Automation and Unemployment in a Fair–Wage Model*

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Abstract

This paper introduces a psychological aspect of human labor into a task-based model of production automation. Unlike machines, workers' productivity is affected by whether they are fairly treated by their employers. Wages are thus raised over the market-clearing level to elicit workers' effort, and unemployment inevitably arises. A key feature of our model is to allow for heterogeneity across tasks in regard to how much workers' effort contributes to labor productivity, so that wages and labor employment levels are not uniform across tasks. In this setting, we show that (i) the progress of automation takes jobs in each level of tasks but may or may not decrease the aggregate labor demand depending on the wage settings in the tasks hiring human labor; (ii) the subsidy for machine use is beneficial in terms of welfare even if it worsens unemployment; and (iii) the subsidy for labor employment enhances welfare but unemployment benefits do not, while both policies increase unemployment.

Keywords: Automation, Fair wage, Unemployment.

JEL Classification Numbers: E12, E23, E24.

1 Introduction

> *We are being afflicted with a new disease of which some readers may not have heard the name, but of which they will hear a great deal in the years to come—namely, technological unemployment. (Keynes, 1933, p. 364)*

The recent rapid development of production technology revives Keynes's worries, as we once experienced during the Luddite movement started by the Industrial Revolution in the early 19th century. Now more than ever, manufacturing machines are being installed in production

*We would like to thank Yuichi Fukuta, Tetsugen Haruyama, Ken-ichi Hashimoto, Shinsuke Ikeda, Masahiko Shibamoto, Ken Tabata, and seminar participants at Kobe University and Osaka University for their helpful comments and suggestions. This research is financially supported by JSPS KAKENHI [grant numbers 19K01767, 20K01669], the Institute of Economic Research at Aoyama Gakuin University and the Program of the Joint Usage/Research Center for Behavioral Economics at the Institute of Social and Economic Research (ISER), Osaka University.

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processes. An influential work by Frey and Osborne (2017) stresses that approximately 47% of total US labor employment is potentially automatable within two decades. This sparks a debate about whether automation advances, especially by artificial intelligence and industrial robots, legitimately take jobs away from human labor. Against the estimates of Frey and Osborne, for example, Arntz et al. (2017) point out that the substitutability of US jobs to machines sizably declines by taking into consideration the heterogeneity of tasks within an occupation. However, most estimates ignore wage responses to changes in employment conditions. To accurately access overall effects on employment and welfare, we develop a general equilibrium model with Keynes's technological unemployment, as well as production automation, and show that automation advances may or may not decrease the aggregate labor demand.

A useful framework for analyzing production automation is a task-based model proposed by Zeira (1998) and Acemoglu and Restrepo (2018, 2019). In their setting, final output is assembled by combining various job tasks.[1] Since human labor and industrial machines are assumed to be perfectly substitutable within a task, some tasks are performed by employing only human labor while others are fully automated, depending on the factor productivity and the factor prices. Notably, most of the task-based models suppose a full-employment situation a priori. In this study, we distinguish human labor from industrial machines by considering a psychology aspect about fairness in manner that is described by Akerlof (1982) and Akerlof and Yellen (1990). More precisely, the productivity of human labor is affected by whether workers are fairly treated by their employers. To increase worker's effort, wages are raised over the market-clearing level; thereby, unemployment inevitably arises.[2]

A key feature of our model is that it allows for heterogeneity across tasks in regard to how much workers' effort improves labor productivity. Consequently, wages vary across tasks; hence, labor employment levels are also task-dependent. Within this framework, we clarify that the progress of automation takes jobs in each level of tasks but may or may not depress aggregate labor demand; if the effectiveness of fairness on labor productivity increases (decreases) on average in the tasks hiring labor as automation is promoted, then wages exceed more (less) than the market-clearing level and unemployment thus becomes more (less) serious in the aggregate level. This ambiguous effect on aggregate employment of automation reconciles inconclusive empirical findings; some studies report no statistically significant impact, while others support an either positive or negative effect as to countries and sample periods.[3] Thus, our model uses

[1] Autor et al. (2003) classify job tasks into routine and non-routine cognitive and manual tasks.

[2] Numerous empirical studies support the idea that fairness concerns are associated with workers' performance and wage settings. In a field experiment, Breza et al. (2018) find that unfair pay disparity decreases workers' performance and attendance when co-workers' productivity is not easily observed. According to Kaur (2019), workers and employers recognize that violating fairness norms by wage cuts induces less work effort. Blinder and Choi (1990), Agell and Lundborg (1995), Bewley (1998, 1999), Fehr et al. (2009), and Fabiani et al. (2010) discuss the importance of fairness concerns for understanding labor markets phenomena such as downward wage rigidity. See Fehr and Gächter (2000) and Bewley (2005) for reviews.

[3] Based on data from 722 commuting zones in the United States from 1990 to 2007, Acemoglu and Restrepo (2020) find that advances in robotics reduce total employment, as well as wages. See Acemoglu et al. (2020) for a negative employment effect in France. Autor and Salomons (2018) use industry-level panel data covering 24

heterogeneity across tasks to explain one possible mechanism underlying the empirical evidence.

Heterogeneity is a critical element in the modern macroeconomic theory.[4] It is no exception in the context of production automation. Autor and Handel (2013) document the evidence that a character of tasks is statistically significant in explaining wage differences among occupations and within an occupation. Arntz et al. (2017) emphasize task heterogeneity within an occupation when assessing the automatability of jobs. Some studies indicate that contributions of work effort to productivity are different across tasks. According to Bloom et al. (2015) and Beckmann and Kräkel (2022), the effectiveness of effort on work performance depends on workplace conditions, including "empowerment," which means that workers are authorized to autonomously decide e.g., when to work and how to solve a task. Moreover, the level of "empowerment" varies across jobs. For example, Wood et al. (2004) report that service sectors adopt more intensively the empowerment than manufacturing sectors do. Based on these facts, we introduce a task-dependent ingredient. This ingredient makes the degree of productivity improvement by work effort non-uniform across tasks, so that wages and labor employment levels differ among tasks. In this economy, automation causes more (less) unemployment in the aggregate level when a task previously performed by human labor sets lower (higher) wages and hires more (less) workers than the social average.

Most studies do not explicitly contain unemployment in the analysis of production automation. For instance, Acemoglu and Autor (2011), Berg et al. (2018), and Acemoglu and Restrepo (2019) focus their attention on wage responses to automation under full employment with inelastic labor supply. Acemoglu and Restrepo (2018, 2020), Acemoglu et al. (2020), and Guerreiro et al. (2022) allow a labor-leisure choice to capture changes in the employment volume. There are some exceptions with regard to embedding unemployment. Prettner and Strulik (2020) build an R&D-driven growth model with two types of labor, namely, low- and high-skilled labor, and focus on an empirically plausible case in which industrial robots are substituted for low-skilled labor easier than for high-skilled labor. Furthermore, in their supplementary material, the authors extend the baseline model by incorporating fair wages. In an equilibrium in which low-skilled labor remains unemployed but high-skilled labor achieves full employment, automa-

developed countries from 1970 to 2007 to demonstrate that automation fully increases aggregate employment while displacing employment in industry levels. Mann and Püttmann (2021) classify utility patents granted from 1976 to 2014 in the United States and adopt them as a measure of automation. They report a positive impact on overall employment, accompanied by more employment in the non-manufacturing sector; however, they find no statistically significant effects in the manufacturing sector. For 27 European countries between 1990 and 2010, Gregory et al. (2020) document that the technologies replacing routine tasks created more jobs than they destroyed. Using 14 industries in 17 countries from 1993 to 2007, Graetz and Michaels (2018) present evidence that industrial robots do not significantly affect the total hours worked but do decrease the employment share of low-skilled workers. Dauth et al. (2021) find that new job creation in services by robot adoption offsets job losses in manufacturing for industrial and regional data in Germany in the period ranging from 1994 to 2014. Aghion et al. (2022) provide a comprehensive survey. See Autor et al. (2003), Goos and Manning (2007), Autor and Handel (2013), Goos et al. (2014), and Michaels et al. (2014) for polarized impacts depending on tasks, jobs, and skill and education levels.

[4]An outstanding example is the heterogeneous agent New Keynesian (HANK) model with uninsurable income shocks and borrowing limits, which is a workhorse of the monetary policy analysis (see e.g., Kaplan and Violante (2018) and Kaplan and Violante (2018)).

tion decreases the labor demand for low-skilled labor.[5] Cords and Prettner (2021) develop a model in which both high-skilled workers and low-skilled workers are unemployed not by fair wages but rather by search frictions in the labor market. In this situation, automation decreases low-skilled workers' employment but increases that of high-skilled workers; thus, it may or may not depress aggregate employment.[6] Pi and Fan (2021) develop a two-sector model with a labor union. They assume that both unskilled and skilled workers are employed in the final-good sector and that robots are manufactured solely by skilled labor. Impacts on individual and aggregate unemployment are shown to be ambiguous as to the production technology available in the robot manufacturing sector. To provide a new insight, we consider a heterogeneity across tasks rather than a skill heterogeneity among workers, which is vital in the existing studies.

More specifically, we complement the literature in three aspects. First, our model is based on a task-based approach, while the work of Prettner and Strulik (2020), Cords and Prettner (2021), and Pi and Fan (2021) is not. Those authors look at a skill heterogeneity, whereas we consider an environment where, even within the same occupation and industry, individuals receive different wages across tasks due to their different effort levels induced by fairness. We point out another root that relates unemployment to production automation—automation takes jobs in each level of tasks but may or may not cause more unemployment in the aggregate level owing to the fair wage settings in the tasks operated by human labor. Second, our model is tractable enough to derive a closed-form solution for transitional dynamics of the economy.[7] We can analytically prove the welfare effects of the subsidy for machine use, the subsidy for labor employment, and unemployment benefits, and we can compare these relative impacts. Third, our fair-wage theory offers a different perspective from that of job search models (e.g., Guimarães and Mazeda Gil 2019 and Cords and Prettner, 2021). These perspectives are mutually important. Actually, the integration of fair wages into the search framework helps explain various business cycle facts.[8]

We also contribute to the debate concerning robot taxes. Guerreiro et al. (2022) claim that a robot tax should be temporally imposed until workers adapt themselves to their new jobs. Gasteiger and Prettner (2020) stress that the robot tax involves an intergenerational income

[5]Despite the decreased labor demand, the unemployment level of low-skilled labor may be reduced. This is because automation increases the high-skilled labor's wages and induces workers to acquire more education, thereby diminishing the number of the low-skilled workers, i.e., the aggregate low-skilled labor supply.

[6]See also Guimarães and Mazeda Gil (2022), who shows a positive employment effect of automation in a search model, although their aim is rather to account for a recent decline in the labor share of income.

[7]Prettner and Strulik (2020) suppose a small-open economy in which the interest rate remains constant at an exogenous level and numerically calculate economic dynamics in response to redistribution policies from high-skilled labor to low-skilled labor when financed by either the labor income tax or the robot tax.

[8]As is well known, Shimer (2005) concludes that the labor search model alone is incapable of producing the observed fluctuations in unemployment and job vacancies. Michaillat (2012) embeds downward real wage rigidity and finds that unemployment attributed to search frictions decreases during recessions and consists of only a small fraction of aggregate unemployment. Similarly, Martin and Wang (2018) numerically show that the majority of unemployment, which arises in a steady state in the model, stems from the unemployment associated with fair wages rather than from search frictions. However, we view the two theories of unemployment as complementary. For example, Kuang and Wang (2017) and Martin and Wang (2018) report that fair wages help generate realistic volatilities of unemployment in a job search model. Vasilev (2021) demonstrates that the fair wage accounts for cyclical behavior in wages and the labor search captures unemployment volatility.

transfer from retirees to the working aged and leads to higher long-run welfare by encouraging savings. Our model indicates that the subsidy for machine use is rather desirable even if it causes more employment. The difference is due to productivity improvement induced by fairness concerns. In our model, the machine subsidy promotes capital accumulation, which increases the wage rate and elicits more work effort, which in turn enhances welfare through an improvement of labor productivity. In a similar manner, the subsidy for human labor employment is also beneficial; it improves welfare by directly inducing workers to exert more effort although the increase in the fair wage unambiguously exacerbates unemployment. The underlying mechanism for our beneficial labor subsidy is distinguished from the suggestions of Acemoglu et al. (2020), who propose a labor tax cut to remedy excessive production automation. As an alternative measure against unemployment, one may be concerned about the role of unemployment benefits. However, we find that unemployment benefits drive up fair wages while simultaneously increasing unemployment, so that the incentive to exert more effort remains unchanged; higher levels of unemployment deteriorate welfare. The reason for harmful unemployment benefits is distinguished from the findings of labor search models in which unemployment benefits disincentivize job search efforts (Pissarides, 1985).

The rest of the paper is organized as follows. Section 2 presents a task-based automation model of workers' fairness concerns about wages. Section 3 derives the equilibrium properties of the model and clarifies an effect of automation advances on unemployment. Sections 4 and 5 present policy implications. Section 4 shows that when financed by lump-sum taxes, the subsidy for machine use and the subsidy of labor employment improve welfare but unemployment benefits do not. Section 5 considers a situation where lump-sum taxes are unavailable and finds that the labor subsidy is welfare-superior to the machine subsidy. Section 6 concludes the paper.

2 The Model

The analytical framework is a hybrid between the task-based model of production automation by Acemoglu and Autor (2011) and the fair-wage model of unemployment by Akerlof (1982) and Akerlof and Yellen (1990). Production is performed by combining various tasks, each of which employs either machines or workers. Hence, machines will take jobs away from workers at each level of tasks. However, the aggregate labor demand may or may not decrease because the wage rate changes simultaneously. To examine a general-equilibrium effect of automation, we introduce a wage adjustment governed by a fair-wage setting. Unlike machines, workers care about whether they are fairly treated by their employers, and they reciprocate with higher work effort when higher wages are paid relative to what they perceive as a fair wage. The fairness concern induces firms to raise wage rates above the market-clearing level; thus, unemployment persists. A key feature of our model is that the markup of wages over the fair wage differs across tasks due to the different degree of labor productivity improvement induced by workers' effort. Within this framework of heterogeneous wage settings, we examine the effect of automation on aggregate labor employment and welfare.

2.1 Household

There exists a mass one of infinitely-lived households. Each household is willing to inelastically supply a unit of labor to one of production tasks, but it may not be realized due to wage stickiness. Hence, the aggregate labor demand L_t may be less than the full-employment level as follows:

$$L_t = \int_0^1 l_t(j)\mathrm{d}j \; (\leq 1), \tag{1}$$

where $l_t(j)$ denotes labor demand in task j at time t, and we normalize a continuous variety of production tasks to be unity.

[Figure 1 about here]

Each household chooses a level of work effort, which positively contributes to labor productivity but yields disutility. As described in Figure 1, the disutility from putting in effort depends on a reference level, $e_t^F(j)$, and is measured by the distance of effort from it as follows:

$$\left[e_t(j) - e_t^F(j)\right]^2,$$

where $e_t(j)$ represents the effort level toward task j at time t, and $e_t^F(j)$ is called the fair level of effort. This formulation means that owing to fairness concern, not only higher effort but also lower effort than the fair level $e_t^F(j)$ increases disutility, therefore implying the optimal level of effort to be $e_t(j) = e_t^F(j)$ (see de la Croix et al. (2009), Raurich and Sorolla (2014) for this formulation). We assume that the fair level of effort in task j is affected by the wage payment in the relevant task, $w_t(j)$, relative to the social norm of labor income, ω_t^F, which is called the fair wage—

$$e_t^F(j) = w_t(j) - \omega_t^F \tag{2}$$

—and that the fair wage is given by the social average of wage income and unemployment benefit, b_t:

$$\omega_t^F = \int_0^1 w_t(j)l_t(j)\mathrm{d}j + b_t(1 - L_t). \tag{3}$$

According to (2), when the higher wage over the social average is paid, workers think that they are being fairly treated and their fair level of effort rises, thereby exerting the more work effort.[9]

[9]de la Croix et al. (2009) allow non-linearity in (2) as follows:

$$e_t^F = \frac{w_t(j)^\psi - (\omega_t^F)^\psi}{\psi}, \quad \psi > 0,$$

In the presence of competitive insurance markets, the risks arising from wage differentials across tasks and unemployment are perfectly diversified among households (Collard and de la Croix, 2000; Danthine and Donaldson, 1990; Raurich and Sorolla, 2014). While the employed in task j earns wage income $w_t(j)$, and the unemployed receive unemployment benefit b_t, they in advance take out insurance to avoid the income risks and eventually obtain the same level of income net of insurance payments and receipts (see Appendix A for a proof). All households thus face an identical budget constraint as follows:

$$\dot{K}_t = r_t K_t + \int_0^1 w_t(j) l_t(j) \mathrm{d}j + b_t(1 - L_t) - c_t - \tau_t,$$

where K_t, r_t, c_t, and τ_t are ownership of machines, the interest rate, consumption, and a lump-sum tax, respectively. Being subject to this budget constraint and taking the $e_t^F(j)$ in (2) as given, a household maximizes lifetime utility as follows:

$$U_0 = \int_0^\infty \left[u(c_t) - \int_0^1 d_t(j) \left[e_t(j) - e_t^F(j) \right]^2 \mathrm{d}j \right] \mathrm{e}^{-\rho t} \mathrm{d}t, \tag{4}$$

with $u'(c_t) > 0$, $u''(c_t) < 0$, and $\rho > 0$. $d_t(j)$ is a dummy variable that takes 1 if employed in task j and takes 0 otherwise. The optimality conditions of utility maximization are as follows:

$$\eta(c_t) \frac{\dot{c}_t}{c_t} = r_t - \rho, \tag{5}$$

$$e_t(j) = w_t(j) - \omega_t^F, \tag{6}$$

along with the transversality condition, where $\eta(c_t) \equiv -u''(c_t) c_t / u'(c_t) > 0$.

2.2 Production

Following Acemoglu and Autor (2011) and Acemoglu and Restrepo (2018, 2019), a representative firm assembles output Y_t by combining various tasks $y_t(j)$ as follows:

$$Y_t = \exp\left[\int_0^1 \ln y_t(j) \mathrm{d}j \right]. \tag{7}$$

which is reduced to (2) when $\psi = 1$. Raurich and Sorolla (2014) define the fair wage by the discount average of past, as well as current labor income, as follows:

$$\omega_t^F = \delta \int_{-\infty}^t \left[\int_0^1 w_s(j) l_s(j) \mathrm{d}j + b_s(1 - L_s) \right] \mathrm{e}^{-\delta(t-s)} \mathrm{d}s, \qquad \delta > 0,$$

which is identical to (3) in steady state. If we consider these more general formulations, the main implications of this paper are not altered.

Each task is performed by using machine $k_t(j)$ and labor $l_t(j)$ as follows:

$$y_t(j) = \theta^K(j)k_t(j) + \theta_t^L(j)l_t(j),$$

where $\theta^K(j)$ and $\theta_t^L(j)$ measure productivities of each input. The machine productivity $\theta^K(j)$ is time-invariant but varies across tasks. We position $\theta^K(j)$ such that it continuously declines as the index j rises as follows:

$$\theta^{K\prime}(j) < 0, \quad \theta^K(1) \geq 0.$$

The labor productivity $\theta_t^L(j)$ is improved by work effort. In particular, we consider that the degree of productivity improvement by effort differs across tasks, as in Meckl (2001, 2004):

$$\theta_t^L(j) = e_t(j)^{\phi(j)}, \quad \text{with} \quad 0 < \phi(j) < 1, \quad \phi'(j) \gtrless 0, \tag{8}$$

where the parameter $\phi(j)$ is task-dependent and measures the contribution of effort to labor productivity. In a task with a higher $\phi(j)$, work effort is more effective in improving the productivity. Thus, the relative productivity between the two inputs in each task, i.e., comparative advantage, endogenously changes depending on the effort level chosen by employees.

Owing to the linearity of task technology, each task is performed by employing either of the two inputs. As we will formally offer the precise condition below, we assume the following:

Assumption 1. *A machine (labor) has comparative advantage in lower (higher)-indexed tasks.*

Let I_t be the threshold such that the firm conducts task $j \leq (>) I_t$ by using machines (labor). With flexible prices, machines and labor are employed at least in some tasks, implying that I_t lies between 0 and 1 in equilibrium. As we will see later, this property holds even in the present fair-wage model.

The firm knows the workers' effort function (6), in which the effort increases with the wage payment $w_t(j)$ but takes the social norm of fair wage, ω_t^F, as given. From (6)–(8) and assumption 1, the profit maximization problem is given by the following:

$$\begin{aligned}\max_{k_t(j), l_t(j), I_t, w_t(j)} \quad & \exp\left[\int_0^{I_t} \ln \theta^K(j)k_t(j)\mathrm{d}j + \int_{I_t}^1 \ln \theta_t^L(j)l_t(j)\mathrm{d}j\right] \\ & - (1 - s^K)r_t \int_0^{I_t} k_t(j)\mathrm{d}j - (1 - s^L) \int_{I_t}^1 w_t(j)l_t(j)\mathrm{d}j,\end{aligned}$$

where s^K and s^L denote the subsidy rates, and the depreciation of machines is assumed away, for simplicity.

The optimality conditions of profit maximization are as follows:

$$k_t(j) = \frac{Y_t}{(1-s^K)r_t} \quad \text{for } j \in [0, I_t], \tag{9}$$

$$l_t(j) = \frac{Y_t}{(1-s^L)w_t(j)} \quad \text{for } j \in (I_t, 1], \tag{10}$$

$$\frac{(1-s^K)r_t/\theta^K(I_t)}{(1-s^L)w_t(I_t)/\theta_t^L(I_t)} = 1, \tag{11}$$

$$\frac{\partial \ln \theta_t^L(j)}{\partial \ln e_t(j)} \frac{\partial \ln e_t(j)}{\partial \ln w_t(j)} = 1. \tag{12}$$

The first and second equations provide the factor demand function for each input. The third equation indicates that the threshold I_t is determined at the point at which the factor price of machines equals that of labor in an efficiency unit net of the subsidy. The last equation is known as the Solow (1979) condition and, in combination with the effort function (6) and the labor productivity function (8), solves $w_t(j)$, $e_t(j)$, and $\theta_t^L(j)$ as a function of the fair wage:[10]

$$w_t(j) = \frac{1}{1-\phi(j)}\omega_t^F, \quad e_t(j) = \frac{\phi(j)}{1-\phi(j)}\omega_t^F, \quad \theta_t^L(j) = \left[\frac{\phi(j)}{1-\phi(j)}\omega_t^F\right]^{\phi(j)}. \tag{13}$$

As long as $\phi'(j) \neq 0$, the $w_t(j)$, $e_t(j)$, and $\theta_t^L(j)$ differ across tasks. As seen from the first equation in (13), each wage is a markup over the fair wage. With higher (lower) $\phi(j)$, labor productivity is more (less) improved by work effort; thus, the wage is set higher (lower) to induce a higher effort.

The $w_t(j)$ and $\theta_t^L(j)$ in (13) rewrite the interest rate relative to the wage rate in an efficiency unit net of the subsidy as follows:

$$\frac{(1-s^K)r_t/\theta^K(j)}{(1-s^L)w_t(j)/\theta_t^L(j)} = \frac{(1-s^K)r_t}{1-s^L} \frac{\phi(j)^{\phi(j)}\left[1-\phi(j)\right]^{1-\phi(j)}}{\theta^K(j)\left(\omega_t^F\right)^{1-\phi(j)}},$$

where $(1-s^K)r_t/(1-s^L)$ and ω_t^F are independent of the index j. Assumption 1 is equivalent to postulating that this relative marginal cost is strictly increasing with respect to the index j; i.e., the mathematical condition that ensures assumption 1 is given by the following:

Assumption 1. $$-\frac{\theta^{K\prime}(j)}{\theta^K(j)} + \frac{\phi'(j)}{\phi(j)} \ln \theta_t^L(j) > 0.$$

[10]The Solow condition means that the optimizing firm chooses the wage rate that minimizes the cost per unit of effective labor, that is,

$$\min_{w_t(j)} \frac{(1-s^L)w_t(j)}{\theta_t^L(j)}, \quad \text{subject to (6) and (8).}$$

If we consider $\theta_t^L(j) = e_t(j)$ as is usually assumed in the literature, the Solow condition is simplified as $\partial \ln e_t(j)/\partial \ln w_t(j) = 1$. See Solow (1979).

Throughout this paper, we suppose that this condition holds before and after policy changes.

3 Equilibrium

3.1 Market Equilibrium Conditions

We now see market equilibrium conditions. The government finances unemployment benefits and subsidies to the firm by imposing lump-sum taxes on households as follows:

$$b_t(1 - L_t) + s^K r_t \int_0^{I_t} k_t(j)\mathrm{d}j + s^L \int_{I_t}^1 w_t(j)l_t(j)\mathrm{d}j = \tau_t. \tag{14}$$

Since the demand for machines in (9) is symmetric across tasks, the equilibrium condition in the machine market requires the following:

$$k_t(j) = \frac{K_t}{I_t} \quad \text{for } j \in [0, I_t]. \tag{15}$$

In contrast, the labor demand allocated to each task is not uniform since the wage setting in (13) is task-dependent as long as $\phi'(j) \neq 0$. To show this, we substitute the $w_t(j)$ in (13) into the labor demand function (10) and integrate the $l_t(j)$ by (1) to obtain the following:

$$\begin{aligned} l_t(j) &= \Phi_t(j)\frac{L_t}{1 - I_t} \quad \text{for } j \in (I_t, 1], \\ \text{where} \quad \Phi_t(j) &\equiv \frac{(1 - I_t)\,[1 - \phi(j)]}{\int_{I_t}^1 [1 - \phi(i)]\,\mathrm{d}i}(> 0), \qquad \int_{I_t}^1 \Phi_t(j)\mathrm{d}j = 1. \end{aligned} \tag{16}$$

The weight function $\Phi_t(j)$ governs the labor allocation to each task. In a task with a higher $\phi(j)$ than average, which implies a lower $\Phi_t(j)$, the wage rate is set relatively higher and the labor demand is lower than the average. Note that if $\phi'(j) = 0$, the wage rates are identical among tasks, and the labor demand is uniformly allocated as follows: $l_t(j) = L_t/(1 - I_t)$ for $j \in (I_t, 1]$.

Using the $\theta_t^L(j)$ in (13), the $k_t(j)$ in (15), and the $l_t(j)$ in (16), we can rewrite the production function (7) as the Cobb-Douglas form as follows:

$$\begin{aligned} Y_t &= A_t K_t^{I_t} L_t^{1-I_t}, \\ \text{where} \quad A_t = \underset{(+)}{A}(\omega_t^F, \underset{(?)}{I_t}) &\equiv \frac{\exp\left\{\int_0^{I_t} \ln\theta^K(j)\mathrm{d}j + \int_{I_t}^1 \ln\left[\frac{\phi(j)}{1-\phi(j)}\omega_t^F\right]^{\phi(j)} \Phi_t(j)\mathrm{d}j\right\}}{I_t^{I_t}(1 - I_t)^{1-I_t}}. \end{aligned} \tag{17}$$

The total factor productivity (TFP), A_t, and the capital share of income, I_t, are endogenously determined. A_t can be expressed as a function of ω_t^F and I_t, which has the following property:

Lemma 1. *The TFP in (17) is increasing with respect to the fair wage, but it may or may not be* *increasing with respect to the frontier of automation as follows:*

$$\frac{\partial A_t}{\partial \omega_t^F} > 0, \qquad \frac{\partial A_t}{\partial I_t} \gtreqless 0.$$

Proof. See Appendix B.

The higher fair wage induces the firm to set the higher wage, which improves labor productivity by encouraging more work effort. In contrast, automation advances do not always lead to a productivity improvement. Actually, the TFP decreases if the machine-labor ratio is large and/or if the degree of labor productivity improvement by work effort lowers as some tasks become automated ($\phi'(j) < 0$).

From the factor demand functions shown in (9) and (10), in which $k_t(j)$, $l_t(j)$, and Y_t are replaced by (15) through (17), the factor prices are represented by the following:

$$r_t = \frac{I_t A_t}{1 - s^K} \left(\frac{K_t}{L_t}\right)^{I_t - 1}, \tag{18}$$

$$w_t(j) = \frac{w_t}{\Phi_t(j)} \quad \text{for } j \in (I_t, 1], \tag{19}$$

$$w_t \equiv \frac{(1 - I_t) A_t}{1 - s^L} \left(\frac{K_t}{L_t}\right)^{I_t}, \tag{20}$$

where the weight function $\Phi_t(j)$ is given in (16). The interest rate equals the marginal product of machines net of the subsidy, whereas the average wage rate, w_t, equals that of the aggregate labor.

Each wage rate in (19) is either larger or smaller than the average wage rate depending on the level of $\phi(j)$ relative to the average, i.e., the level of $\Phi_t(j)$. By equating the $w_t(j)$ in (13) with that in (19), we can interpret the average wage rate as to be a markup over the fair wage as follows:

$$w_t = m(I_t)\omega_t^F, \qquad \text{where} \qquad m(I_t) \equiv \frac{1 - I_t}{\int_{I_t}^1 [1 - \phi(j)]\, \mathrm{d}j} (> 1). \tag{21}$$

$m(I_t)$ is the social average of markups and has the following property:

Lemma 2. *As the frontier of automation rises, the average wage markup over the fair wage, defined by (21), increases (decreases) if* $\phi'(j) > (<) 0$.

Proof. Differentiating the $m(I_t)$ in (21) with respect to I_t generates the following:

$$\frac{m'(I_t)}{m(I_t)} = \frac{\int_{I_t}^1 [\phi(j) - \phi(I_t)]\, \mathrm{d}j}{(1 - I_t) \int_{I_t}^1 [1 - \phi(j)]\, \mathrm{d}j} \begin{cases} > 0 & \text{if } \phi'(j) > 0, \\ = 0 & \text{if } \phi'(j) = 0, \\ < 0 & \text{if } \phi'(j) < 0. \end{cases} \tag{22}$$

The denominator on the right-hand side of the first equality is positive due to $0 < \phi(j) < 1$, whereas the numerator is ambiguous depending on the sign of $\phi'(j)$. $\square$

As the number of tasks employing machines increases, only the tasks with higher (lower) wage markups survive and thus the social average of markups rises (falls) if $\phi'(j) > (<)0$. In the case of $\phi'(j) = 0$, this effect disappears; thus, the average markup is constant and unaffected by the degree of automation.

3.2 Fair Wage and Labor Employment

We next examine the relation between the fair wage and labor employment. Suppose that unemployment benefit is a constant proportion of the average wage rate as follows:

Assumption 2. $b_t = \beta w_t, \quad 0 \leq \beta < 1.$

Applying the $l_t(j)$ in (16) and the $w_t(j)$ in (19) to the fair wage (3) and using (20) to eliminate w_t from the result, we obtain the following:

$$\omega_t^F = \omega^F(\underset{(+)}{K_t}, \underset{(+)}{L_t}, \underset{(?)}{I_t}; \underset{(+)}{s^L}, \underset{(+)}{\beta}) \equiv \frac{(1-I_t)A(\omega_t^F, I_t)}{1-s^L}\left(\frac{K_t}{L_t}\right)^{I_t}[L_t + \beta(1-L_t)]. \tag{23}$$

Lemma 3. *The fair wage function in (23) has the following properties:*

$$\frac{\partial \omega_t^F}{\partial K_t} > 0, \quad \frac{\partial \omega_t^F}{\partial L_t} > 0, \quad \frac{\partial \omega_t^F}{\partial I_t} \gtreqless 0, \quad \frac{\partial \omega_t^F}{\partial s^L} > 0, \quad \frac{\partial \omega_t^F}{\partial \beta} > 0.$$

Proof. See Appendix B.

This lemma states that (i) capital accumulation raises the wage rate, thereby increasing the fair wage; (ii) although more labor employment reduces the per capita wage rate, the fair wage rises through the increased number of employees; (iii) automation advances may or may not raise the fair wage because they decrease the labor share of income, $1 - I_t$, but increase the TFP when $\phi' > 0$ (see lemma 1); and (iv) both the labor employment subsidy and the unemployment benefit directly raise the fair wage.

Equation (23) is equivalent to $\omega_t^F = w_t[L_t + \beta(1-L_t)]$. Substituting this relation into the average wage rate in (21) determines aggregate labor demand as follows:

$$L_t = L(\underset{(?)}{I_t}; \underset{(-)}{\beta}) \equiv \frac{1}{(1-\beta)m(I_t)} - \frac{\beta}{1-\beta}. \tag{24}$$

The existence of unemployment, i.e., $1 - L_t > 0$, is always ensured since $m(I_t) > 1$. For L_t to be positive, we assume a sufficiently small β such that $\beta < 1/m(I_t)$. Note that full employment holds ($L_t = 1$) if workers' effort does not contribute to labor productivity ($\phi(j) = 0$ for $\forall j$).

Lemma 4. *The function for aggregate labor demand in (24) satisfies the following property:*

$$\frac{\partial L_t}{\partial I_t} \lesseqgtr 0 \ \ if \ \ \phi'(j) \gtreqless 0, \qquad \frac{\partial L_t}{\partial \beta} < 0.$$

Proof. See Appendix B.

This lemma demonstrates that (i) automation advances worsen (create) the aggregate labor demand if they increase (decrease) the productivity effect of work effort, i.e., if $\phi'(j) > (<)0$, and (ii) the unemployment benefit reduces the aggregate labor demand by increasing the fair wage (see Lemma 3).

With Lemmas 2 and 4, we can establish the following proposition:

Proposition 1. *As the frontier of automation rises, the average wage markup over the fair wage increases (decreases) and then aggregate labor demand falls (expands) if $\phi'(j) > (<)0$.*

Remark 1. *If there is no heterogeneity in the wage setting ($\phi'(j) = 0$), aggregate labor demand is constant and hence the progress of automation has no effect on aggregate labor demand.*

Although automation takes some production tasks away from human labor, the wage rates in turn respond to the labor market condition. We find that whether the progress of automation takes jobs in the aggregate level depends on the wage settings in the surviving tasks. If $\phi'(j) > 0$, then the average wage markup over the fair wage increases in the surviving tasks; consequently, automation advances make the unemployment issue more serious. In contrast, if $\phi'(j) < 0$, then automation creates employment in the aggregate level by lowering the social average markups.

3.3 The Frontier of Automation

Now we turn to the determination of the automation frontier, I_t. By applying the $\theta_t^L(j)$ in (13) and the factor prices (18)–(20) to the boundary condition (12), we have the following relation:

$$\frac{K_t}{L_t} = \frac{I_t \Phi_t(I_t)}{(1-I_t)\theta^K(I_t)} \left[\frac{\phi(I_t)}{1-\phi(I_t)} \omega_t^F \right]^{\phi(I_t)}, \quad \text{or equally,} \quad I_t = I\Big(\underset{(+)}{\frac{K_t}{L_t}}, \underset{(-)}{\omega_t^F} \Big), \tag{25}$$

which satisfies the following:

Lemma 5. *Production automation progresses with an increase in the machine-labor ratio but recedes with an increase in the fair wage.*

$$\frac{\partial I_t}{\partial (K_t/L_t)} > 0, \quad \frac{\partial I_t}{\partial \omega_t^F} < 0.$$

Proof. See Appendix B.

The larger machine-labor ratio, K_t/L_t, implies a lower interest rate relative to the wage rate and leads to an expansion of the number of tasks employing machines. In contrast, a higher fair wage ω_t^F encourages higher levels of effort and increases labor productivity (Lemma 1). These aspects are to workers' advantage over machines and thus discourage the use of machines.

In equilibrium, I_t exists between 0 and 1 for the following reason. Machines are necessarily used in certain places since the interest rate is fully flexible. Although the wage rate is set over the fair wage, the fair wage in (23) sufficiently falls toward 0 as I_t approaches 1. While the decrease in the fair wage simultaneously pushes down labor productivity, its decrease in speed is slower than that of the wage decline due to $0 < \phi(I_t) < 1$. As a result, labor is employed in at least some higher-indexed tasks.

3.4 Dynamics

With the production function (17), the equilibrium condition in the commodity market requires the following:

$$\dot{K}_t = A(\omega_t^F, I_t) K_t^{I_t} L_t^{1-I_t} - c_t. \tag{26}$$

The consumption dynamics (5) in which r_t is eliminated by (18) follows:

$$\eta(c_t)\frac{\dot{c}_t}{c_t} = \frac{I_t A(\omega_t^F, I_t)}{1-s^K}\left(\frac{K_t}{L_t}\right)^{I_t-1} - \rho. \tag{27}$$

These two equations, in which ω_t^F, L_t, and I_t are given by (23) through (25), formulate an autonomous dynamic system with respect to K_t and c_t. In Appendix C, we prove that the dynamic path toward a steady state is saddle-point stable.

4 Policy Implications

This section investigates the welfare effects of policy changes in s^K, s^L, and β around $s^K = s^L = \beta = 0$. In what follows, the steady-state value of each variable is denoted by an asterisk. This section intuitively explains the mechanism of policy effects. The formal proofs of the following arguments are provided in Appendices B and C.

Evaluating the Euler equation (27) in steady state, we can find that the machine-labor ratio, K^*/L^*, is determined so that the interest rate equals the time preference:

$$\frac{I^* A(\omega^{F*}, I^*)}{1-s^K}\left(\frac{K^*}{L^*}\right)^{I^*-1} = \rho, \qquad \text{or equally,} \qquad \frac{K^*}{L^*} = K(\underset{(+)}{\omega^{F*}}, \underset{(?)}{I^*}; \underset{(+)}{s^K}). \tag{28}$$

Lemma 6. *In steady state, the machine-labor ratio satisfies the following relations:*

$$\frac{\partial(K^*/L^*)}{\partial\omega^{F*}} > 0, \qquad \frac{\partial(K^*/L^*)}{\partial I^*} \gtreqless 0, \qquad \frac{\partial(K^*/L^*)}{\partial s^K} > 0.$$

Proof. See Appendix B.

For a given I^*, an increase in the fair wage heightens the intensity of machines by raising the TFP (see Lamma 1). Automation advances may hamper capital accumulation if they reduce the social average of work effort effectiveness, i.e., if $\phi'(j) < 0$. It is straightforward that the subsidy for machine use directly encourages capital accumulation.

The equilibrium relations represented by (23)–(25) hold also in a steady state. Together with the steady-state determination of the machine-labor ratio (28), they solve four undetermined steady-state variables $(K^*, \omega^{F*}, L^*, I^*)$ as a function of three policy instruments (s^K, s^L, β). Given these solutions, the capital dynamics (26) in which $\dot{K}_t = 0$ gives the steady-state consumption as follows:

$$c^* = c(\underset{(+)}{K^*}, \underset{(+)}{L^*}, \underset{(+)}{\omega^{F*}}, \underset{(?)}{I^*}) \equiv A(\omega^{F*}, I^*)(K^*)^{I^*}(L^*)^{1-I^*}. \tag{29}$$

Lemma 7. *The steady-state consumption has the following relations:*

$$\frac{\partial c^*}{\partial K^*} > 0, \qquad \frac{\partial c^*}{\partial L^*} > 0, \qquad \frac{\partial c^*}{\partial \omega^{F*}} > 0, \qquad \frac{\partial c^*}{\partial I^*} \gtreqless 0 \;\; \textit{if} \;\; \phi'(j) \gtreqless 0.$$

Proof. See Appendix B.

All other things being equal, a higher level of consumption is attained by the use of more production factors, K^* and L^*, and by a higher TFP increased by a higher fair wage ω^{F*} (Lemma 1). As shown in Lemma 2, the progress of automation does not always improve the TFP and hence may depress the steady-state consumption.

We can prove that the lifetime utility (4) is expressed as a function of c^* and K^*:

$$U_0 = U(\underset{(+)}{c^*}, \underset{(-)}{K^*}). \tag{30}$$

Lemma 8. *The lifetime utility increases with respect to the steady-state consumption but decreases with respect to the steady-state capital:*

$$\frac{\partial U_0}{\partial c^*} > 0, \qquad \frac{\partial U_0}{\partial K^*} < 0.$$

Proof. See Appendix C.

A higher c^* enhances the steady-state utility, while the accumulation of more K^* decreases the transitional consumption for a given level of c^* and harms the lifetime utility.

4.1 Subsidy for Machine Use

Let us first examine an effect of the subsidy for machine use. Governments in developed and developing counties have implemented various policies to utilize robots as a growth engine (see de Backer et al., 2018). An increase in s^K directly encourages capital accumulation per labor (Lemma 6). This raises the average wage rate and, in turn, the fair wage (Lemma 3), thereby heightening labor productivity and consumption (Lemmas 1 and 7). As a result, welfare is improved. An increase in capital accumulation simultaneously lowers the interest rate and accelerates automation (Lemma 5). While automation advances make unemployment more serious if $\phi'(j) > 0$ from Lemma 4, this possible harmful effect on c^* is definitely outweighed by the beneficial effect on c^* of both capital accumulation and productivity improvement. The overall effects are summarized as follows:

$$\frac{\hat{K}^*}{\hat{s}^K} > 0, \quad \frac{\hat{\omega}^{F*}}{\hat{s}^K} > 0, \quad \frac{\hat{L}^*}{\hat{s}^K} \lesseqgtr 0 \ \text{ if } \ \phi'(j) \gtreqless 0, \quad \frac{\hat{I}^*}{\hat{s}^K} > 0, \quad \frac{\hat{c}^*}{\hat{s}^K} > 0, \quad \frac{\hat{U}_0}{\hat{s}^K} > 0,$$

where we define the deviation of valuable x from an original steady state x^* to a new steady state x^{**} by the following:

$$\hat{x}^* \equiv x^{**} - x^*.$$

Proposition 2. *The subsidy for machine use promotes production automation, which decreases (increases) the aggregate labor employment if $\phi'(j) > (<)0$. However, welfare definitely improves even if labor employment declines.*

See Appendix C for the formal proof.

The welfare benefit of a capital subsidy essentially comes from productivity improvement that is either directly induced by a higher fair wage or indirectly induced by machine accumulation. This mechanism is distinguished from the analysis of distortionary capital taxation in a representative-agent framework with exogenous production technology (Chamley, 1981, 1985; Judd, 1987). Chamley (1981) demonstrates the welfare cost of capital taxation by using the second-order Taylor expansion around a non-distortionary state. Taking account of the endogenous labor supply, Chamley (1985) and Judd (1987) find that capital taxation remains costly. In the context of production automation, Guerreiro et al. (2022) point out that it is optimal to temporally impose a tax on machines until the skill adaptation of workers has been completed but ultimately abolish the tax in the long run. We do not deal with skill adaptation; rather, we consider unemployment. The consideration of unemployment leads to our conclusion that welfare is better off according to the machine subsidy rather than the machine taxation.

Remark 2. *In Chamley (1981, 1985) and Judd (1987), the machine subsidy/taxation has no first-order welfare effect in the case of full employment, which is the same as the case with* $\phi(j) = 0$ *in our context (see Appendix C).*

We must state that our result may alter when the machine tax involves income redistribution among individuals. In an overlapping-generations model, Gasteiger and Prettner (2020) point out a possibility that the machine tax increases the wage of the working aged who are substitutable for machines, thereby raising savings and welfare in the long run. Using a fair-wage model with low- and high-skilled labor, Prettner and Strulik (2020) stress that the income transfer to the low-skilled workers via the machine tax discourages education and leads to an increase in the existing high-skilled workers' wages, which lifts the perceived fair wage and depresses labor employment of the low-skilled worker. Acknowledging the importance of redistribution policies, we believe that our analysis is valuable in offering a new perspective for rationalizing the machine subsidy.

4.2 Subsidy for Labor Employment

We next analyze an impact of subsidizing human labor employment. While the labor subsidy is usually intended to stimulate labor employment, we will show that while it has the opposite effect, it nevertheless enhances welfare.

An increase in s^L directly drives up the fair wage (Lemma 3) and then increases the TFP (Lemma 1). The increased fair wage promotes capital accumulation (Lemma 6) but hampers automation (Lemma 5). These opposite effects have repercussions on each other; by lowering the interest rate, more capital induces automation (Lemma 5), which affects capital accumulation again (Lemma 6). Owing to these interactions, a total effect on K^*, I^*, and c^* is generically ambiguous. However, taking account of transitional dynamics, we can show that the positive effect of productivity improvement dominates other opposite effects in terms of welfare.

$$\frac{\hat{K}^*}{\hat{s}^L} \gtreqless 0, \quad \frac{\hat{\omega}^{F*}}{\hat{s}^L} > 0, \quad \frac{\hat{L}^*}{\hat{s}^L}\begin{cases} < 0 & \text{if } \phi'(j) \gtrless 0, \\ = 0 & \text{if } \phi'(j) = 0, \end{cases} \quad \frac{\hat{I}^*}{\hat{s}^L} \gtreqless 0 \text{ if } \phi'(j) \gtreqless 0, \quad \frac{\hat{c}^*}{\hat{s}^L} \gtreqless 0, \quad \frac{\hat{U}_0}{\hat{s}^L} > 0.$$

Our result in the dynamic setting can be interpreted as a generalization of Blumkin et al. (2020), who clarify the benefits of wage subsidies in a static fair-wage framework with no capital accumulation and no production automation.[11] On the premise that the US tax system is biased to labor, Acemoglu et al. (2020) propose a reduction in labor taxation toward the neutral level so as to remedy excessive production automation. In contrast, our result suggests that a positive labor subsidy is rather desirable in the presence of unemployment. Note that the wage subsidy

[11] In a laboratory experiment, Blumkin et al. (2020) demonstrate that social welfare is more improved by a wage subsidy made directly to workers than by that to employers, who are indifferent in the absence of workers' fairness concerns.

involves no income redistribution across workers in our representative-agent model with perfect insurance markets. This is distinguished from Prettner and Strulik (2020), who claim that the redistribution from high- to low-skilled workers, financed by the wage income tax, reduces the labor demand for low-skilled workers by heightening what they perceive to be a fair wage.

It is noteworthy to mention that against expectations, the subsidy for labor employment ends up depressing employment. Intuitively, this is due to an increase in the fair wage. To understand the reason in greater details, we first see the determination of automation in (25): I_t is positively influenced by K_t/L_t, whereas I_t is negatively related to ω_t^F by the power of $\phi(I_t)$. The difference of these impacts is thus $\frac{\partial(K_t/L_t)}{K_t/L_t} - \phi(I_t)\frac{\partial\omega_t^F}{\omega_t^F}$. In the steady state, we have $\frac{\partial(K_t/L_t)}{K_t/L_t} = \frac{1}{1-I^*}\frac{\partial A^*}{A^*}$ from the first equation in (28) and $\frac{\partial A^*}{A^*} = \int_{I^*}^1 \phi(j)\mathrm{d}j\frac{\partial\omega^{F*}}{\omega^{F*}}$ from (17). In the end, the impact on I^* is measured by the following:

$$\frac{\partial(K^*/L^*)}{K^*/L^*} - \phi(I^*)\frac{\partial\omega^{F*}}{\omega^{F*}} = \frac{\int_{I^*}^1[\phi(j)-\phi(I^*)]\mathrm{d}j}{1-I^*}\frac{\partial\omega^{F*}}{\omega^{F*}}\begin{cases} >0 & \text{if } \phi'(j)>0, \\ =0 & \text{if } \phi'(j)=0, \\ <0 & \text{if } \phi'(j)<0. \end{cases}$$

Hence, when $\phi'(j) > 0$, the positive effect through capital accumulation dominates the negative effect from the increased fair wage; consequently, I^* rises. From proposition 1, an increase in I^* increases the average markup and reduces employment. In contrast, when $\phi'(j) < 0$, the opposite happens—automation is scaled back and employment therefore decreases. In sum, an increase in s^L reduces employment irrespective of the sign of $\phi'(j)$. Note that these two effects are just canceled out and employment is unaffected when $\phi'(j) = 0$.

We can summarize the result in the following proposition (see Appendix C for the formal proof):

Proposition 3. *The subsidy for labor employment promotes (scales back) production automation if $\phi'(j) > (<)0$. However, welfare definitely improves even though labor employment unambiguously declines.*

Remark 3. *Similar to the machine subsidy, the labor subsidy has no first-order welfare impact in the situation with full employment, which is interpreted as $\phi(j) = 0$ in our context (see Appendix C).*

4.3 Unemployment Benefits

Policymakers may choose to enrich unemployment benefits instead of to subsidize labor employment when unemployment exists. However, unemployment benefits drive up the fair wage (Lemma 3) and cause more unemployment (Lemma 4), which eventually depresses the fair wage to the original level. For this reason, the equilibrium fair wage and labor productivity remain unchanged; however, less employment leads to less capital accumulation, both of which harm welfare.

$$\frac{\hat{K}^*}{\hat{\beta}} < 0, \quad \frac{\hat{\omega}^{F*}}{\hat{\beta}} = 0, \quad \frac{\hat{L}^*}{\hat{\beta}} < 0, \quad \frac{\hat{I}^*}{\hat{\beta}} = 0, \quad \frac{\hat{c}^*}{\hat{\beta}} < 0, \quad \frac{\hat{U}_0}{\hat{\beta}} < 0.$$

Proposition 4. *Unemployment benefits deteriorate employment and welfare, while keeping the degree of production automation unchanged.*

The labor subsidy and unemployment benefits both cause more unemployment, as seen from Propositions 3 and 4. However, the welfare impacts are opposite—the labor subsidy is beneficial, whereas the unemployment benefit is harmful due to lack of productivity improvement.

Remark 4. *There is no first-order welfare effect of the unemployment benefit if workers' effort does not contribute to labor productivity ($\phi(j) = 0$ for $\forall j$), or equivalently, if full employment holds (see Appendix C).*

As is well known, the theory of job search emphasizes that unemployment benefits lift the equilibrium wage rate, so that the number of vacancies decreases and unemployment increases (see, e.g., Pissarides, 1985). Although our underlying mechanisms are fairly different from those of the search theory, our analysis is not a substitute for but a complement to this theory. In particular, our model captures involuntary unemployment that arises from real wage rigidity, whereas job search theory accounts for frictional unemployment, according to Meckl (2004) and Martin and Wang (2018). When the progress of automation causes unemployment, both types of unemployment are probably involved. In fact, Kuang and Wang (2017), Martin and Wang (2018), and Vasilev (2021) demonstrate that the combination of labor searching and fair wages helps explain business cycle features, especially, the observed volatilities of unemployment and wages.

5 Policy Combination

The previous section supposes that each of the three policies is financed by a lump-sum tax. In this section, we investigate policy implications in the absence of lump-sum taxes. When policymakers implement one of those policies, they cannot help but decrease the other policies to meet the government's budget constraint. Hence, we have to consider an overall effect through policy combinations, by which an impact of a single policy may be offset or be amplified.

We numerically calculate the policy effects around the steady state with $s^K = s^L = \beta = 0$. Linearizing the government's budget equation (14) generates the following:

$$\hat{\beta} w^*(1 - L^*) + \hat{s}^K r^* K^* + \hat{s}^L w^* L^* = 0, \tag{31}$$

where $\hat{\tau} = 0$ is imposed since lump-sum taxes are assumed to be unavailable. The budget constraint indicates that, for instance, an increase in $\hat{s}^K$ leads to a decrease at least in either $\hat{s}^L$ or $\hat{\beta}$.

For computing the model, we set $\rho = 0.03$, which implies the steady-state interest rate to be 3% per annum (Christiano et al., 2005). The functional forms of $\theta^K(j)$ and $\phi(j)$ are respectively specified by the following:

$$\theta^K(j) = \gamma \exp(-j), \qquad \phi(j) = \begin{cases} \zeta \exp(j) & \text{if } \phi'(j) > 0, \\ \zeta \exp(-j) & \text{if } \phi'(j) < 0. \end{cases} \tag{32}$$

The values of γ and ζ are calculated as those that fit the labor share of income ($w^* L^*/Y^*$) and the unemployment rate ($1 - L^*$), respectively, on average, from 2000 to 2019 for each of five countries, namely, China, Germany, Japan, the United Kingdom (UK), and the United States of America (USA).[12] Table 1a lists the matched values regarding the labor share of income and the unemployment rate in the five target countries. Table 1b reports the calibrated parameters γ and ζ, which vary according to the sign of $\phi'(j)$. With these γ and ζ, we confirm that assumption 1 is satisfied around the steady state. Given the parameter values of ρ, γ, and ζ, we linearize the model and obtain an effect of policy interventions. See Appendix D for details of calibration.

[Table 1a about here]

[Table 1b about here]

Figures 2a–5b present numerical results.[13] The policy effect is evaluated on a percentage deviation from the original steady state as follows:

$$\tilde{I}^* \equiv \frac{\hat{I}^*}{I^*} \times 100, \qquad \tilde{L}^* \equiv \frac{\hat{L}^*}{L^*} \times 100, \qquad \tilde{U}_0 \equiv \frac{\hat{U}_0}{U_0} \times 100.$$

[Figure 2a about here]

[Figure 2b about here]

5.1 Cost of Machine Subsidy

Let us begin with policy combinations regarding the subsidy for machine use. From propositions 2–4, we know that on the premise that each policy instrument is financed by a lump-sum tax,

[12] As for the data on the labor share of income, we employ the share of labor compensation in GDP at current national prices (variable name: labsh), provided by the *Penn World Table* version 10.0 (Feenstra et al., 2015). The unemployment rate is drawn from the *World Economic Outlook Database* (version April 2022), compiled by the International Monetary Fund. According to the *World Robotics 2021 Industrial Robots* by the International Federation of Robotics, China, Japan, the United States of America, Korea, and Germany are the 5 largest markets for the robotic industry, as measured by the annual installations of industrial robots. Thus, we focus on these countries, substituting the United Kingdom for Korea.

[13] In Figures 2a–4b, the red line, the green line (round), the blue line (asterisk), the cyan line (square), and the magenta line (diamond) show the results for China, Germany, Japan, the UK, and the USA, respectively.

we have the following:

$$\frac{\hat{L}^*}{\hat{s}^K} \gtreqqless 0 \ \text{ if } \ \phi'(j) \lesseqqgtr 0, \qquad \frac{\hat{I}^*}{\hat{s}^K} > 0, \qquad \frac{\hat{U}_0}{\hat{s}^K} > 0;$$

$$\frac{\hat{L}^*}{\hat{s}^L} \begin{cases} < 0 & \text{if } \phi'(j) \gtrless 0, \\ = 0 & \text{if } \phi'(j) = 0, \end{cases} \qquad \frac{\hat{I}^*}{\hat{s}^L} \gtreqqless 0 \ \text{ if } \ \phi'(j) \lesseqqgtr 0, \qquad \frac{\hat{U}_0}{\hat{s}^L} > 0;$$

$$\frac{\hat{L}^*}{\hat{\beta}} < 0, \qquad \frac{\hat{I}^*}{\hat{\beta}} = 0, \qquad \frac{\hat{U}_0}{\hat{\beta}} < 0.$$

The machine subsidy covered by lump-sum taxes definitely improves welfare. However, if lump-sum taxes are unavailable, the machine subsidy involves a cut in either the labor subsidy and/or the unemployment benefit, thereby either offsetting or amplifying the welfare-enhancing effect.

We first explore a combination between the machine subsidy, s^K, and the labor subsidy, s^L. Figure 2a illustrates the case for $\phi'(j) > 0$. The top left panel indicates that an increase in s^K diminishes s^L subject to the government's budget constraint.[14] The effects on L^*, I^*, and U_0 demonstrate a conflict between an increase in s^K and a decrease in s^L. In the bottom left panel, a contractionary effect on employment of the machine subsidy outweighs the expansionary effect of the labor taxation (or a negative labor subsidy). Similarly, the machine subsidy dominantly encourages automation advances (top right panel). However, the impact of the machine subsidy is quantitatively offset in terms of welfare; consequently, welfare worsens by subsidizing machine use (bottom right panel). This welfare implication contrasts with proposition 2. In the absence of lump-sum taxes, the machine subsidy indirectly impacts the welfare cost by cutting down the labor subsidy. As seen in the bottom right panel of Figure 2b, the welfare-enhancing effect of the machine subsidy is also outweighed when $\phi'(j) < 0$. The positive effects on employment and automation of the machine subsidy are amplified by the labor taxation (Figure 2b, bottom left and top right panels) because there is no opposite effect on L^* and I^* between an increase in s^K and a decrease in s^L. In our setting, the relative impacts, described in Figures 2a and 2b, are robust for all five countries.

We next combine the machine subsidy, s^K, with the unemployment benefit, β. The results in Figures 3a–3b are visually similar for $\phi'(j) > 0$ and $\phi'(j) < 0$.[15] As for welfare, an increase in s^K and a decrease in β have no conflicting effects; thus, the machine subsidy is desirable independently of the sign of $\phi'(j)$ (see bottom right panels). Production automation also simply progresses by subsidizing machine use (top right panels) because β has no impact on I^*. Within our parameter values, L^* increases with s^K irrespective of the sign of $\phi'(j)$ (bottom left panels).

[Figure 3a about here]

[14] We can interpret a negative s^K (s^L) as the taxation on machine use (labor employment).

[15] Since we impose $s^K = s^L = \beta = \tau = 0$ in the initial steady state, β becomes negative for a positive s^K. We can keep β positive if τ and β are initially assumed to be positive.

[Figure 3b about here]

The calibration results demonstrate that the overall effect of the machine subsidy hinges heavily on its financing instrument; when financed by lump-sum taxes and a reduction in the unemployment benefit, the machine subsidy is beneficial. However, it is harmful in combination with labor taxation.

5.2 Choices of Policy Measures

As unemployment becomes more serious, policymakers may be confronted with a choice of employment measures. In the present context, these measures consist of the labor subsidy and the unemployment benefit. From propositions 3–4, it is apparent that the labor subsidy that is supported by decreasing the unemployment benefit is definitely welfare-improving (see bottom right panels in Figures 4a and 4b). Owing to the reduced unemployment benefit, employment expands even if labor employment is subsidized (see bottom left panels). Based on the welfare implications shown in Figures 2 and 4, we can see that the labor subsidy is superior to the other two policies when unemployment exists and lump-sum taxes are unavailable. It is worthwhile to mention the effects on employment and automation advances. Contrary to proposition 3, an increase in s^L eventually creates employment if such an increase is financed by downsizing the unemployment benefit. Since I^* is not influenced by β, an impact on I^* of s^L (top right panels) agrees with proposition 3.

[Figure 4a about here]

[Figure 4b about here]

What is the best policy mix when s^K, s^L, and β are simultaneously controlled? Figures 5a and 5b describe the case in which β is adjusted in response to exogenous shifts in s^K and s^L. We can regard these shifts as a combination of Figures 3 and 4. Since the impacts are qualitatively the same across the object countries, we present only the US case. Obviously, it is the best from the welfare viewpoint to subsidize both machine use and labor employment, irrespective of the sign of $\phi'(j)$ (see bottom right panels in Figures 5a and 5b).

[Figure 5a about here]

[Figure 5b about here]

6 Conclusion

A prevailing fear of automation is the advancing of job displacement from human labor to machines, thereby causing more unemployment. However, this view may or may not be supported by the existing empirical studies. In this paper, we introduce unemployment induced by workers' concern about fairness to clarify whether production automation worsens unemployment. Automation advances increase (decrease) unemployment in an economy in which the wage markup over the fair wage increases (decreases) on average for tasks that employ labor. We also demonstrate the welfare effects of policy interventions. The subsidies for production factors including machines and human labor enhance welfare by encouraging work effort; however, the unemployment benefit is harmful since it does not involve labor productivity improvement. Furthermore, it is shown that these policy implications rely on a financial scheme—in particular, the machine subsidy may deteriorate welfare when financed not by lump-sum taxation but rather by labor taxation; in other words, the labor subsidy is desirable over the machine subsidy in the absence of lump-sum taxes.

There are many directions for future research. According to Autor et al. (2003) and Acemoglu and Autor (2011), the substitutability of production factors from human labor to machines differs across tasks. The authors divide tasks into some categories according to, for example, routine manual and non-routine cognitive tasks. Since routine tasks are easier to be computerized, only some workers will be damaged by advancing job displacement. To analyze the resulting income differences across individuals, we have to incorporate skill heterogeneity into the model, as seen in Acemoglu and Autor (2011) and Acemoglu and Restrepo (2018). Second, we can precisely assess quantitative impacts on unemployment of automation advances by introducing various causes of unemployment, including not only fair wages but also search frictions (Cords and Prettner, 2021), skill adaptation (Guerreiro et al., 2022), etc. The third direction is to consider international spillovers of automation technology. The technological progress that arises in one country may influence the degree of production automation in other countries, thereby changing the terms of trade (Momoda et al., 2022). Imperfections in the labor market yield additional spillover effects that extend beyond countries.

Appendices

Appendix A: Diversification of Income Risks by Insurance

This appendix shows that all households share the same level of income when a competitive insurance market exists. The labor income net of insurance payments and receipts is as follows:

$$\begin{cases} w_t(j) - p_t(j) & \text{if employed in task } j, \\ b + q_t & \text{if unemployed,} \end{cases}$$

where $p_t(j)$ is an insurance payment when employed in task j, and q_t is an insurance receipt when unemployed. Since risk-averse households desire to smooth income across states of employment, the following holds:

$$w_t(j) - p_t(j) = w_t(i) - p_t(i) = b_t + q_t \quad \text{for } j \neq i. \tag{A.1}$$

Perfect competition in the insurance market requires a zero-profit condition of risk-neutral insurance companies as follows:

$$\int_0^1 p_t(j)l_t(j)\mathrm{d}j - q_t(1 - L_t) = 0. \tag{A.2}$$

The relations (A.1) and (A.2) solve $p_t(j)$ and q_t as follows:

$$p_t(j) = w_t(j) - \left[\int_0^1 w_t(i)l_t(i)\mathrm{d}i + b_t(1 - L_t)\right], \quad q_t = \int_0^1 w_t(i)l_t(i)\mathrm{d}i - b_t L_t.$$

As a result, all households gain an identical net income that equals the social average as follows:

$$w_t(j) - p_t(j) = b_t + q_t = \int_0^1 w_t(j)l_t(j)\mathrm{d}j + b_t(1 - L_t).$$

This relation provides the budget equation of households shown in the text, implying that the risks caused by wage differentials in tasks and unemployment are perfectly diversified across households.

Appendix B: Proof of Lemmas 1 and 3–7

This appendix provides the Proof of Lemmas 1 and 3–7. All derivatives are evaluated in the neighborhood of $s^K = 0$, $s^L = 0$, and $\beta = 0$.

Proof of lemma 1

Differentiate the A_t in (17) in which the $\Phi_t(j)$ is replaced by (16). This derives the following:

$$\frac{\partial A_t}{A_t} = \int_{I_t}^{1} \phi(j)\mathrm{d}j \frac{\partial \omega_t^F}{\omega_t^F} + \left\{ \frac{\int_{I_t}^{1} [\phi(j) - \phi(I_t)]\,\mathrm{d}j}{\int_{I_t}^{1} [1 - \phi(j)]\,\mathrm{d}j} - \ln \frac{I_t \Phi_t(I_t)}{(1 - I_t)\theta^K(I_t)} \left[\frac{\phi(I_t)}{1 - \phi(I_t)} \omega_t^F \right]^{\phi(I_t)} \right\} \partial I_t.$$

From the function for aggregate labor demand in (24) with $\beta = 0$, the following holds:

$$L_t = \frac{\int_{I_t}^{1} [1 - \phi(j)]\,\mathrm{d}j}{1 - I_t}, \quad \text{or equivalently,} \quad (1 - I_t)(1 - L_t) = \int_{I_t}^{1} \phi(j)\mathrm{d}j.$$

We use this relation to rewrite the coefficient of $\partial \omega_t^F / \omega_t^F$ and use $m'(I_t)/m(I_t)$ in (22) and the boundary condition in (25) to rewrite the coefficient of ∂I_t as follows:

$$\frac{\partial A_t}{A_t} = \underbrace{(1 - I_t)(1 - L_t)}_{(+)} \frac{\partial \omega_t^F}{\omega_t^F} + \underbrace{\left[(1 - I_t) \frac{m'(I_t)}{m(I_t)} - \ln \frac{K_t}{L_t} \right]}_{(?)} \partial I_t. \tag{B.1}$$

Thus, A_t increases with respect to ω_t^F but may or may not increase with respect to I_t because the signs of $m'(I_t)/m(I_t)$ and $\ln K_t/L_t$ are both ambiguous.

Proof of Lemma 3

Differentiating the fair wage function (23) around $s^L = 0$ and $\beta = 0$ and using (B.1) to eliminate $\partial A_t / A_t$ from the result, we obtain the following:

$$\underbrace{[(1 - I_t)L_t + I_t]}_{(+)} \frac{\partial \omega_t^F}{\omega_t^F} = \underbrace{I_t}_{(+)} \frac{\partial K_t}{K_t} + \underbrace{(1 - I_t)}_{(+)} \frac{\partial L_t}{L_t} - \underbrace{\left[\frac{1}{1 - I_t} - (1 - I_t) \frac{m'(I_t)}{m(I_t)} \right]}_{(?)} \partial I_t$$
$$+ \partial s^L + \underbrace{\frac{1 - L_t}{L_t}}_{(+)} \partial \beta.$$

ω_t^F is positively related to K_t, L_t, s^L, and β. The relation between ω_t^F and I_t is ambiguous because the sign of $m'(I_t)/m(I_t)$ is undetermined from (22).

Proof of Lemma 4

We differentiate the aggregate labor demand function (24) around $\beta = 0$ as follows:

$$\frac{\partial L_t}{L_t} = -\underbrace{\frac{m'(I_t)}{m(I_t)}}_{(?)}\partial I_t - \underbrace{\frac{1-L_t}{L_t}}_{(-)}\partial \beta. \tag{B.2}$$

Keeping in mind the property of $m'(I_t)/m(I_t)$ in (22), we can find that $\partial L_t/\partial I_t \lesseqgtr 0$ if $\phi'(j) \gtreqless 0$; whereas L_t certainly decreases with respect to β.

Proof of Lemma 5

Take the logarithm of the boundary condition (25) and use (16), in which $j = I_t$, to substitute for $\Phi_t(I_t)$. Differentiating the result yields the following:

$$\partial I_t = \underbrace{\frac{1}{\Gamma_t}}_{(+)} \frac{\partial\,(K_t/L_t)}{K_t/L_t} - \underbrace{\frac{\phi(I_t)}{\Gamma_t}}_{(-)} \frac{\partial \omega_t^F}{\omega_t^F},$$

$$\text{where} \quad \Gamma_t \equiv -\frac{\theta^{K\prime}(I_t)}{\theta^K(I_t)} + \frac{\phi'(I_t)}{\phi(I_t)} \ln \theta_t^L(I_t) + \frac{1}{I_t} + \frac{\Phi_t(I_t)}{1-I_t} > 0.$$

The sign of Γ_t is positive under Assumption 1. Thus, I_t is positively associated with K_t/L_t and negatively associated with ω_t^F.

Proof of Lemma 6

Differentiate the machine-labor ratio in (28) and eliminate $\partial A^*/A^*$ by using (B.1) evaluated in steady state. Then, we have the following:

$$\frac{\partial K^*/L^*}{K^*/L^*} = \underbrace{(1-L^*)}_{(+)} \frac{\partial \omega^{F*}}{\omega^{F*}} + \underbrace{\left[\frac{1}{I^*(1-I^*)} + \frac{m'(I^*)}{m(I^*)}\right]}_{(?)} \partial I^* + \underbrace{\frac{1}{1-I^*}}_{(+)} \partial s^K.$$

K^*/L^* is positively related to both ω^{F*} and s^K. However, the relation between K^*/L^* and I^* is ambiguous due to $m'(I^*)/m(I^*) \gtreqless 0$.

Proof of Lemma 7

Using (B.1) evaluated in the steady state to substitute for $\partial A^*/A^*$, the differentiation of the steady-state consumption in (29) generates the following:

$$\frac{\partial c^*}{c^*} = \underbrace{I^*}_{(+)} \frac{\partial K^*}{K^*} + \underbrace{(1-I^*)}_{(+)} \frac{\partial L^*}{L^*} + \underbrace{(1-I^*)(1-L^*)}_{(+)} \frac{\partial \omega^{F*}}{\omega^{F*}} + \underbrace{(1-I^*)\frac{m'(I^*)}{m(I^*)}}_{(?)} \partial I^*.$$

c^* increases with respect to K^*, L^*, and ω^{F*}. From the sign of $m'(I_t)/m(I_t)$ in (22), it holds that $\partial c^*/\partial I^* \gtreqless 0$ if $\phi'(j) \gtreqless 0$.

Appendix C: Dynamic Stability and Comparative Statics

This appendix examines the dynamic stability around a steady state with $s^K = 0$, $s^L = 0$, and $\beta = 0$ and conducts comparative statics regarding s^K, s^L, and β.

Dynamic Stability

Define the deviation of valuable x_t from an original steady state x^* by the following:

$$\hat{x}_t \equiv x_t - x^*.$$

The relations shown in (B.1) and (B.2) hold on any equilibrium path:

$$\frac{\hat{A}_t}{A^*} = (1-I^*)(1-L^*)\frac{\hat{\omega}_t^F}{\omega^{F*}} + \left[(1-I^*)\frac{m'(I^*)}{m(I^*)} - \ln\frac{K^*}{L^*}\right]\hat{I}_t, \tag{C.1}$$

$$\frac{\hat{L}_t}{L^*} = -\frac{m'(I^*)}{m(I^*)}\hat{I}_t - \frac{1-L^*}{L^*}\hat{\beta}. \tag{C.2}$$

The dynamics of a fair wage are obtained by differentiating (23) around $\beta = 0$ and applying the L-dynamics in (C.2) to the result as follows:

$$[(1-I^*)L^* + I^*]\frac{\hat{\omega}_t^F}{\omega^{F*}} = I^*\frac{\hat{K}_t}{K^*} - \frac{\hat{I}_t}{1-I^*} + \hat{s}^L + \frac{I^*(1-L^*)}{L^*}\hat{\beta}.$$

The frontier of automation evolves according to the following:

$$\left[-\frac{\theta^{K\prime}(I^*)}{\theta^K(I^*)} + \frac{\phi'(I^*)}{\phi(I^*)}\ln\theta^{L*}(I^*) + \frac{1}{I^*(1-I^*)}\right]\hat{I}_t = \frac{\hat{K}_t}{K^*} - \phi(I^*)\frac{\hat{\omega}_t^F}{\omega^{F*}} + \frac{1-L^*}{L^*}\hat{\beta},$$

which comes from linearizing the boundary condition (25) and replacing the L-dynamics by

(C.2). These two equations are solved to obtain the following:

$$\begin{aligned}\frac{\hat{\omega}_t^F}{\omega^{F*}} &= \frac{I^*}{\Omega^*}\frac{\hat{K}_t}{K^*} + \frac{1+\Theta^*}{\Omega^*}\hat{s}^L + \frac{I^*(1-L^*)}{\Omega^* L^*}\hat{\beta},\\ \frac{\hat{I}_t}{I^*(1-I^*)} &= \frac{\Lambda^*}{\Omega^*}\frac{\hat{K}_t}{K^*} - \frac{\phi(I^*)\Theta^*}{\Omega^*}\hat{s}^L + \frac{\Lambda^*(1-L^*)}{\Omega^* L^*}\hat{\beta},\end{aligned} \tag{C.3}$$

where

$$\begin{aligned}\Theta^* &\equiv \frac{1}{I^*(1-I^*)\left[-\frac{\theta^{K\prime}(I^*)}{\theta^K(I^*)} + \frac{\phi'(I^*)}{\phi(I^*)}\ln\theta^{L*}(I^*)\right]} > 0,\\ \Lambda^* &\equiv \{(1-I^*)L^* + I^*[1-\phi(I^*)]\}\,\Theta^* > 0,\\ \Omega^* &= (1-I^*)L^* + I^* + \Lambda^* > 0.\end{aligned}$$

Assumption 1 ensures that Θ^*, Λ^*, and Ω^* are positive.

Taking account of (C.1) through (C.3), we linearize the dynamic equations, (26) and (27), in the neighborhood of $K_t = K^*$ and $c_t = c^*$:

$$\begin{pmatrix}\dot{K}_t\\ \dot{c}_t\end{pmatrix} = M_1\begin{pmatrix}\hat{K}_t\\ \hat{c}_t\end{pmatrix} + M_2\begin{pmatrix}\hat{s}^K\\ \hat{s}^L\\ \hat{\beta}\end{pmatrix}, \tag{C.4}$$

where

$$\begin{aligned}M_1 &\equiv \begin{pmatrix}\frac{\rho(1+\Lambda^*)}{\Omega^*} & -1\\ -\frac{\rho^2 L^*}{\Omega^*\eta(c^*)} & 0\end{pmatrix},\\ M_2 &\equiv \begin{pmatrix}0 & \frac{(1+\Theta^*)(1-I^*)Y^*(1-L^*)}{\Omega^*} & -\frac{(L^*+\Lambda^*)(1-I^*)Y^*(1-L^*)}{\Omega^* L^*}\\ \frac{\rho c^*}{\eta(c^*)} & \left[\frac{1-L^*}{L^*} + \frac{m'(I^*)}{m(I^*)}(1-I^*)\Theta^*\right]\frac{\rho(1-I^*)Y^*}{\Omega^*\eta(c^*)} & -\frac{\rho(1-I^*)Y^*(1-L^*)}{\Omega^*\eta(c^*)}\end{pmatrix}.\end{aligned}$$

The characteristic roots of coefficient matrix M_1 are one negative $\lambda_1(< 0)$ and one positive $\lambda_2(> 0)$ because the determinant of M_1 is negative:

$$\lambda_1\lambda_2 = -\frac{\rho^2 L^*}{\Omega^*\eta(c^*)} < 0.$$

Since K_t is unjumpable and c_t is jumpable, the dynamic path is saddle-point stable.

Comparative Statics

When the valuable x shifts from an original steady state x^* to a new steady state x^{**}, we express its deviation by the following:

$$\hat{x}^* \equiv x^{**} - x^*.$$

The steady-state effect of policy changes is derived by solving (C.4) with $\dot{K}_t = \dot{c}_t = 0$:

$$\begin{aligned}
\frac{\hat{K}^*}{K^*} &= \underbrace{\frac{\Omega^*}{(1-I^*)L^*}}_{(+)}\hat{s}^K + \underbrace{\left[\frac{1-L^*}{L^*} + (1-I^*)\Theta^*\frac{m'(I^*)}{m(I^*)}\right]}_{(?)}\hat{s}^L - \underbrace{\frac{1-L^*}{L^*}}_{(-)}\hat{\beta}, \\
\frac{\hat{c}^*}{c^*} &= \underbrace{\frac{I^*(1+\Lambda^*)}{(1-I^*)L^*}}_{(+)}\hat{s}^K + \underbrace{\left[\frac{1-L^*}{L^*} + I^*(1-I^*)\Theta^*\frac{m'(I^*)}{m(I^*)}\right]}_{(?)}\hat{s}^L - \underbrace{\frac{1-L^*}{L^*}}_{(-)}\hat{\beta}.
\end{aligned} \tag{C.5}$$

Applying the abovementioned first equation to (C.2) and (C.3) provides the following:

$$\begin{aligned}
\frac{\hat{\omega}^{F*}}{\omega^{F*}} &= \underbrace{\frac{I^*}{(1-I^*)L^*}}_{(+)}\hat{s}^K + \underbrace{\frac{1}{L^*}}_{(+)}\hat{s}^L, \\
\frac{\hat{I}^*}{I^*(1-I^*)} &= \underbrace{\frac{\Omega^*}{(1-I^*)L^*}}_{(+)}\hat{s}^K + \underbrace{(1-I^*)\Theta^*\frac{m'(I^*)}{m(I^*)}}_{(?)}\hat{s}^L, \\
\frac{\hat{L}^*}{L^*} &= \underbrace{-\frac{\Omega^*}{(1-I^*)L^*}\frac{m'(I^*)}{m(I^*)}}_{(?)}\hat{s}^K \underbrace{-(1-I^*)\Theta^*\left[\frac{m'(I^*)}{m(I^*)}\right]^2}_{(-)}\hat{s}^L - \underbrace{\frac{1-L^*}{L^*}}_{(-)}\hat{\beta}.
\end{aligned} \tag{C.6}$$

Let us turn to the welfare analysis. The instantaneous utility linearized around the steady state is given by $u(c_t) = u(c^*) + u'(c^*)(c_t - c^*)$. Using the characteristic roots of matrix M_1, the consumption dynamics are characterized by $c_t - c^* = \lambda_2(K_0 - K^*)e^{\lambda_1 t}$, where $\lambda_1 < 0$ and $\lambda_2 > 0$. Keeping in mind these relations and $e_t(j) = e_t^F(j)$ in equilibrium, the lifetime utility (4) is reduced to the following:

$$U_0 = \frac{u(c^*)}{\rho} + \frac{u'(c^*)\lambda_2(K_0 - K^*)}{\rho - \lambda_1},$$

which satisfies the following:

$$\frac{\partial U_0}{\partial c^*} = \frac{u'(c^*)}{\rho} > 0, \qquad \frac{\partial U_0}{\partial K^*} = -\frac{u'(c^*)\lambda_2}{\rho - \lambda_1} < 0.$$

The first term on the right-hand side stands for the steady-state level of welfare, which increases with c^*. The second term represents the transitional effect, meaning that it is necessary to lower the transitional consumption to accumulate more K^*. This proves Lemma 8.

Totally differentiating U_0 and using (C.5) and $\lambda_1 + \lambda_2 = \rho(1+\Lambda^*)/\Omega^*$, we find the welfare effects to be as follows:

$$\hat{U}_0 = \underbrace{-\lambda_1 \Omega^* I^* B}_{(+)} \hat{s}^K + \underbrace{[\rho(1+\Theta^*)L^* - \lambda_1\Omega^*](1-I^*)B}_{(+)} \hat{s}^L \underbrace{- [\rho(L^* + \Lambda^*) - \lambda_1\Omega^*](1-I^*)B}_{(-)} \hat{\beta}, \tag{C.7}$$

where

$$B \equiv \frac{u'(c^*)Y^*(1-L^*)}{\rho(\rho-\lambda_1)\Omega^* L^*} > 0.$$

The subsidies for employment of machines and labor are both beneficial. In contrast, an increase in the unemployment benefit is harmful. When workers' effort does not influence labor productivity ($\phi(j) = 0$ for $\forall j$), then full employment holds ($L^* = 1$ and $B = 0$); thus, changes in s^K, s^L, and β have no welfare impact:[16]

$$\frac{\hat{U}_0}{\hat{s}^K} = \frac{\hat{U}_0}{\hat{s}^L} = \frac{\hat{U}_0}{\beta} = 0 \quad \text{if} \quad \phi(j) = 0 \quad \text{for} \quad \forall j.$$

Appendix D: Calibration Method

This appendix explains how to calibrate the model. We first derive the value of ζ in (32). From (17) and (20), the labor share of income, which is evaluated in the steady state with $s^L = 0$, satisfies the following:

$$\frac{w^* L^*}{Y^*} = 1 - I^*, \quad \text{or equivalently,} \quad I^* = 1 - \frac{w^* L^*}{Y^*}.$$

We match the labor share of income, $w^* L^*/Y^*$, to the actual data of the object countries, as listed in Table 1a. Hence, the steady-state value I^* is pinned down. Taking account of the definition of $m(I_t)$ in (21) and the functional form of $\phi(j)$ in (32), we obtain the steady-state employment from (24) in which $\beta = 0$:

$$L^* = \begin{cases} 1 - \frac{\zeta[\exp(1) - \exp(I^*)]}{1-I^*} & \text{if } \phi(j) = \zeta \exp(j), \\ 1 - \frac{\zeta[\exp(-I^*) - \exp(-1)]}{1-I^*} & \text{if } \phi(j) = \zeta \exp(-j). \end{cases}$$

For a given value of I^*, we choose the value of ζ so as to replicate the unemployment rate, $1 - L^*$, in the object countries, as reported in Table 1a.

We next determine the value of γ in (32). Since $\theta^K(j)$ and $\phi(j)$ are specified by (32), the TFP in (17) also depends on γ and ζ: $A_t = A(\omega_t^F, I_t; \gamma, \zeta)$. Setting $s^K = s^L = \beta = 0$ in (23) and

[16]The neutrality on welfare of a capital subsidy when $\phi(j) = 0$ is due to the linearization of our analysis in the neighborhood of $s^K = 0$. See, e.g., Ikeda and Gombi (1999) for the linearized welfare analysis. Chamley (1981) adopts the second-order Taylor expansion around steady state to show the welfare cost of capital taxation.

(28) provides the following steady-state relations:

$$\omega^{F*} = (1 - I^*)A(\omega^{F*}, I^*; \gamma, \zeta)\left(\frac{K^*}{L^*}\right)^{I^*} L^*, \quad I^* A(\omega^{F*}, I^*; \gamma, \zeta)\left(\frac{K^*}{L^*}\right)^{I^*-1} = \rho, \tag{D.1}$$

which solves ω^{F*} as follows:

$$\omega^{F*} = \frac{\rho L^*(1 - I^*)}{I^*}\left(\frac{K^*}{L^*}\right).$$

Eliminating K^*/L^* from this equation by using (25) and rearranging the result, we can express ω^{F*} and K^* as follows:

$$\omega^{F^*} = \omega^{F^*}(I^*, L^*; \gamma, \zeta, \rho) \equiv \left[\frac{\rho L^* \Phi(I^*)}{\theta^K(I^*)}\right]^{\frac{1}{1-\phi(I^*)}} \left[\frac{\phi(I^*)}{1 - \phi(I^*)}\right]^{\frac{\phi(I^*)}{1-\phi(I^*)}},$$

$$K^* = K^*(I^*, L^*; \gamma, \zeta, \rho) \equiv (\rho L^*)^{\frac{\phi(I^*)}{1-\phi(I^*)}} \frac{I^*}{1 - I^*}\left[\frac{\Phi^*(I^*)}{\theta^K(I^*)}\right]^{\frac{1}{1-\phi(I^*)}} \left[\frac{\phi(I^*)}{1 - \phi(I^*)}\right]^{\frac{\phi(I^*)}{1-\phi(I^*)}} L^*,$$

where $\theta^K(j)$ and $\phi(j)$ are given by (32). Substituting these two relations into the second equation in (D.1) obtains the following:

$$I^* A\left(\omega^{F*}(I^*, L^*; \gamma, \zeta, \rho), I^*; \gamma, \zeta\right)\left[\frac{K^*(I^*, L^*; \gamma, \zeta, \rho)}{L^*}\right]^{I^*-1} = \rho,$$

where ρ is set at 0.03; $I^*(= w^* L^*/Y^*)$ and L^* are chosen to replicate the actual data shown in Table 1a; and ζ has been already calculated as shown above. We use MATLAB (R2020a) to compute the value of γ from this non-linear equation. Table 1b summarizes the calibrated values of γ and ζ.

Given the values of ρ, γ, and ζ, we calculate the effect of policy changes as a percentage deviation from the initial steady state as follows:

$$\tilde{I}^* \equiv \frac{\hat{I}^*}{I^*} \times 100, \quad \tilde{L}^* \equiv \frac{\hat{L}^*}{L^*} \times 100, \quad \tilde{U}_0 \equiv \frac{\hat{U}_0}{U_0} \times 100,$$

by using (C.6) and (C.7) subject to the government's budget constraint (31).

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Table 1a: Matched values

	China	Germany	Japan	UK	USA
Labor share of income ($w^* L^* / Y^*$)	0.573	0.625	0.564	0.593	0.605
Unemployment rate ($1 - L^*$)	0.040	0.068	0.041	0.057	0.059

Table 1b: Calibrated parameters

	$\phi'(j) > 0$		$\phi'(j) < 0$	
	γ	ζ	γ	ζ
China	0.040817154527765	0.019331508234667	0.041320833381385	0.080538579314616
Germany	0.038415429201586	0.033642303887026	0.038181608681426	0.133057893008995
Japan	0.041087063396563	0.019734197096850	0.041705826332374	0.082959540414689
UK	0.039791747082266	0.027797384249975	0.040071568646525	0.113515787542850
USA	0.039329771033756	0.028928655760472	0.039432004572790	0.116726390413934

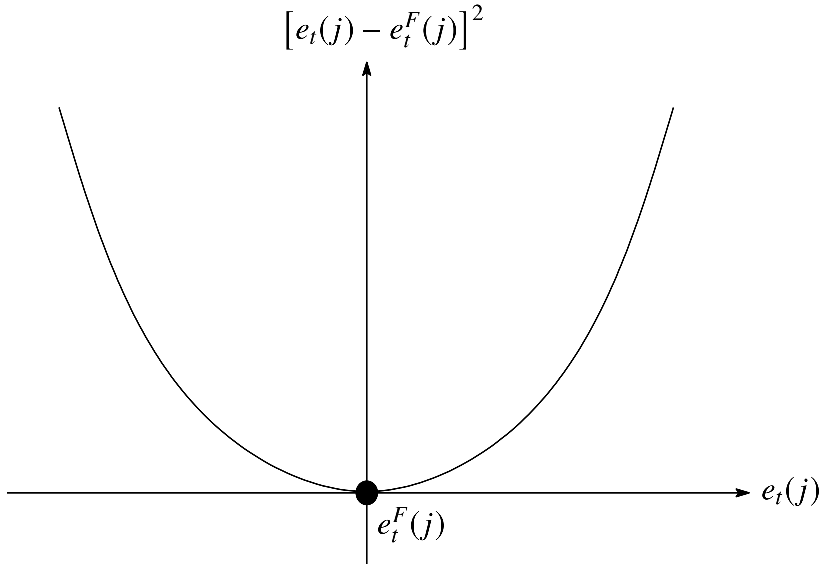

Figure 1: Disutility from work effort

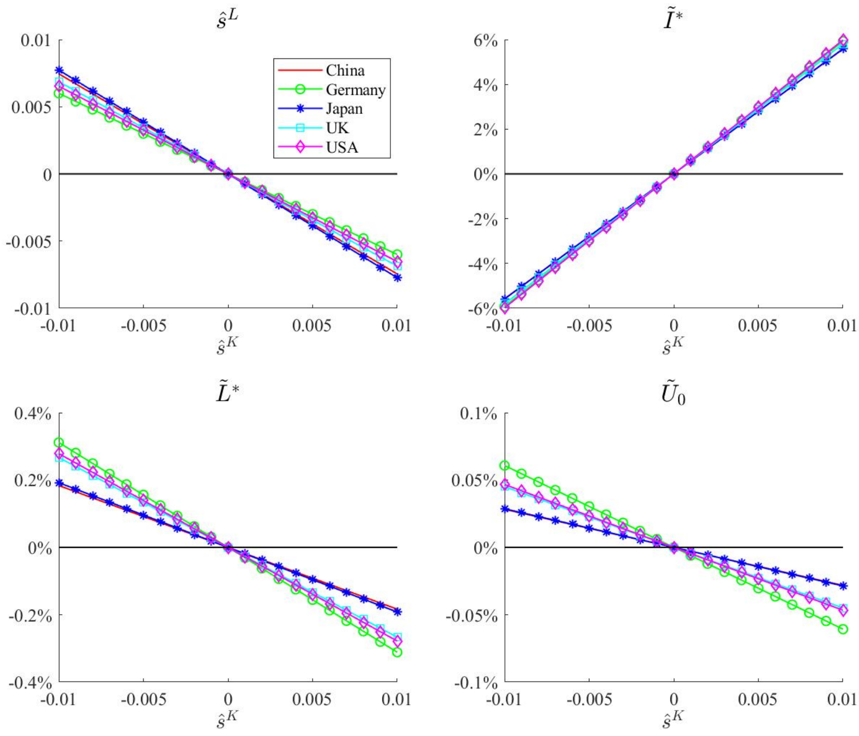

Figure 2a: Policy combination between s^K and s^L when $\phi'(j) > 0$

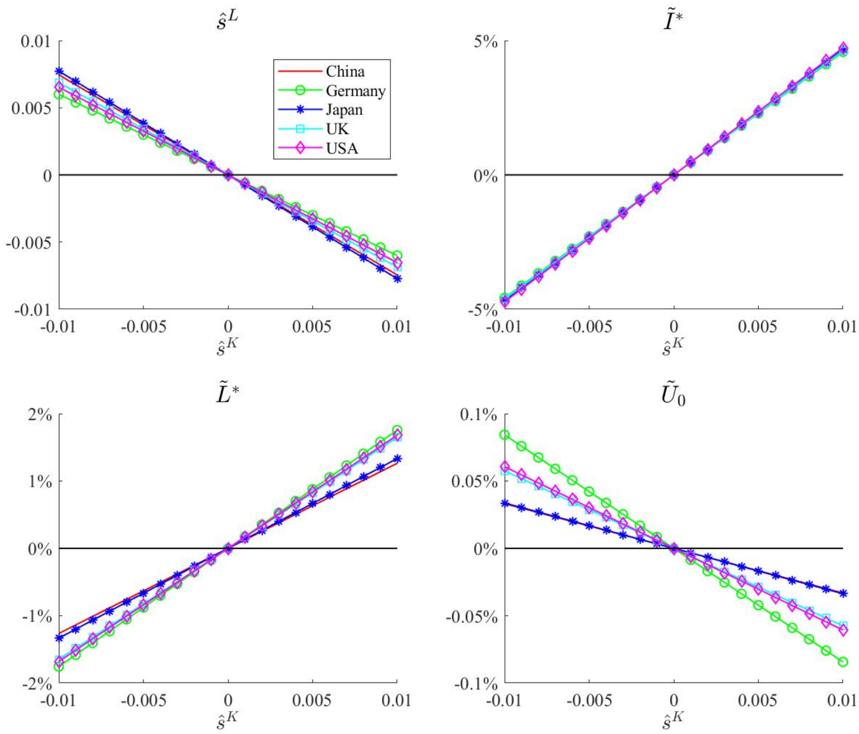

Figure 2b: Policy combination between s^K and s^L when $\phi'(j) < 0$

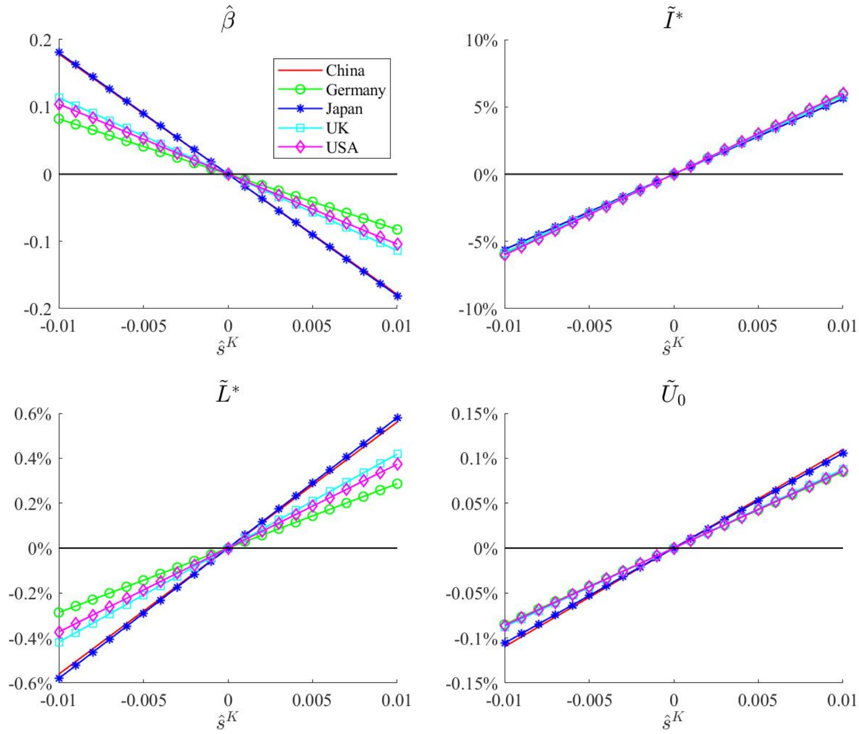

Figure 3a: Policy combination between s^K and β when $\phi'(j) > 0$

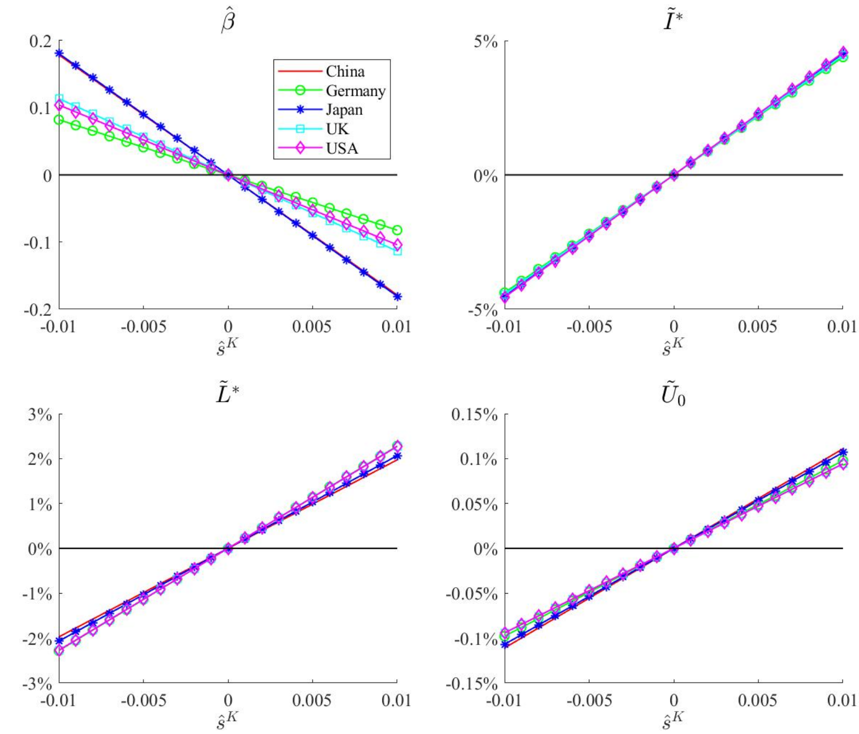

Figure 3b: Policy combination between s^K and β when $\phi'(j) < 0$

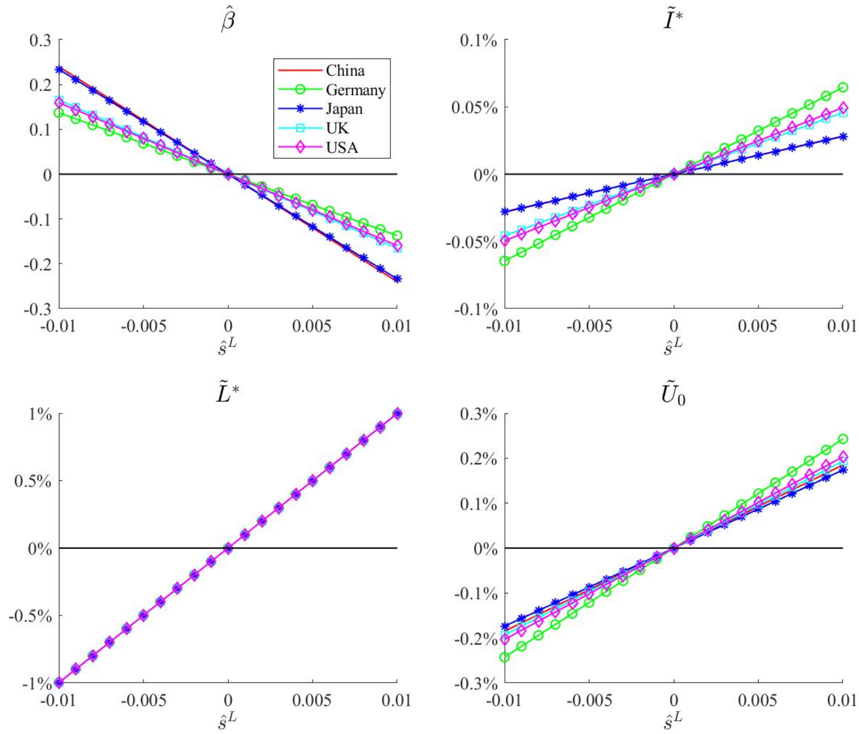

Figure 4a: Policy combination between s^L and β when $\phi'(j) > 0$

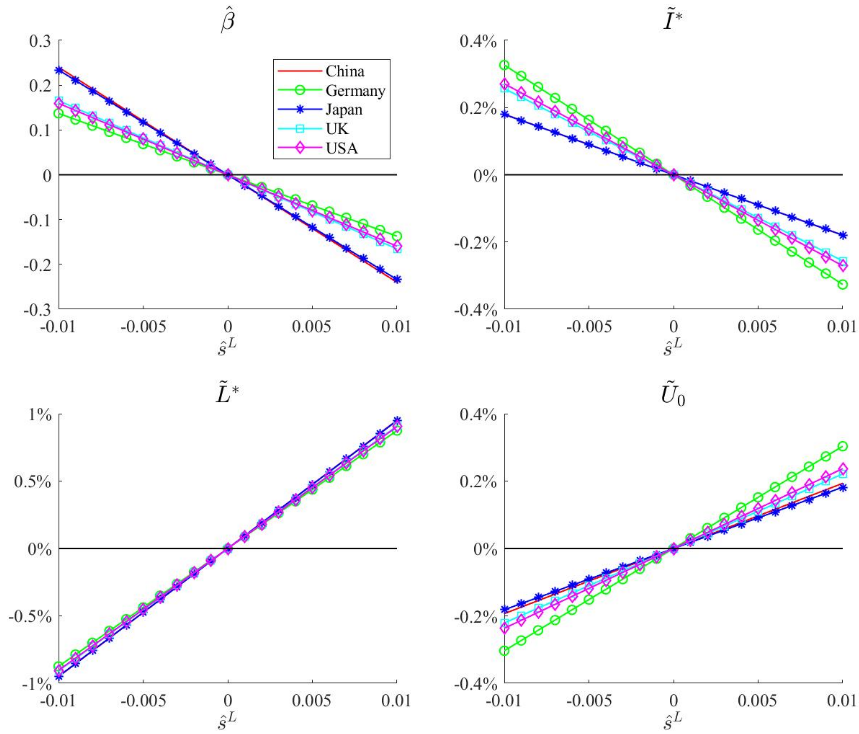

Figure 4b: Policy combination between s^L and β when $\phi'(j) < 0$

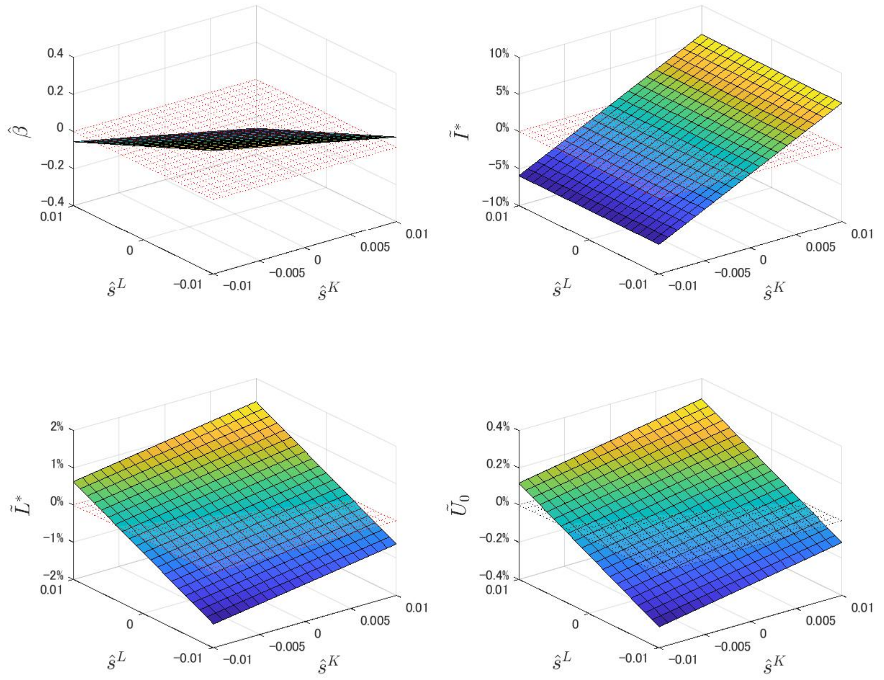

Figure 5a: Policy combination among s^K , s^L, and β when $\phi'(j) > 0$ (for the USA)

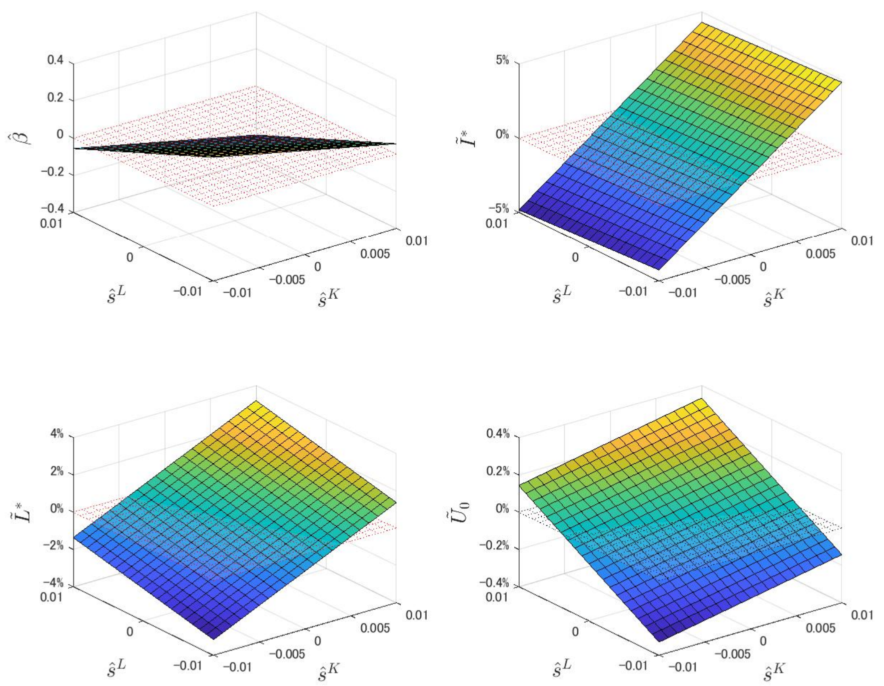

Figure 5b: Policy combination among s^K , s^L, and β when $\phi'(j) < 0$ (for the USA)