# Dissecting Return Regression: The Role of Pre-Investment Values

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October 8th, 2022

**Abstract**

Firms are heterogenous in their pre-investment values, so are the investment costs and the realized returns. What is the relationship between pre-investment firm values, the investment costs they pay, and the realized returns? We derive a formula that decomposes the marginal impact of pre-investment values on returns into an *economic* effect and a *mechanical* effect, taking into account the endogeneity of investment. It reveals that regressing realized returns on pre-investment values leads to a biased estimate of the economic effect, with the bias direction and magnitude depending on relative investment size and realized returns. Correcting bias is straightforward for data with only positive returns. For data in which returns take both sings, such as takeovers, stronger assumptions are necessary to make a meaningful inference. We conclude with suggestions for circumventing this issue. (*JEL* G11, G14, G34)

**Key words**: Investment, Return, Size Effects, Takeovers.

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# 1 Introduction

Firms and individuals make investments, expecting some forms of returns. Firms are heterogenous in their pre-investment values, so are the incurred investment costs and the realized returns. What is the relationship between pre-investment firm values, the investment costs they pay, and the realized returns? The importance of understanding how firm values affect their investment decisions and associated returns has long been recognized. See, for example, a large empirical literature on determinants of *acquirer returns in takeovers*, recently reviewed by Schneider and Spalt (2022). However, no simple, theoretical derivation of the relationship among pre-investment firm values, investment costs, and realized returns that can be used to guide empirical works seems to be available. We attempt to fill this gap.

We frame the problem as follows. Each transaction is identified by three numbers, a pre-investment firm value $V$, investment cost $P$, and the post-investment firm value $\widehat{V}$. From $\left(V, P, \widehat{V}\right)$, a return on investment $\pi \equiv \frac{\widehat{V}-V}{P}$ (ROI for short) and a realized return $R \equiv \frac{\widehat{V}-V}{V}$ can be computed. If we believe that a firm value $V$ and an investment cost $P$ matter *at the individual investment level*, we should let $\pi$ depend on $V$ and $P$. Then, we can ask the following question: *Is $R$ informative about how $V$ affects $\pi$?* In particular, *does regressing $R$ on $V$ reveal an economic impact of $V$ on $\pi$?*

The difficulty of answering this question stems from the fact that there are three channels through which $V$ affects $R = \pi\frac{P}{V}$:

1. $V$ affects $R$ through $\pi$ (*a firm-value effect*).
2. $V$ affects $R$ through the choice of $P$ because:
   (a) $P$ affects $\pi$ (*a matched investment-cost effect*).
   (b) $P$ multiplies $\frac{\pi}{V}$ (*a matched investment-scale effect*).
3. $V$ divides $\pi P$ (*a firm-scale effect*).

The firm-value effect is the extent to which a pre-investment firm value affects returns *by improving a particular return on investment*, taking other factors fixed. The two matched investment effects come from firms' investment choice. For example, if firms with larger $V$ optimally choose more costly investments (i.e., $P$ and $V$ are positively associated cross-sectionally), then such an investment may directly affect $\pi$ (i.e., 2.(a)) or it may scale up returns for a given level of $\pi$ (i.e., 2.(b)). These effects admit economic interpretations.

In contrast, the firm-scale effect is purely mechanical, as it follows from *the way we define returns*. As a simple example, suppose that $\pi P$ are positive but random across firms independent of $V$, i.e., both an investment and its ROI are determined by factors unrelated to $V$, such as *luck*. Then, we should find that, *on average*, $R$ would be smaller for firms with larger $V$. Clearly, this relationship should *not* be interpreted as an *economic* impact of $V$ on $R$. If we would like to learn how $V$ affects $\pi$, we need to isolate the firm-value effect from the other effects of $V$ on $R$.

The paper is organized as follows. Section 2 introduces a model of investment. Section 3 contains our main analysis. Section 4 concludes with suggestions for empirical work.

# 2 A Model of Investment

We describe a model in a context of *heterogenous firms making financial investments*, but it can be applied to any individuals or organizations making costly investment, expecting some form of returns. Each transaction is identified by three numbers: a pre-investment firm value $V$, its investment spending $P$, and the post-investment firm value $\widehat{V}$. *A return on investment* (ROI) is defined by changes in firm values per unit of investment cost:

$$\pi \equiv \frac{\widehat{V} - V}{P}. \tag{1}$$

ROI is positive if investment increases the firm value (i.e., $\widehat{V} > V$), and negative otherwise. *A realized return* from the investment is

$$R \equiv \frac{\widehat{V} - V}{V} = \pi \frac{P}{V}. \tag{2}$$

By assuming $P > 0$ and $V > 0$, (1) and (2) imply that $R$ and $\pi$ have the same sign.

So far, we only used two *definitions* to construct a new data $(\pi, R)$ from a raw data $\left(V, P, \widehat{V}\right)$. What can we learn from these data? We need to assume some economic structure. We make two simple assumptions. First, we assume that both $P$ and $V$ may affect $\pi$. For example, for each investment opportunity, firms can take various actions to increase ROI, and the extent to which such actions are effective may depend on pre-investment firm value $V$. Without imposing a particular form of these dependences, we write $\pi(V, P; \varepsilon)$, where a vector of random variables $\varepsilon$ captures other factors that affect $\pi$ *independent of* $(V, P)$. We call $\varepsilon$ *ROI shocks*, or *shocks* for short. Second, we assume that firms with $V$ *choose* $P$ *to maximize* $\widehat{V} = \pi P + V$, knowing how $\pi$ depends on $P$. The optimal choice by firm implies some *equilibrium relationship* between $P$ and $V$, which we denote by $P^*(V; \varepsilon)$.

One might argue that what matters for investors should be a realized return $R$. However, it depends on many factors beyond their control, captured by $\varepsilon$ in our model.[1] If we are interested in return variations *attributable to firms' characteristics*, it is best captured by $\pi_V \equiv \frac{\partial \pi(V,P;\varepsilon)}{\partial V}$. Accordingly, we ask the following questions: can regressing $R$ on $V$ reveal information about $\pi_V$? How is the estimated regression coefficient biased relative to $\pi_V$?

**Observable ROI shocks.** Suppose that $\varepsilon$ is observable at the time of investment. A firm with $V$ solves

$$\max_P \left\{ \pi\left(V, P; \varepsilon\right) P + V \right\}.$$

If $\pi\left(V, P; \varepsilon\right) \leq 0$ for any $P$, then the firm would not invest. To interpret observed $P$ as a solution to this problem, we must assume $\pi\left(V, P; \varepsilon\right) > 0$. The first order condition is

$$\pi\left(V, P; \varepsilon\right) + P\pi_P = 0, \tag{3}$$

where $\pi_P \equiv \frac{\partial \pi(V,P;\varepsilon)}{\partial P}$. The next assumption ensures the interior optimum $P^*\left(V; \varepsilon\right) \in (0, \infty)$.

**Assumption 1** *When $\varepsilon$ is observable, $\pi_P < 0 < \pi$ and $\frac{\partial \pi_P}{\partial P} < -2\frac{\pi_P}{P}$ for any $V, P, \varepsilon$.*

**Assumption 1** states that investment is profitable, ROI is decreasing in investment cost $P$, and $\widehat{V} = \pi P + V$ is well-behaved as a function of $P$. From a theoretical point of view, it is a simple set of assumptions that connects data with a rational investment choice.[2] From an empirical point of view, it can be checked with data $\pi$ and $P$.[3]

A realized return can be written as

$$R\left(V; \varepsilon\right) \equiv \pi\left(V, P^*\left(V; \varepsilon\right); \varepsilon\right) \frac{P^*\left(V; \varepsilon\right)}{V}. \tag{4}$$

In the next section, we study the implication of (3) and (4) for the interpretation of regression coefficient obtained by regressing $R$ on $V$.

**Unobservable ROI shocks.** A drawback of the above model is that it is not consistent with *negative* realized returns. To rationalize a data with negative returns, we must

[1] Typically, return regressions leave the overall variation in returns largely unexplained. For example, Golubov et al. (2015) find that unobservable firm-specific factors mostly drive acquirer returns.

[2] Recall that $\pi \equiv \frac{\widehat{V}-V}{P}$ and $\pi > 0 \Leftrightarrow \widehat{V} > V$. Therefore, $\pi_P = \frac{\widehat{V}_P P - \left(\widehat{V}-V\right)}{P^2} < 0$ is equivalent to $\widehat{V}_P < \pi$, and it is implied by $\widehat{V}_P \leq 0$ for $\pi > 0$. If we further assume $\widehat{V} \equiv F\left(V, P; \varepsilon\right) - P$, then a sufficient condition for $\pi_P < 0$ is $F_P \leq 1$.

[3] In a context of M&As, where $V$ is a bidder firm value and $P$ a price of a target firm, Schneider and Spalt (2022) report that they cannot reject $\pi_P = 0$ in their data. However, while statistically insignificant, their Table 6 exhibits $\pi_P < 0$ (especially for Non-Public firms, for which $\pi > 0$). We discuss their work in more details below.

posit that at least some element of $\varepsilon$ must be unobservable to firms at the time of investment and also that some realization of those shocks makes returns negative. With such unobservable ROI shocks, a firm with $V$ solves

$$\max_P \left\{ \overline{\pi} (V, P) P + V \right\},$$

where $\overline{\pi}(V,P) \equiv E[\pi(V,P;\varepsilon)]$. To rationalize observed $P$, we assume $\overline{\pi}(V,P) > 0$, but this does not exclude $\pi(V,P;\varepsilon) < 0$ for some realizations of $\varepsilon$. The first order condition is

$$\overline{\pi}(V,P) + P\overline{\pi}_P = 0, \tag{5}$$

where $\overline{\pi}_P \equiv \frac{\partial \overline{\pi}(V,P)}{\partial P}$. We make the following assumption.

**Assumption 2** *When $\varepsilon$ is unobservable, $\overline{\pi}_P < 0 < \overline{\pi}$ and $\frac{\partial \overline{\pi}_P}{\partial P} < -2\frac{\overline{\pi}_P}{P}$ for any $V, P$.*

With the interior optimal choice $\overline{P}^* (V)$, a realized return is

$$\overline{R}(V;\varepsilon) \equiv \pi \left( V, \overline{P}^* (V); \varepsilon \right) \frac{\overline{P}^* (V)}{V}. \tag{6}$$

Realized returns (6) depend on $\pi$, but **Assumption 2** does not determine the sign of $\pi$ and $\pi_P$. Therefore, we need more assumptions to study the implication of (5) and (6) for the regression analysis. We defer the further discussion to Section 3.2.

**Remark 1.** Jansen et al. (2013) study M&A data, where $V$ is a bidder firm value, $P$ a price of a target firm, and *a significant fraction of realized returns are negative*. They note that preceding studies which *regress $R$ on relative size $\frac{P}{V}$* found the positive coefficient estimate (e.g., Asquith et al. (1983)) as well as the negative estimate (e.g., Fuller et al. (2002)). Importantly, they show that when they split their sample by the sign of $R$, the coefficient estimate has the sign of $R$ (p. 535, Figure 1). This is consistent with (6), if changes in $\frac{\overline{P}^*}{V}$ do not affect $\pi$ much.[4]

The above discussion indicates that the sign of realized returns matters for the regression analysis. In Section 3, we focus on the regression of $R$ on $V$, which is more prevalent in the empirical literature.

[4] More formally, $\frac{dR}{d\left(\frac{P}{V}\right)} = \pi + \frac{\partial \pi}{\partial\left(\frac{P}{V}\right)} \frac{P}{V}$, which has the sign of $\pi$ if $\frac{\partial \pi}{\partial\left(\frac{P}{V}\right)} \frac{P}{V} \approx 0$.

# 3 Implications for Regression Analyses

It is common practice to regress realized returns $R$ on pre-investment firm values $V$. It is expected to reveal *economic* effects of pre-investment firm values on the post-investment firm values, such as negotiation powers and agency problems. In this section, we show that when realized returns are largely driven by forces that are independent of pre-investment firm values and investment costs (in our model, $\varepsilon$ being a dominant force), then one may obtain statistically significant regression coefficients, but they do not reveal economic effects.

## 3.1 Observable ROI shocks

We take a total derivative of (4) with respect to $V$.

$$\frac{dR}{dV} = \underbrace{\pi_V \frac{P^*}{V}}_{1} + \underbrace{\frac{P_V^*}{V}\left(P^* \pi_P + \pi\right)}_{2(a)(b)} \underbrace{-\frac{R}{V}}_{3}, \tag{7}$$

where $\pi_V \equiv \frac{\partial \pi(V,P;\varepsilon)}{\partial V}$ and $P_V^* \equiv \frac{\partial P^*(V;\varepsilon)}{\partial V}$. The first term $\pi_V \frac{P^*}{V}$ is the firm-value effect. The second term is the matched investment effects. The third term $-\frac{R}{V}$ is the firm-scale effect. We combine (7) with the firm's investment choice analyzed in the previous section.

**Lemma 1** *If the optimality condition* (3) *characterizes* $P^*(V;\varepsilon)$*, then* (7) *becomes*

$$\frac{dR}{dV} = \pi_V \frac{P^*}{V} - \frac{R}{V}. \tag{8}$$

**Proof.** The optimality condition (3) implies that the second term in (7) is zero. ■

**Lemma 1** shows that in the data in which $P^*$ and $V$ are connected by the optimality condition (3), the total marginal impact of $V$ on $R$ is the sum of two terms: (i) $\pi_V$ multiplied by $\frac{P^*}{V}$, and (ii) $-\frac{R}{V}$. This has two implications.

First, because realized returns $R$ must be positive in this model, the firm-scale effect $-\frac{R}{V}$ creates a downward bias, regardless of the sign of $\pi_V$. The magnitude of this *additive bias* increases in $R$ and decreases in $V$, suggesting more significant downward bias for data in which $R$ is large relative to $V$.

Second, $\frac{P^*}{V}$ is smaller than one in most applications, because $V$ is a stock variable (e.g. market capitalization), while $P$ is a flow variable (e.g. investment expenditure). Therefore, $\left|\pi_V \frac{P^*}{V}\right| < |\pi_V|$ holds. The direction of this *multiplicative bias* depends on the sign of $\pi_V$, while its magnitude depends on *relative investment size* $\frac{P^*}{V}$. Given $\frac{P^*}{V} < 1$, $\pi_V \frac{P^*}{V}$ *underestimates* *positive* $\pi_V$ and *overestimates negative* $\pi_V$. **Table 1** summarizes these implications.

Table 1

Bias of $\frac{dR}{dV} = \pi_V \frac{P^*}{V} - \frac{R}{V}$ relative to $\pi_V$.

| Multiplicative bias $\frac{P^*}{V}$ | | Total direction of bias |
|---|---|---|
| $\frac{P^*}{V} < 1$ | $\ominus$ for $\pi_V > 0$ | $\frac{dR}{dV} < \pi_V$ |
| | $\oplus$ for $\pi_V < 0$ | $\frac{dR}{dV} \lesseqgtr ? \; \pi_V$ |
| $\frac{P^*}{V} > 1$ | $\oplus$ for $\pi_V > 0$ | $\frac{dR}{dV} \lesseqgtr ? \; \pi_V$ |
| | $\ominus$ for $\pi_V < 0$ | $\frac{dR}{dV} < \pi_V$ |

For $R > 0$, the additive bias $-\frac{R}{V}$ is negative and its magnitude is decreasing in $V$. On the other hand, whether the multiplicative bias $\frac{P^*}{V}$ inflates or deflates $\pi_V$ depends on the sign of $\pi_V$. This indicates the following trade-off. Suppose $\frac{P^*}{V} < 1$. Then, *if* $\frac{P^*}{V}$ *decreases in* $V$, the multiplicative bias becomes more significant as $V$ increases. This condition is likely to hold in many applications where $\frac{P^*}{V}$ is a ratio of particular investment expenditure to the overall wealth. Then, for firms with larger $V$, the magnitude of the additive bias $-\frac{R}{V}$ would be smaller, but the multiplicative bias $\frac{P^*}{V}$ makes $\frac{dR}{dV}$ further away from $\pi_V$.

The model is simple and lacks many realistic features. However, precisely because it does not depend on particular features of investment, we expect that the basic logic persists through time, across industry sectors, and regardless of institutional details such as regulations and payment methods.

## 3.2 Unobservable ROI shocks

By taking a total derivative of the realized return (6) with respect to $V$,

$$\frac{dR}{dV} = \underbrace{\pi_V \frac{\overline{P}^*}{V}}_{1} + \underbrace{\frac{\overline{P}^*_V}{V}\left(\overline{P}^* \pi_P + \pi\right)}_{2(a)(b)} \underbrace{-\frac{R}{V}}_{3}. \tag{9}$$

By using the optimality condition (5), we obtain the following result.

**Lemma 2** *If the optimality condition* (5) *characterizes* $\overline{P}^*(V)$, *then* (9) *becomes*

$$\frac{dR}{dV} = \begin{cases} \frac{\overline{P}^*}{V}\pi_V - \frac{R}{V}\left\{1 + \overline{P}^*_V \frac{V}{\overline{P}^*}\left(\frac{\pi_P/\pi}{\overline{\pi}_P/\overline{\pi}} - 1\right)\right\} & for \quad R \neq 0, \\ \frac{\overline{P}^*}{V}\left(\pi_V + \overline{P}^*_V \pi_P\right) & for \quad R = 0. \end{cases} \tag{10}$$

**Proof.** For $R = 0$, substitute $R = \pi = 0$ in (9). For $R \neq 0$, use $\overline{P}^* = -\frac{\overline{\pi}}{\overline{\pi}_P}$ to write

$\overline{P}^* \pi_P + \pi$ in (9) as $-\pi\left(\frac{\pi_P/\pi}{\overline{\pi}_P/\overline{\pi}} - 1\right)$. Use $\frac{\pi}{V} = \frac{R}{P^*}$ to obtain the result. ■

In (10), the additive bias for $R \neq 0$ can be written as

$$-\frac{R}{V}\left\{1 + \eta_V^{\overline{P}^*}\left(\frac{\eta_P^{\pi}}{\eta_P^{\overline{\pi}}} - 1\right)\right\}, \tag{11}$$

where $\eta_V^{\overline{P}^*} \equiv \overline{P}^*_V \frac{V}{\overline{P}^*}$ is the elasticity of the optimal investment $\overline{P}^*(V)$, $\eta_P^{\pi} \equiv \pi_P \frac{P}{\pi}$ is the elasticity of $\pi$ with respect to $P$, and $\eta_P^{\overline{\pi}} \equiv \overline{\pi}_P \frac{P}{\overline{\pi}}$ is the elasticity of $\overline{\pi}$ with respect to $P$. If $\eta_P^{\pi} = \eta_P^{\overline{\pi}}$, then (11) collapses to $-\frac{R}{V}$. Otherwise, we must sign (11) to determine the direction of bias. By **Assumption 2**, $\eta_P^{\overline{\pi}} < 0$. The sign of $\eta_V^{\overline{P}^*}$ can be assessed empirically. However, the term $\left(\frac{\eta_P^{\pi}}{\eta_P^{\overline{\pi}}} - 1\right)$ is difficult to sign empirically and theoretically.[5]

**Remark 2.** Schneider and Spalt (2022) assume $\pi_P = \pi_V = 0$ in their "pure scaling model". This implies $\frac{dR}{dV} = -\frac{R}{V}$ and they empirically study the extent to which regression coefficients obtained for different quantiles of data, sorted by $R$ and $V$, replicate the pattern implied by $-\frac{R}{V}$. However, assuming $\pi_P = \pi_V = 0$ is problematic from a perspective of our model. First, $\pi_P = 0$ implies $\overline{\pi}_P = 0$, which is inconsistent with $P$ as an optimal choice by firms. Second, additionally assuming $\pi_V = 0$ implies $\overline{P}^*_V = 0$.[6] However, $\overline{P}^*_V > 0$ better describes their data (see their Table 3 and 5). While $-\frac{R}{V}$ plays a major role in their data, their analysis does not directly support $\pi_V = 0$.

In the next section we propose a way to make an inference about $\pi_V$ when the additive bias in (10) is significant and difficult to sign.

# 4 Discussion

## 4.1 Alternative regression specification

One may wonder if simply regressing $\pi$ on $V$ can reduce biases. To show that this is not necessarily the case, we focus on a case with observable shocks. Define $\eta_V^{P^*} \equiv P^*_V \frac{V}{P^*}$.

**Lemma 3** *If the optimality condition* (3) *characterizes* $P^*(V;\varepsilon)$*, then the total marginal effect of* $V$ *on* $\pi$ *is*

$$\frac{d\pi}{dV} = \pi_V - \eta_V^{P^*}\frac{R}{P^*}. \tag{12}$$

[5] By the implicit function theorem applied to (5), $\overline{P}^*_V = \frac{1}{V}\frac{\eta_V^{\overline{\pi}} - \eta_V^{\overline{\pi}_P}}{1 - \eta_P^{\overline{\pi}} - \eta_P^{\overline{\pi}_P}}$. Thus, the sign of $\overline{P}^*_V$ (i.e., whether $\overline{P}^*(V)$ increases or decreases in $V$) is informative about the elasticities of $\overline{\pi}$ and $\overline{\pi}_P$ with respect to $V$ and $P$. However, it does not sign $\left(\frac{\eta_P^{\pi}}{\eta_P^{\overline{\pi}}} - 1\right)$.

[6] Use $\overline{\pi}_P = \overline{\pi}_V = 0$ in the expression of $\overline{P}^*_V$ in the previous footnote.

*The additive bias* $-\eta_V^{P^*}\frac{R}{P^*}$ *in* (12) *has the same sign as* $-\frac{R}{V}$ *in* (8), *and is smaller in magnitude, if and only if*

$$0 < \eta_V^{P^*} < \frac{P^*}{V}. \tag{13}$$

**Proof.** To obtain (12), substitute $\pi_P = -\frac{\pi}{P^*}$ into $\frac{d\pi}{dV} = \pi_V + \pi_P P_V^*$ and use $\eta_V^{P^*} \equiv P_V^* \frac{V}{P^*}$ and $\frac{\pi}{V} = \frac{R}{P^*}$. The inequalities (13) follow by comparing $-\frac{\eta_V^{P^*}}{P^*}R$ and $-\frac{R}{V}$. ■

In (12), the sign of the additive bias $-\eta_V^{P^*}\frac{R}{P^*}$ depends not only on $R$ but also on $\eta_V^{P^*}$. Hence, this regression specification adds an additional layer of uncertainty associated with $\eta_V^{P^*}$. If it can be precisely estimated and one is confident that the condition (13) holds, then for the purpose of learning $\pi_V$, regressing $\pi$ on $V$ is more desirable than a usual return regression of $R$ on $V$. We note, however, that (13) can be written as $0 < P_V^* < \left(\frac{P^*}{V}\right)^2$. For applications where $\frac{P^*}{V}$ is very small, this condition may be easily violated.

## 4.2 Bias correction in return regression

The analysis of the case with observable shocks suggests a straightforward way to make an inference about $\pi_V$. On the other hand, the bias in the case of unobservable shocks is harder to sign, hence more difficult to correct.

**Observable ROI shocks.** From (8),

$$\frac{V}{P^*}\left(\frac{dR}{dV}\right) + \frac{R}{P^*} = \pi_V. \tag{14}$$

This suggests the following bias correction procedure. First, for each quantile of data sorted by $V$, obtain an estimate of $\left(\frac{dR}{dV}\right)$ by regressing $R$ on $V$. Second, adjust the coefficient by (14) using $\frac{V}{P^*}$ and $\frac{R}{P^*}$ for the corresponding quantile of data. This procedure allows for the estimation of $\pi_V$ for each quantile of $V$.

**Unobservable ROI shocks.** In general, the direction and the magnitude of the additive bias (11) are difficult to know. To minimize the effect of the additive bias, we propose *a local regression with observations* $R \approx 0$. Specifically, (10) for $R = 0$ implies

$$\frac{V}{\overline{P}^*}\left(\frac{dR}{dV}\right)|_{R=0} = \left(\pi_V + \overline{P}_V^*\pi_P\right)|_{\pi=0}. \tag{15}$$

This suggests a local inference using observations with $R \approx 0$. We still need to know $\overline{P}_V^*\pi_P|_{\pi\approx 0}$ to estimate $\pi_V|_{\pi\approx 0}$. However, if we can sign $\overline{P}_V^*\pi_P$ around $\pi \approx 0$, (15) could be used to sign $\pi_V$ around $\pi \approx 0$. More precisely,

$$\frac{V}{\overline{P}^*}\left(\frac{dR}{dV}\right)|_{R\approx 0} \;>\; 0 \;\Rightarrow\; \pi_V|_{\pi\approx 0} > -\overline{P}^*_V \pi_P|_{\pi\approx 0},$$
$$\frac{V}{\overline{P}^*}\left(\frac{dR}{dV}\right)|_{R\approx 0} \;<\; 0 \;\Rightarrow\; \pi_V|_{\pi\approx 0} < -\overline{P}^*_V \pi_P|_{\pi\approx 0}.$$

For example, if one finds $\frac{V}{\overline{P}^*}\left(\frac{dR}{dV}\right)|_{R\approx 0} > 0$, $\overline{P}^*_V > 0$, and $\pi_P|_{\pi\approx 0} < 0$, then $\pi_V|_{\pi\approx 0}$ has *a positive lower bound* $-\overline{P}^*_V \pi_P|_{\pi\approx 0}$.

**Remark 3.** For the local estimation based on (15), Figure 2 in Schneider and Spalt (2022) is suggestive. They plot estimated $\frac{dR}{dV}$ for each quantile (decile) of $R$. If $\frac{dR}{dV} = -\frac{R}{V}$, as they argue, then the estimated coefficient should be *close to zero for the quantile including* $R = 0$. However, they find $\frac{dR}{dV}|_{R\approx 0} < 0$ for a subsample with non-public targets, and $\frac{dR}{dV}|_{R\approx 0} > 0$ for the remaining subsample with public targets.[7] They do not report the value of $\pi_P|_{\pi\approx 0}$ for the corresponding quantile of data. However, if $\pi_P|_{\pi\approx 0} < 0$ holds for the subsample with public targets, then (15) indicates $\pi_V|_{\pi\approx 0} > -\overline{P}^*_V \pi_P|_{\pi\approx 0} > 0$, i.e., firms with larger $V$ have higher ROI when taking over public targets.

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[7] Both are statistically significant at the 5% confidence level. According to their Table 7, $R = 0$ occurs between 40th percentile and 50th percentile for the non-public target sample, while it occurs between 50th percentile and 60th percentile for the public target sample. See the corresponding part of their Figure 2.