}(A)=\frac{1}{\beta} \frac{x(A)}{A}\left(\eta_{P^{\prime}}(A)+1-\alpha\right) . \tag{17}
\end{equation*}
$$

With $F(A, X)=A^{\alpha} X^{\beta},(4),(9),(11)$ can be explicitly solved:

$$
\begin{gather*}
x(A)=\left(\frac{1+\tau_{B}}{1-c} \frac{P^{\prime}(A)}{\alpha A^{\alpha-1}}\right)^{\frac{1}{\beta}}  \tag{18}\\
A_{B}(x(A))=\left\{(1-c) A^{\alpha}-\left(1+\tau_{B}\right) \frac{P(A)}{(x(A))^{\beta}}\right\}^{\frac{1}{\alpha}}  \tag{19}\\
X_{T}(A)=\left\{\left(1-\tau_{T}\right) \frac{P(A)}{A^{\alpha}}\right\}^{\frac{1}{\beta}} \tag{20}
\end{gather*}
$$

Substituting (17)-(20) into (16), using $\eta_{P}(A) \equiv \frac{P^{\prime}(A) A}{P(A)}$ and simplifying,

$$
\begin{equation*}
\frac{1-\tau_{T}}{1+\tau_{B}} \alpha \beta^{\beta}=\left(\frac{\eta_{P}(A)}{1-c}\right)^{1-\frac{\beta}{\alpha}}\left(\eta_{P}(A)-\alpha\right)^{\frac{\beta}{\alpha}}\left(\eta_{P^{\prime}}(A)+1-\alpha\right)^{\beta} \tag{21}
\end{equation*}
$$

We make a conjecture $P(A)=c_{1} A^{c_{0}}$ to find a solution to (21), which also satisfies (8) (i.e., to find an equilibrium with takeovers). Proposition 3 describes this equilibrium.

## Proposition 3 (Closed form solution)

Suppose $\Phi_{A}(A)=\frac{A}{A_{\max }}, \Phi_{X}(X)=\frac{X}{X_{\max }}$, and $F(A, X)=A^{\alpha} X^{\beta}$.
(a) An equilibrium with

$$
\begin{aligned}
P(A) & =\frac{1-c}{1+\tau_{B}} \frac{X_{\max }^{\beta}}{A_{\max }^{c_{0}}} \frac{\alpha}{c_{0}} A^{c_{0}}, \\
x(A) & =X_{\max }\left(\frac{A}{A_{\max }}\right)^{\frac{c_{0}-\alpha}{\beta}} \Leftrightarrow a(X)=A_{\max }\left(\frac{X}{X_{\max }}\right)^{\frac{\beta}{c_{0}-\alpha}}, \\
X_{T}(A) & =\left\{\frac{1-\tau_{T}}{1+\tau_{B}}(1-c) \frac{\alpha}{c_{0}}\right\}^{\frac{1}{\beta}} x(A) \text { and } A_{B}(X)=\left\{(1-c)\left(1-\frac{\alpha}{c_{0}}\right)\right\}^{\frac{1}{\alpha}} a(X)
\end{aligned}
$$

exists, where $c_{0} \in(\alpha, \infty)$ is a unique solution to

$$
\begin{equation*}
\frac{c_{0}}{\alpha}=1+\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{1+\alpha}}\left\{\left(\frac{1-\tau_{T}}{1+\tau_{B}}\right)^{\frac{\alpha}{\beta}}\left(\frac{1}{1-c} \frac{c_{0}}{\alpha}\right)^{1-\frac{\alpha}{\beta}}\right\}^{\frac{1}{1+\alpha}} \tag{22}
\end{equation*}
$$

(b) Takeover volume is $V=\left(\frac{1-\tau_{T}}{1+\tau_{B}} \frac{1-c}{\frac{c_{0}}{\alpha}}\right)^{\frac{1}{\beta}} \frac{1}{1+\frac{1}{c_{0}-\alpha}}=\left\{(1-c)\left(1-\frac{\alpha}{c_{0}}\right)\right\}^{\frac{1}{\alpha}} \frac{\frac{c_{0}-\alpha}{\beta}}{\frac{c_{0}-\alpha}{\beta}+1}$.
(c) Relative bidder size is $\frac{\Pi_{B}(x(A))}{\Pi_{T}(A)}=\frac{1+\tau_{B}}{1-\tau_{T}}\left(\frac{c_{0}}{\alpha}-1\right)$.

Propositions 4 and 5 gather comparative statics of volume and the relative bidder size.

## Proposition 4 (Volume)

(a) $V$ decreases in $\tau_{T}, \tau_{B}, c$.
(b) $\lim _{\beta \rightarrow 0} V=0$ and $\lim _{\alpha \rightarrow 0} V=0$.
(c) $\lim _{\beta \rightarrow \infty} V=\frac{(1-c)^{\frac{1}{\alpha}}}{1+(1-c)^{\frac{1}{\alpha}}}$ and $\lim _{\alpha \rightarrow \infty} V=\frac{\left(\frac{1-\tau_{T}}{1+\tau_{B}}(1-c)\right)^{\frac{1}{\beta}}}{1+\left(\frac{1-\tau_{T}}{1+\tau_{B}}(1-c)\right)^{\frac{1}{\beta}}}$.

Proposition 4(a) shows that transactions costs, whether it is fees $\tau_{T}, \tau_{B}$ or integration $\operatorname{costs} c$, decrease the volume of takeovers. Proposition $4(\mathbf{b})$ is also intuitive. Given the complementarity in production technology, if either one of the two factors loses its productivity, takeovers disappear. ${ }^{13}$ These results are not so surprising. Proposition 4(c) is slightly more subtle. First, a technological improvement in non-tradeable alone (i.e., $\beta \rightarrow \infty$ ) eliminates the negative effect of fees $\tau_{T}, \tau_{B}$. However, when $(1-c)^{\frac{1}{\alpha}}$ is small, a takeover volume is small no matter how large $\beta$ is. In contrast, a technological improvement in tradeable alone (i.e., $\alpha \rightarrow \infty$ ) does not eliminate the negative effect of $\tau_{T}, \tau_{B}, c$. Intuitively, frictions associated with matching (i.e., a distorted choice of tradeable due to fees) disappear as the productivity of the complementary non-tradeable goes up, while frictions associated with integration (i.e., loss of values after matching) do not.

## Proposition 5 (Relative bidder size)

(a) $R B(A) \equiv \frac{\Pi_{B}(x(A))}{\Pi_{T}(A)}$ is increasing in $\tau_{T}, \tau_{B}$ if $\beta \geq \frac{\alpha}{1+\alpha}$, while it is increasing (decreasing) in $c$ if and only if $\beta>(<) \alpha$.
(b) $R B(A)$ is increasing in $\beta$ and decreasing in $\alpha$.
(c) $\lim _{\beta \rightarrow 0} R B(A)=\lim _{\alpha \rightarrow \infty} R B(A)=0$, while $\lim _{\beta \rightarrow \infty} R B(A)=\lim _{\alpha \rightarrow 0} R B(A)=\infty$.

[^9]Proposition 5 relates the market value of the bidder relative to its matched target firm to transaction cost parameters $\left(\tau_{T}, \tau_{B}, c\right)$ and technology parameters $(\alpha, \beta)$. To our knowledge, these are the new results in the literature. Proposition 5(b) shows that the relative size of bidders increases in $\left(\tau_{T}, \tau_{B}\right)$ given that $\beta$ is not too small relative to $\alpha .{ }^{14}$ To understand why, first notice that $\tau_{T}$ directly decreases target size because $\Pi_{T}(A)=\left(1-\tau_{T}\right) P(A)$. Second, recall the first order condition of bidders' problem (4) is

$$
(1-c) \alpha A^{\alpha-1} X^{\beta}=\left(1+\tau_{B}\right) P^{\prime}(A) .
$$

This means that for a given $A, x(A)$ must be higher for higher $\tau_{B}$, implying a larger $\Pi_{B}(x(A))$. Intuitively, the matching distortion demands better bidders for a given target. In contrast, the effect of $c$ depends on the relative importance of $(\alpha, \beta)$. Similar to the effect of $\tau_{B}$, a higher $c$ demands better bidders for a given $A$. In addition, a higher $c$ directly reduces the post-takeover value $\Pi_{B}(X)$. For a large $\beta$, the former effect dominates, because an increase in $\beta$ amplifies the matching distortion due to $c .{ }^{15}$

Finally, Proposition 5(c) shows that the relative size of bidders increases in $\beta$ and decreases in $\alpha$. This is intuitive because a higher $\beta(\alpha)$ means the higher productivity of non-tradeable (tradeable), which favors bidders (targets) more as they are rich in nontradeable (tradeable).

For the sake of completeness, we present double limits result.

## Proposition 6 (Double limits)

For a fixed $\frac{\beta}{\alpha}=k>0$, consider taking limits $\alpha, \beta \rightarrow \infty$ or $\alpha, \beta \rightarrow 0$.
(a) $V$ approaches $\frac{1}{2}$ as $\alpha, \beta \rightarrow \infty$, while it approaches 0 as $\alpha, \beta \rightarrow 0$.
(b) $R B(A)$ approaches $\frac{1+\tau_{B}}{1-\tau_{T}} k$ as $\alpha, \beta \rightarrow \infty$, while it approaches $\frac{1+\tau_{B}}{1-\tau_{T}}(\chi(k)-1)$ as $\alpha, \beta \rightarrow$ 0 , where $\chi(k)$ is an increasing function defined by $\chi=1+\left(\frac{1-\tau_{T}}{1+\tau_{B}}\right)^{\frac{1}{k}}\left(\frac{1}{1-c} \chi\right)^{1-\frac{1}{k}}$ and satisfies $\frac{1+\tau_{B}}{1-\tau_{T}}(\chi(k)-1) \gtrless 1 \Leftrightarrow k \gtrless 1$.

Proposition 6(a) shows that a level of technology matters for the takeover volume: as both factors become more and more productive (i.e., $\alpha, \beta \rightarrow \infty$ ), all firms participate in takeovers, and vice versa. Proposition 6(b) shows that in such limits, the relative size of bidders still depends on $\frac{\beta}{\alpha}$ in a way characterized by Proposition 5(c).

[^10]Discussion. To relate the above results to data, we need to take a stand on empirical contents of $X$. Our preferred interpretation is that $X$ is an organization capital. Li et al. (2018) estimates firms' organization capital and find that bidders own more of it and also that it is relevant for takeover performances. They also find that target firms' organization capital is irrelevant for takeover performances. These findings are consistent with our model.

Given this interpretation, Propositions 4 and 5 offer cross-industry predictions. Other things equal, our model predicts that the takeover volume should be larger in industries where transaction costs of takeovers are smaller and organization capital is more important in production. The model also predicts that a relative size of bidders should be larger in industries where organization capital is relatively more important. Finally, the effect of integration costs on the relative bidder size should be positive (negative) in industries where organization capital plays a major (minor) role in production.

## 5 Conclusion

We presented a competitive model of takeovers in which projects (tradeable) and organizations (non-tradeable) are complements. We showed that the signs of bidder and target announcement returns identify the pre-announcement public knowledge about firms. Because the signs of announcement returns change depending on what aspect of firms is publicly known before the announcement, it is critically important to control investors' information to make a correct inference from announcement returns. We also showed that the takeover volume and the relative bidder size can provide useful information about production technologies that combine tradeable and non-tradeable. While there are some suggestive evidences supporting our model, more empirical work is needed to confirm its validity.

## 6 Appendix: Proofs for Section 4

## Proof of Proposition 3

(a) With $F(A, X)=A^{\alpha} X^{\beta}$ and the conjectured price $P(A)=c_{1} A^{c_{0}}$, the condition (8) becomes $\alpha<c_{0}$. Substituting $P^{\prime}(A)=c_{1} c_{0} A^{c_{0}-1}$ into (18) yields $x(A)=\left(\frac{1+\tau_{B}}{1-c} \frac{c_{1} c_{0}}{\alpha}\right)^{\frac{1}{\beta}} A^{\frac{c_{0}-\alpha}{\beta}}$. The boundary condition $x\left(A_{\max }\right)=X_{\max }$ pins down $c_{1}$ so that $P(A)=\frac{1-c}{1+\tau_{B}} \frac{X_{\max }^{\beta}}{A_{\max }^{\sigma_{0}}} \frac{\alpha}{c_{0}} A^{c_{0}}$ and $x(A)=X_{\max }\left(\frac{A}{A_{\max }}\right)^{\frac{c_{0}-\alpha}{\beta}}$. Finally, substituting $\left(\eta_{P}(A), \eta_{P^{\prime}}(A)\right)=\left(c_{0}, c_{0}-1\right)$ into (21) yields

$$
\begin{equation*}
\frac{1-\tau_{T}}{1+\tau_{B}} \alpha \beta^{\beta}=\left(\frac{c_{0}}{1-c}\right)^{1-\frac{\beta}{\alpha}}\left(c_{0}-\alpha\right)^{\frac{1+\alpha}{\alpha} \beta} . \tag{23}
\end{equation*}
$$

Solving this for $c_{0}-\alpha$ and rewriting it as an equation in $\frac{c_{0}}{\alpha},(22)$ is obtained.
For $\beta=\alpha$, (22) is $\frac{c_{0}}{\alpha}=1+\left(\frac{1-\tau_{T}}{1+\tau_{B}}\right)^{\frac{1}{1+\alpha}}>1$. For $\beta<\alpha$, the right hand side of (22) is decreasing in $\frac{c_{0}}{\alpha}$ and approaches one as $\frac{c_{0}}{\alpha} \rightarrow \infty$. For $\beta>\alpha$, the right hand side is increasing and concave in $\frac{c_{0}}{\alpha}$. Therefore, (22) has a unique solution $\frac{c_{0}}{\alpha} \in(1, \infty)$. Finally, $x(A), X_{T}(A)$, $A_{B}(X)$ are obtained by computing (18)-(20) with $P(A)=\frac{1-c}{1+\tau_{B}} \frac{X_{\max }^{\beta}}{A_{\max }^{\alpha} \alpha} \frac{\alpha}{c_{0}} A^{c_{0}}$.
(b) To derive the volume $V \equiv \int_{0}^{A_{\max }} \phi_{A}(A) \Phi_{X}\left(X_{T}(A)\right) d A=\frac{1}{A_{\max } X_{\max }} \int_{0}^{A_{\max }} X_{T}(A) d A$, substitute $P(A)=\frac{1-c}{1+\tau_{B}} \frac{X_{\max }^{\beta}}{A_{\max }^{\circ}-\alpha} \frac{\alpha}{c_{0}} A^{c_{0}}$ into $X_{T}(A)=\left\{\left(1-\tau_{T}\right) \frac{P(A)}{A^{\alpha}}\right\}^{\bar{\beta}}:$

$$
\begin{aligned}
V & =\frac{1}{A_{\max } X_{\max }} \int_{0}^{A_{\max }}\left\{\frac{1-\tau_{T}}{1+\tau_{B}}(1-c) \frac{X_{\max }^{\beta}}{A_{\max }^{c_{0}-\alpha}} \frac{\alpha}{c_{0}}\right\}^{\frac{1}{\beta}} A^{\frac{c_{0}-\alpha}{\beta}} d A \\
& =\left(\frac{1-\tau_{T}}{1+\tau_{B}} \frac{1-c}{\frac{c_{0}}{\alpha}}\right)^{\frac{1}{\beta}} \frac{1}{A_{\max }^{1+\frac{c_{0}-\alpha}{\beta}}} \int_{0}^{A_{\max }} A^{\frac{c_{0}-\alpha}{\beta}} d A \\
& =\left(\frac{1-\tau_{T}}{1+\tau_{B}} \frac{1-c}{\frac{c_{0}}{\alpha}}\right)^{\frac{1}{\beta}} \frac{1}{1+\frac{c_{0}-\alpha}{\beta}} .
\end{aligned}
$$

To obtain the second expression, using (22), $\left(\frac{1-\tau_{T}}{1+\tau_{B}} \frac{1-c}{\frac{c}{0}_{\alpha}^{\alpha}}\right)^{\frac{1}{\beta}}=\frac{\alpha}{\beta} \frac{\left(\frac{c_{0}}{\alpha}-1\right)^{\frac{1+\alpha}{\alpha}}}{\left(\frac{1}{1-c} \frac{c_{0}}{\alpha}\right)^{\frac{1}{\alpha}}}$. Substituting this,

$$
\begin{aligned}
V & =\frac{\alpha}{\beta} \frac{\left(\frac{c_{0}}{\alpha}-1\right)^{\frac{1+\alpha}{\alpha}}}{\left(\frac{1}{1-c} \frac{c_{0}}{\alpha}\right)^{\frac{1}{\alpha}}} \frac{\frac{\beta}{\alpha}}{\frac{\beta}{\alpha}+\frac{c_{0}}{\alpha}-1} \\
& =\left\{(1-c)\left(1-\frac{\alpha}{c_{0}}\right)\right\}^{\frac{1}{\alpha}} \frac{\frac{c_{0}}{\alpha}-1}{\frac{c_{0}}{\alpha}-1+\frac{\beta}{\alpha}} \\
& =\left\{(1-c)\left(1-\frac{\alpha}{c_{0}}\right)\right\}^{\frac{1}{\alpha}} \frac{\frac{c_{0}-\alpha}{\beta}}{\frac{c_{0}-\alpha}{\beta}+1}
\end{aligned}
$$

(c) To derive $R B(A) \equiv \frac{\Pi_{B}(x(A))}{\Pi_{T}(A)}$, compute

$$
\begin{aligned}
\Pi_{T}(A) & =\left(1-\tau_{T}\right) P(A)=\frac{\left(1-\tau_{T}\right)(1-c)}{1+\tau_{B}} \frac{X_{\max }^{\beta}}{A_{\max }^{c_{0} \alpha}} \frac{\alpha}{c_{0}} A^{c_{0}}, \\
\Pi_{B}(x(A)) & =(1-c) A^{\alpha}(x(A))^{\beta}-\left(1+\tau_{B}\right) P(A) \\
& =(1-c) A^{\alpha} X_{\max }^{\beta}\left(\frac{A}{A_{\max }}\right)^{c_{0}-\alpha}-(1-c) \frac{X_{\max }^{\beta}}{A_{\max }^{c_{0}-\alpha}} \frac{\alpha}{c_{0}} A^{c_{0}} \\
& =(1-c) \frac{X_{\max }^{\beta}}{A_{\max }^{c_{0}-\alpha}}\left(1-\frac{\alpha}{c_{0}}\right) A^{c_{0}} .
\end{aligned}
$$

Therefore,

$$
R B(A)=\frac{1+\tau_{B}}{1-\tau_{T}} \frac{1-\frac{\alpha}{c_{0}}}{\frac{\alpha}{c_{0}}}=\frac{1+\tau_{B}}{1-\tau_{T}}\left(\frac{c_{0}}{\alpha}-1\right) .
$$

To prove some of the comparative statics results presented below, we use an alternative expression of (22). First, (23) can be written as

$$
\frac{1-\tau_{T}}{1+\tau_{B}} \beta^{\beta}=\left(\frac{1}{1-c} \frac{c_{0}}{\alpha}\right)^{1-\frac{\beta}{\alpha}}\left(\frac{c_{0}}{\alpha}-1\right)^{\frac{\beta}{\alpha}}\left(c_{0}-\alpha\right)^{\beta}
$$

We multiply $\frac{c_{0}-\alpha}{\frac{c_{0}}{\alpha}-1} \frac{1}{\alpha}$ to the right hand side to get

$$
\frac{1-\tau_{T}}{1+\tau_{B}} \beta^{\beta}=\frac{1}{\alpha}\left(\frac{1}{1-c} \frac{\frac{c_{0}}{\alpha}}{\frac{c_{0}}{\alpha}-1}\right)^{1-\frac{\beta}{\alpha}}\left(c_{0}-\alpha\right)^{\beta+1}
$$

Then solving for $c_{0}-\alpha$ and rewriting as an equation in $\frac{c_{0}}{\alpha}$,

$$
\begin{equation*}
\frac{c_{0}}{\alpha}=1+\left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{1+\beta}}\left\{\frac{1-\tau_{T}}{1+\tau_{B}}\left(\frac{1}{1-c} \frac{\frac{c_{0}}{\alpha}}{\frac{c_{0}}{\alpha}-1}\right)^{\frac{\beta}{\alpha}-1}\right\}^{\frac{1}{1+\beta}} \tag{24}
\end{equation*}
$$

## Lemma 6 (Comparative statics of $\frac{c_{0}}{\alpha}$ )

(a) $\frac{c_{0}}{\alpha}$ increases in $\beta$ and decreases in $\alpha, \tau_{T}, \tau_{B}$.
(b) $\frac{c_{0}}{\alpha}$ increases (decreases) in $c$ if and only if $\beta>(<) \alpha$.
(c) $\lim _{\beta \rightarrow \infty} \frac{c_{0}}{\alpha}=\lim _{\alpha \rightarrow 0} \frac{c_{0}}{\alpha}=\infty$ and $\lim _{\beta \rightarrow 0} \frac{c_{0}}{\alpha}=\lim _{\alpha \rightarrow \infty} \frac{c_{0}}{\alpha}=1$.
(d) For a fixed $\frac{\beta}{\alpha}=k>0, \frac{c_{0}}{\alpha}$ approaches $1+k$ as $\alpha, \beta \rightarrow \infty$.
(e) For a fixed $\frac{\beta}{\alpha}=k>0, \frac{c_{0}}{\alpha}$ approaches $\chi(k)$ as $\alpha, \beta \rightarrow 0$, where $\chi(k)>1$ is a unique solution to $\chi=1+\left(\frac{1-\tau_{T}}{1+\tau_{B}}\right)^{\frac{1}{k}}\left(\frac{1}{1-c} \chi\right)^{1-\frac{1}{k}}$ that increases in $k$ without $a$ bound.

## Proof.

(a) The right hand side of (22) as a function of $\frac{c_{0}}{\alpha}$ is either increasing and concave (for $\alpha<\beta$ ) or constant (for $\alpha=\beta$ ) or decreasing (for $\alpha>\beta$ ) in $\frac{c_{0}}{\alpha}$. Therefore, when changes in parameters increase (decrease) the right hand side, $\frac{c_{0}}{\alpha}$ as a unique solution to (22) increases (decreases). Because the right hand side of (22) increases in $\beta$ while decreases in $\tau_{T}, \tau_{B}$, the results for $\beta, \tau_{T}, \tau_{B}$ follow.

For $\alpha$, consider (24). The right hand side of (24) is either increasing and concave (for $\alpha>\beta$ ) or constant (for $\alpha=\beta$ ) or decreasing (for $\alpha<\beta$ ) in $\frac{c_{0}}{\alpha}$. Also, for a fixed value of $\frac{c_{0}}{\alpha}$, the right hand side decreases in $\alpha$. This implies that $\frac{c_{0}}{\alpha}$ is decreasing in $\alpha$.
(b) The right hand side of (22) is increasing (decreasing) in $c$ if and only if $\beta>(<) \alpha$.
(c)

For $\lim _{\beta \rightarrow \infty} \frac{c_{0}}{\alpha}$, note that the right hand side of (22) (as a function of $\frac{c_{0}}{\alpha}$ ) shifts up without a bound.

For $\lim _{\beta \rightarrow 0} \frac{c_{0}}{\alpha}$, note that the right hand side of (22) (as a function of $\frac{c_{0}}{\alpha}$ ) becomes 1 .
For $\lim _{\alpha \rightarrow \infty} \frac{c_{0}}{\alpha}$, note that the right hand side of (22) (as a function of $\frac{c_{0}}{\alpha}$ ) becomes 1.
For $\lim _{\alpha \rightarrow 0} \frac{c_{0}}{\alpha}$, note that the right hand side of (22) (as a function of $\frac{c_{0}}{\alpha}$ ) approaches an increasing linear function with a slope no smaller than 1.
( $\mathbf{d}, \mathbf{e}$ ) By fixing $\frac{\beta}{\alpha}=k$ in (22),

$$
\frac{c_{0}}{\alpha}=1+k^{\frac{\alpha}{1+\alpha}}\left\{\left(\frac{1-\tau_{T}}{1+\tau_{B}}\right)^{k}\left(\frac{1}{1-c} \frac{c_{0}}{\alpha}\right)^{1-k}\right\}^{\frac{1}{1+\alpha}}
$$

Taking the limits $\alpha \rightarrow \infty$ or $\alpha \rightarrow 0$ yields the results.

## Proof of Proposition 4

(a) Comparative statics with respect to $\left(\tau_{B}, \tau_{T}\right)$ directly follow from Lemma 6(a) and the expression of $V$.

Consider $c$. For $\beta=\alpha$, (22) becomes $\frac{c_{0}}{\alpha}=1+\left(\frac{1-\tau_{T}}{1+\tau_{B}}\right)^{\frac{1}{1+\alpha}}$. Hence $V$ is obviously decreasing in $c$. For $\beta<\alpha, \frac{c_{0}}{\alpha}$ decreases in $c\left(\right.$ Lemma 6(b)) which implies that $\frac{\frac{c_{0}}{\alpha}-1}{\frac{c_{0}}{\alpha}-1+\frac{\beta}{\alpha}}$ decreases in $c$. Therefore, it is sufficient to show that $(1-c)\left(1-\frac{\alpha}{c_{0}}\right)=\frac{\frac{c_{0}}{\alpha}-1}{\frac{1}{1-c} \frac{c_{0}}{\alpha}}$ decreases in $c$. The numerator $\frac{c_{0}}{\alpha}-1$ decreases in $c$. Notice that the right hand side of (22) is a decreasing function of $\frac{1}{1-c} \frac{c_{0}}{\alpha}$ for $\beta<\alpha$. Because the right hand side of (22) for $\beta<\alpha$ must decrease in response to an increase in $c, \frac{1}{1-c} \frac{c_{0}}{\alpha}$ must increase.

For $\beta>\alpha, \frac{c_{0}}{\alpha}$ increases in $c$. We use (24) to evaluate $(1-c)\left(1-\frac{\alpha}{c_{0}}\right)=\frac{\frac{c_{0}}{\alpha}-1}{\frac{1}{1-c} \frac{c_{0}}{\alpha}}$ in the expression of $V$. Because the right hand side of (24) contains $\left(\frac{\frac{c_{0}}{\alpha}-1}{\frac{1}{1-c} \frac{c_{0}}{\alpha}}\right)^{1-\frac{\beta}{\alpha}}$, solving (24) for $\frac{\frac{c_{0}}{\alpha}-1}{\frac{1}{1-c} \frac{c_{0}}{\alpha}}$ yields $\frac{\frac{c_{0}}{\alpha}-1}{\frac{1}{1-c} \frac{c_{0}}{\alpha}}=C\left(\frac{c_{0}}{\alpha}-1\right)^{\frac{\alpha(1+\beta)}{\alpha-\beta}}$, where $C$ is a collection of parameters independent of c. This implies that

$$
V=C^{\frac{1}{\alpha}} \frac{\left(\frac{c_{0}}{\alpha}-1\right)^{\frac{1+\beta}{\alpha-\beta}+1}}{\frac{c_{0}}{\alpha}-1+\frac{\beta}{\alpha}}=C^{\frac{1}{\alpha}} \frac{\left(\frac{c_{0}}{\alpha}-1\right)^{\frac{1+\alpha}{\alpha-\beta}}}{\frac{c_{0}}{\alpha}-1+\frac{\beta}{\alpha}} .
$$

Because $\frac{1+\alpha}{\alpha-\beta}<0$, the numerator is decreasing in $c$.
(b) To show $\lim _{\beta \rightarrow 0} V=0$, first note that $\lim _{\beta \rightarrow 0} \frac{c_{0}}{\alpha}=1\left(\right.$ Lemma 6(c)) implies $\lim _{\beta \rightarrow 0}\left\{(1-c)\left(1-\frac{\alpha}{c_{0}}\right)\right\}^{\frac{1}{\alpha}}=$

0 . Because $\frac{\frac{c_{0}-\alpha}{\beta}}{\frac{c_{0}-\alpha}{\beta}+1}$ is bounded above by 1 , the result follows.
To show $\lim _{\alpha \rightarrow 0} V=0$, we use $V=\left(\frac{1-\tau_{T}}{1+\tau_{B}} \frac{1-c}{\frac{c_{0}}{\alpha}}\right)^{\frac{1}{\beta}} \frac{1}{1+\frac{\varepsilon_{0}-\alpha}{\beta}}$ (see the derivation of $V$ above). Because $\lim _{\alpha \rightarrow 0} \frac{c_{0}}{\alpha}=\infty($ Lemma $6(\mathbf{c}))$ and $\frac{1}{1+\frac{c_{0}-\alpha}{\beta}}$ is bounded above by 1 , the result follows.
(c) For $\lim _{\beta \rightarrow \infty} V$, we use $V=\left\{(1-c)\left(1-\frac{\alpha}{c_{0}}\right)\right\}^{\frac{1}{\alpha}} \frac{\frac{c_{0}-\alpha}{\beta}}{\frac{c_{0}-\alpha}{\beta}+1}$. Because $\lim _{\beta \rightarrow \infty} \frac{\alpha}{c_{0}}=0$ (Lemma $\mathbf{6 ( c )})$, showing that $\lim _{\beta \rightarrow \infty} \frac{c_{0}-\alpha}{\beta}=\left(\frac{1}{1-c}\right)^{\frac{1}{\alpha}}$ proves the result. By multiplying $\frac{\alpha}{\beta}$ to both sides of (22) and subtracting $\frac{\alpha}{\beta}$,

$$
\begin{aligned}
\frac{c_{0}-\alpha}{\beta} & =\left\{\frac{\alpha}{\beta}\left(\frac{1-\tau_{T}}{1+\tau_{B}}\right)^{\frac{\alpha}{\beta}}\left(\frac{1}{1-c} \frac{c_{0}}{\alpha}\right)^{1-\frac{\alpha}{\beta}}\right\}^{\frac{1}{1+\alpha}} \\
& =\left\{\left(\frac{1-\tau_{T}}{1+\tau_{B}} \frac{\alpha}{\beta}\right)^{\frac{\alpha}{\beta}}\left(\frac{1}{1-c} \frac{c_{0}}{\beta}\right)^{1-\frac{\alpha}{\beta}}\right\}^{\frac{1}{1+\alpha}} \\
& =\left\{\left(\frac{1-\tau_{T}}{1+\tau_{B}} \frac{\alpha}{\beta}\right)^{\frac{\alpha}{\beta}}\left(\frac{1}{1-c}\left(\frac{c_{0}-\alpha}{\beta}+\frac{\alpha}{\beta}\right)\right)^{1-\frac{\alpha}{\beta}}\right\}^{\frac{1}{1+\alpha}} .
\end{aligned}
$$

As an equation in $\frac{c_{0}-\alpha}{\beta}$, this has a unique solution $\frac{c_{0}-\alpha}{\beta}$ greater than $\frac{\alpha}{\beta}$ for any $\beta \geq \alpha$. Taking the limit $\beta \rightarrow \infty$,

$$
\lim _{\beta \rightarrow \infty} \frac{c_{0}-\alpha}{\beta}=\left(\frac{1}{1-c} \lim _{\beta \rightarrow \infty} \frac{c_{0}-\alpha}{\beta}\right)^{\frac{1}{1+\alpha}} \Leftrightarrow \lim _{\beta \rightarrow \infty} \frac{c_{0}-\alpha}{\beta}=\left(\frac{1}{1-c}\right)^{\frac{1}{\alpha}}
$$

For $\lim _{\alpha \rightarrow \infty} V$, we use $V=\left(\frac{1-\tau_{T}}{1+\tau_{B}} \frac{1-c}{\frac{c_{0}}{\alpha}}\right)^{\frac{1}{\beta}} \frac{1}{1+\frac{c_{0}-\alpha}{\beta}}$. We first show that $\lim _{\alpha \rightarrow \infty} \frac{c_{0}-\alpha}{\beta}=\left(\frac{1-\tau_{T}}{1+\tau_{B}}(1-c)\right)^{\frac{1}{\beta}}$.

By multiplying $\frac{\alpha}{\beta}$ to both sides of (24) and subtracting $\frac{\alpha}{\beta}$,

$$
\begin{aligned}
\frac{c_{0}-\alpha}{\beta} & =\left\{\frac{\alpha}{\beta} \frac{1-\tau_{T}}{1+\tau_{B}}\left(\frac{1}{1-c} \frac{\frac{c_{0}}{\alpha}}{\frac{c_{0}}{\alpha}-1}\right)^{\frac{\beta}{\alpha}-1}\right\}^{\frac{1}{1+\beta}} \\
& =\left\{\frac{\alpha}{\beta} \frac{1-\tau_{T}}{1+\tau_{B}}(1-c)\left(\frac{1}{1-c}\right)^{\frac{\beta}{\alpha}}\left(\frac{\frac{c_{0}-\alpha}{\beta}+\frac{\alpha}{\beta}}{\frac{c_{0}-\alpha}{\beta}}\right)^{\frac{\beta}{\alpha}-1}\right\}^{\frac{1}{1+\beta}} \\
& =\left\{\frac{1-\tau_{T}}{1+\tau_{B}}(1-c)\left(\frac{1}{1-c} \frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha}}\left(\frac{\frac{c_{0}-\alpha}{\beta} \frac{\alpha}{\beta}}{\frac{c_{0}-\alpha}{\beta}+\frac{\alpha}{\beta}}\right)^{1-\frac{\beta}{\alpha}}\right\}^{\frac{1}{1+\beta}} \\
& =\left\{\frac{1-\tau_{T}}{1+\tau_{B}}(1-c)\left(\frac{1}{1-c} \frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha}}\left(\frac{1}{\frac{\beta}{\alpha}+\frac{\beta}{c_{0}-\alpha}}\right)^{1-\frac{\beta}{\alpha}}\right\}^{\frac{1}{1+\beta}} .
\end{aligned}
$$

As an equation in $\frac{c_{0}-\alpha}{\beta}$, this has a unique positive solution for any $\alpha>\beta$. Taking the limit $\alpha \rightarrow \infty$,

$$
\lim _{\alpha \rightarrow \infty} \frac{c_{0}-\alpha}{\beta}=\left(\frac{1-\tau_{T}}{1+\tau_{B}}(1-c) \lim _{\alpha \rightarrow \infty} \frac{c_{0}-\alpha}{\beta}\right)^{\frac{1}{1+\beta}} \Leftrightarrow \lim _{\alpha \rightarrow \infty} \frac{c_{0}-\alpha}{\beta}=\left(\frac{1-\tau_{T}}{1+\tau_{B}}(1-c)\right)^{\frac{1}{\beta}}
$$

Therefore, showing $\lim _{\alpha \rightarrow \infty}\left\{(1-c)\left(1-\frac{\alpha}{c_{0}}\right)\right\}^{\frac{1}{\alpha}}=1$ completes the proof. Using $\lim _{\alpha \rightarrow \infty} \frac{c_{0}}{\alpha}=1$ (Lemma 6(c)),

$$
\begin{aligned}
\lim _{\alpha \rightarrow \infty}\left\{(1-c)\left(1-\frac{\alpha}{c_{0}}\right)\right\}^{\frac{1}{\alpha}} & =\lim _{\alpha \rightarrow \infty}(1-c)^{\frac{1}{\alpha}} \lim _{\alpha \rightarrow \infty}\left(1-\frac{\alpha}{c_{0}}\right)^{\frac{1}{\alpha}}=\lim _{\alpha \rightarrow \infty} \frac{\left(\frac{c_{0}}{\alpha}-1\right)^{\frac{1}{\alpha}}}{\left(\frac{c_{0}}{\alpha}\right)^{\frac{1}{\alpha}}} \\
& =\lim _{\alpha \rightarrow \infty}\left(\frac{\beta}{\alpha}\right)^{\frac{1}{1+\alpha}}\left\{\left(\frac{1-\tau_{T}}{1+\tau_{B}}\right)^{\frac{1}{\beta}}\left(\frac{1}{1-c} \frac{c_{0}}{\alpha}\right)^{\frac{1}{\alpha}-\frac{1}{\beta}}\right\}^{\frac{1}{1+\alpha}} \\
& =\lim _{\alpha \rightarrow \infty}\left(\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha}}\right)^{\frac{\alpha}{1+\alpha}} \lim _{\alpha \rightarrow \infty}\left\{\beta\left(\frac{1-\tau_{T}}{1+\tau_{B}}(1-c)\right)^{\frac{1}{\beta}}\right\}^{\frac{1}{1+\alpha}} \lim _{\alpha \rightarrow \infty}\left(\frac{1}{1-c} \frac{c_{0}}{\alpha}\right)^{\frac{1}{1++\alpha) \alpha}} \\
& =1 .
\end{aligned}
$$

## Proof of Proposition 5

(a) The comparative statics with respect to $c$ directly follows from Lemma $\mathbf{6 ( b )}$ ) and the expression of $R B(A) \equiv \frac{\Pi_{B}(x(A))}{\Pi_{T}(A)}=\frac{1+\tau_{B}}{1-\tau_{T}}\left(\frac{c_{0}}{\alpha}-1\right)$. Consider $\left(\tau_{T}, \tau_{B}\right)$. Evaluating $R B(A)=$
$\frac{1+\tau_{B}}{1-\tau_{T}}\left(\frac{c_{0}}{\alpha}-1\right)$ by using (22) yields

$$
R B(A)=\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{1+\alpha}}\left\{\left(\frac{1-c}{\frac{c_{0}}{\alpha} \frac{1+\tau_{B}}{1-\tau_{T}}}\right)^{\frac{\alpha}{\beta}-1}\left(\frac{1+\tau_{B}}{1-\tau_{T}}\right)^{\alpha}\right\}^{\frac{1}{1+\alpha}} .
$$

For $\beta=\alpha, R B(A)=\left(\frac{1+\tau_{B}}{1-\tau_{T}}\right)^{\frac{\alpha}{1+\alpha}}$ and the result obviously holds.
For $\beta<\alpha$, because $\frac{c_{0}}{\alpha}$ is decreasing in $\left(\tau_{T}, \tau_{B}\right)$ (Lemma $\left.6 \mathbf{( a )}\right)$, the result holds if $\left(\frac{1+\tau_{B}}{1-\tau_{T}}\right)^{\alpha-\left(\frac{\alpha}{\beta}-1\right)}$ is weakly increasing in $\frac{1+\tau_{B}}{1-\tau_{T}}$. Therefore, a sufficient condition is $\alpha-\left(\frac{\alpha}{\beta}-1\right) \geq$ $0 \Leftrightarrow \beta \geq \frac{\alpha}{1+\alpha}$.

For $\beta>\alpha$, it is sufficient to show that $\frac{c_{0}}{\alpha} \frac{1+\tau_{B}}{1-\tau_{T}}$ is increasing in $\frac{1+\tau_{B}}{1-\tau_{T}}$. Using (22),

$$
\frac{c_{0}}{\alpha} \frac{1+\tau_{B}}{1-\tau_{T}}=\frac{1+\tau_{B}}{1-\tau_{T}}+\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{1+\alpha}}\left\{\left(\frac{1+\tau_{B}}{1-\tau_{T}}\right)^{\alpha}\left(\frac{1}{1-c} \frac{c_{0}}{\alpha} \frac{1+\tau_{B}}{1-\tau_{T}}\right)^{1-\frac{\alpha}{\beta}}\right\}^{\frac{1}{1+\alpha}}
$$

For $\beta>\alpha$, the right hand side is an increasing and concave function of $\frac{c_{0}}{\alpha} \frac{1+\tau_{B}}{1-\tau_{T}}$, and it is increasing in $\frac{1+\tau_{B}}{1-\tau_{T}}$. Hence, $\frac{c_{0}}{\alpha} \frac{1+\tau_{B}}{1-\tau_{T}}$ is increasing in $\frac{1+\tau_{B}}{1-\tau_{T}}$.
(b) The comparative statics with respect to $\alpha, \beta$ directly follows from Lemma $\mathbf{6 ( a )}$ and the expression of $R B(A)$.
(c) The limits of $R B(A)$ follow from Lemma $\mathbf{6}(\mathbf{c})$ and the expression of $R B(A)$.

## Proof or Proposition 6

(a) This follows from Lemma $\mathbf{6}(\mathbf{d}, \mathbf{e})$ and the expression of $V$.
(b) This follows from Lemma $\mathbf{6}(\mathbf{d}, \mathbf{e})$ and the expression of $R B(A)$.

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[^1]:    ${ }^{1}$ Renneboog and Vansteenkiste (2019) state that serial bidders' underperformance is one of the most consistent findings in the literature. It holds in the U.S. (Fuller et al. (2000), Klasa and Stegemoller (2007), Billett and Qian (2008), Laamanen and Keil (2008), El-Khativ et al. (2015), Li et al. (2018)), in the U.K. (Doukas and Petmezas (2007), Ismail and Abdallah (2013)), and in Australia (Hossain et al. (2021)).

[^2]:    ${ }^{2}$ If a fraction of a firm can be sold and the remaining part continues as a going concern, we view it as a sale of a division of the firm, not a takeover.

[^3]:    ${ }^{3}$ Independence is not a critical assumption. We only need imperfect correlation to generate takeovers.
    ${ }^{4}$ Without a disclosure, a pooling equilibrium may exist. However, it can be shown that with free disclosure, target firms find it optimal to disclose their $A$. See Kawakami (2022).

[^4]:    ${ }^{5}$ The proof of Lemma 5 shows that a smaller cost $\phi=\min \left\{\Pi_{T}(A), \Pi_{B}(X)\right\}$ is sufficient to deter a buy-and-sell strategy. For high transaction costs $\tau_{T}, \tau_{B}, c, \min \left\{\Pi_{T}(A), \Pi_{B}(X)\right\}$ becomes small, so it is unlikely to observe a firm active on both sides of frictional takeover markets.

[^5]:    ${ }^{6}$ A firm with $A=a>A_{B}\left(X_{\max }\right)$ will never be a bidder. With the observation $A=a \in\left(0, A_{B}\left(X_{\max }\right)\right)$, investors do not know whether this firm will be a target or a bidder.
    ${ }^{7}$ A firm with $X=x>X_{T}\left(A_{\max }\right)$ will not be a target.

[^6]:    ${ }^{8}$ Firms with $F(A, X)>\max \left\{\bar{f}_{A}, \bar{f}_{X}\right\}$ will not participate in takeovers. For firms with $F(A, X) \in$ $\left(\bar{f}, \max \left\{\bar{f}_{A}, \bar{f}_{X}\right\}\right)$, investors know which side of the market firms will participate (if they ever do).
    ${ }^{9}$ Formally, $\widehat{\Pi}_{T}(f)$ and $\widehat{\Pi}_{B}(f)$ are defined by a line integral on a curve $F(A, X)=f$.

[^7]:    ${ }^{10}$ For a given $f \in(0, \bar{f})$, if a measure of non-participating firms approaches zero (i.e., the likelihood of takeover approaches one), Stage-1 price $q(f)$ approaches the weighted average of $\widehat{\Pi}_{T}(f)$ and $\widehat{\Pi}_{B}(f)$. In this case, $\widehat{\Pi}_{T}(f)>(<) \widehat{\Pi}_{B}(f)$ implies target premia (discounts) and bidder discounts (premia). If $\widehat{\Pi}_{T}(f)=\widehat{\Pi}_{B}(f)$, then announcement returns are positive for targets and bidders for any positive likelihood of takeovers. However, this symmetry holds only under a special circumstance.
    ${ }^{11}$ Netter et al. (2011) report that U.S. public bidders made 8 takeovers on average from 1992 to 2009.

[^8]:    ${ }^{12}$ The following analogy may be useful. Observing a night sky, we say that stars are moving. Pointing out that the earth is moving does not invalidate that stars are moving. However, we need to take into account the movement of the earth to correctly interpret our observations of stars.

[^9]:    ${ }^{13}$ Letting $\widetilde{X} \equiv X^{\beta}, \widetilde{A} \equiv A^{\alpha}$ and $\widetilde{F}(\widetilde{A}, \widetilde{X})=\widetilde{A} \widetilde{X}$, this result can be reinterpreted as changes in the distributions of factors.

[^10]:    ${ }^{14}$ Note that $\beta$ can be smaller than $\alpha$. The condition $\beta \geq \frac{\alpha}{1+\alpha}$ is equivalent to $\alpha \leq \frac{\beta}{1-\beta}$ for $\beta<1$, or $\beta \geq 1$. Therefore, $\beta \geq 1$ is sufficient (i.e., no decreasing returns in non-tradeable $X$ ). When $\beta<1$, an upper bound on $\left.\alpha, \frac{\beta}{1-\beta} \in \overline{(0}, \infty\right)$, increases in $\beta$ and takes one for $\beta=\frac{1}{2}$.
    ${ }^{15}$ For a small $\beta$, the matching does not respond much to changes in $c$, and the latter effect dominates.

