Trading Costs in Takeover Markets

(Preliminary and incomplete)

Kei Kawakami*

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Abstract

We study the effect of trading costs in takeover markets. Without trading costs, all firms are active on both sides of the market, selling their initial projects and buying a new one. With trading costs, firms self-select into four groups: (i) firms active only on the target side of the market, (ii) firms active only on the bidder side, (iii) firms active on both sides, (iv) firms that do not participate in the market. The interaction between fixed costs and the merger productivity yields testable predictions with respect to the nature of group (iv). Trading costs that depend on transaction prices induce endogenous asymmetry in an otherwise symmetric environment: for a matched pair of firms, the equilibrium value of the target firm is smaller than that of the bidder. This holds even when trading costs exist only on the bidder side. (*JEL* L1, G3)

Key words: Takeovers, Trade costs, Matching.

*Department of Economics, Aoyama Gakuin University, 4-4-25, Shibuya, Tokyo 150-8366, Japan (e-mail: kei.kawakami@gmail.com; https://sites.google.com/site/econkeikawakami/). I thank seminar participants at University of Melbourne for comments and suggestions. All errors are mine. This work was supported by JSPS KAKENHI Grant Number JP18K12745, and Nomura Foundation. Financial support from the College of Economics at Aoyama Gakuin University are gratefully acknowledged.

1 Introduction

Takeover markets are economically large, and so are the associated trading costs. Golubov et al. (2012) report that in 2007, $4.2 trillion was spent on M&A deals and investment banks advised on over 85% of the deals by transaction values. Advisory fees were estimated to be $39.7 billion, approximately 1% of the transaction values.[1] Arguably, these explicit fees are only a part of trading costs. As David (2021) emphasized, takeover markets can have a large aggregate consequence through its role for resource reallocation. Yet, direct empirical evidence on whether and how trading costs affect resource allocation are scant. Given the difficulty of directly observing many sources of trading costs, we take a step back and ask a basic question: how do trading costs affect takeovers?

In this paper, we conduct a systematic analysis of the effect of trading costs in a competitive model of takeover markets. In the model, each firm is endowed with tangible and intangible assets that are complementary in generating profits. The tangible asset is tradeable and we call it a project. The intangible asset is non-tradeable and we call it skill. Project quality and skill level vary across firms. We assume that initial distribution of project quality and skill are independent so that there are gains from reallocating projects across firms. Owners of firms can profit from either selling and/or buying firms by participating in takeovers, or from operating them without participating in takeovers.

In the absence of any trading costs, all firms (except a measure zero firms who start with the first-best allocation) sell their initial projects and buy new ones. For the firms that engage in takeovers on both sides of the market, their net trading profit from buying and selling firms follows a systematic pattern. Firms with initial project quality that is high *relative to their skill level* will sell it at a high price and buy a lower quality project that better matches their skill level. Hence, these firms make a positive trading profit. On the other hand, firms with initial project quality that is low relative to their skill level will buy a better project than their endowed ones, so these firms' trading profit is negative. Thus,

[1] For merger fees, see also Hunter and Jagtiani (2003) and Calomiris and Hitscherich (2005).

the first-best reallocation without trading costs has two features: (i) all firms are active on both sides of the market, and (ii) a negative correlation between the trading profit and the level of post-acquisition production across firms.

Next, we introduce two sets of trading costs. The first set of trading costs are a fixed cost that is independent of firm characteristics. We consider a target-specific fixed cost, a bidder-specific fixed cost, and a fixed cost common to all firms that participate in takeover markets. The second set of trading costs are costs proportional to transaction prices. The key difference between the first set of trading costs and the second set of trading costs is that only the latter distorts a matching between bidder firms and target firms.

We first show that, in the presence of fixed costs, there can be four groups of firms: (i) firms active only on the target side of the market, (ii) firms active only on the bidder side, (iii) firms active on both sides, and (iv) firms that do not participate in the market. The interaction between fixed costs and the merger productivity yields testable predictions. We show that in an industry with the low level of the merger productivity, the most profitable firms (in terms of their pre-acquisition production) tend to stay away from takeovers. On the other hand, in an industry with the high level of the merger productivity, moderately profitable firms find takeovers unattractive.

Second, we show that trading costs that are proportional to transaction prices induce endogenous asymmetry between bidders and targets in an otherwise symmetric environment: for a matched pair of firms, the equilibrium value of the target firm is typically smaller than that of the bidder. Importantly, this holds even when trading costs exist only for bidders. Intuitively, trading costs that distort matching hurts targets relatively more because the matching distortion reduces realized gains, which in turn lowers transaction prices. Empirically, it is well documented that target firms are on average much smaller than bidder firms. While there are many reasons for this asymmetry unrelated to our model (e.g., size-dependent financial constraint), our model identifies a new mechanism that hurts target firms more than bidder firms.

Related literature. Theoretical models of takeovers can be classified into three groups, each corresponding to a different view of takeover motives: resource-based view (Jovanovic and Braguinsky (2004), Nocke and Yeaple (2007, 2008), David (2021)), market-power view (Kamien and Zang (1990), Loertscher and Marx (2019)), managerial view (Gorton et al. (2009)). Our model belongs to the first gourp, and abstracts from other motives of takeovers. Our work is most closely related to Nocke and Yeaple (2008), where takeovers are driven by two-dimensional heterogeneity. They model cross-border mergers and acquisitions, but abstract from trading costs. We do not model cross-border deals, and study the effect of trading costs of various forms.

Section 2 describes our model in a general environment. Section 3 studies a symmetric benchmark. Section 4 analyzes trading costs that are proportional to transaction prices. Section 5 concludes.

2 Model

There is a continuum of firms. Each firm has one project and can manage only one project. Firms are heterogenous in their initial project quality $A \in [A_{\min}, A_{\max}]$. Firms are also heterogeneous in their management skill $X \in [X_{\min}, X_{\max}]$. Distribution of (A, X) is to be specified. A firm with (A, X) solves

$$\max \left\{\Pi_T\left(A\right), \Pi_B\left(X\right), \Pi_{TB}\left(A, X\right), \Pi_P\left(A, X\right)\right\}, \tag{1}$$

where $\Pi_T\left(A\right)$ is a payoff as a target firm, $\Pi_B\left(X\right)$ is a payoff as a bidder firm, $\Pi_{TB}\left(A, X\right)$ is a payoff of being both a bidder and a target, and $\Pi_P\left(A, X\right)$ is a payoff when producing with its original (A, X) (i.e., not participating in takeovers). Firms take a competitive price schedule of projects $P\left(A\right)$, specified for each project quality level, as given.

- Target firms with project of quality A can sell it at price $P\left(A\right)$, but it must pay the trading cost $\tau_T P(A) + \phi_T$. Hence, their payoff is

$$\Pi_T(A) = (1 - \tau_T) P(A) - \phi_T.$$

- Bidder firms with management skill X can buy any project of quality $a \in [A_{\min}, A_{\max}]$ at price $P(a)$, but it must pay the trading cost $\tau_B P(a) + \phi_B$. The production with a new project of quality a yields $za^\alpha X^\beta$. Hence, their payoff is

$$\Pi_B(X) = \max_a \left\{ za^\alpha X^\beta - (1 - \tau_B) P(a) \right\} - \phi_B.$$

- If firms choose to participate in takeovers on both sides of the market, their payoff is

$$\Pi_{TB}(A, X) \equiv \Pi_T(A) + \Pi_B(X).$$

- Producing with initial (A, X) results in the payoff

$$\Pi_P(A, X) = A^\alpha X^\beta - \phi.$$

We interpret (ϕ, z, α, β) as industry-specific parameters. A positive ϕ can be interpreted as a fixed cost of production that is saved by takeovers. We also allow ϕ to take a negative value. In that case, it captures (fixed) opportunity costs of takeovers. For example, a value arising from firms' brand name may be lost in the process of takeovers, regardless of being a bidder or a target. Following David (2021), we call z *the merger productivity*: $z < 1$ can be interpreted as a tax or adjustment costs on recombining factors of production, while $z > 1$ can be interpreted as as a subsidy for bidders. In sum, $\phi < 0$ and $z < 1$ imply the presence of *implicit* costs of takeovers that are industry-specific, while $\phi > 0$ and $z > 1$ capture the industry characteristics ammenable to takeovers. The other four parameters $(\phi_T, \phi_B, \tau_T, \tau_B)$ capture *explicit* trading costs of takeovers which target and bidder firms pay. We assume these parameters are non-negative.

An equilibrium is a price function $P(A)$ and optimal decision of firms (1) taking $P(A)$ as given, such that takeover markets are cleared for all quality level $A \in [A_{\min}, A_{\max}]$.

3 Symmetric benchmark

We first study the benchmark symmetric case where $\tau_T = \tau_B = 0$, $\alpha = \beta$, and the distribution of A and X are identical and independent. We start with bidders' problem

$$\Pi_B(X) = \max_a \{z(aX)^\alpha - P(a)\} - \phi_B.$$

If the first order condition

$$\alpha z a^{\alpha-1} X^\alpha = P'(a).$$

has a unique solution, we denote it as

$$a^*(X) = \arg\max_a \{z(aX)^\alpha - P - \phi_B\}.$$

This is a matching function: $a^*(X)$ is the project quality demanded by firms with skill X. Using $a^*(X)$,

$$\Pi_B(X) = z(a^*(X)X)^\alpha - \phi - P(a^*(X)) - \phi_B.$$

Given that $a^*(X)$ is monotonic (to be verified in equilibrium), its inverse function

$$x^*(A) \equiv \left\{\frac{1}{\alpha z} P'(A) A^{1-\alpha}\right\}^{\frac{1}{\alpha}} \tag{2}$$

determines a skill level of firms which demand projects of quality A. To derive an equilibrium price function $\{P(A)\}_{A \in [A_{\min}, A_{\max}]}$, we make a guess, and verify that it clears the market.

Conjecture

$$P(A) = \frac{z}{2} A^{2\alpha} + C_2$$

with some unknown coefficient C_2 to be determined. This price function yields $P'(A) = \alpha z A^{2\alpha-1}$. Substituting this into (2), the matching function is $x^*(A) = A$ and

$$\Pi_B(X) = \frac{z}{2} X^{2\alpha} - (C_2 + \phi_B),$$

$$\Pi_T(A) = \frac{z}{2} A^{2\alpha} + C_2 - \phi_T.$$

This implies that the payoff from acting on both sides of the market is

$$\Pi_{TB}(A, X) = \frac{z}{2}\left(A^{2\alpha} + X^{2\alpha}\right) - \phi_M,$$

where we defined $\phi_M \equiv \phi_T + \phi_B$. Next, symmetric sharing of created surplus requires

$$\Pi_T(A) = \Pi_B(x^*(A)) \quad \Leftrightarrow \quad C_2 - \phi_T = -(C_2 + \phi_B) \quad \Leftrightarrow \quad C_2 = \frac{\phi_T - \phi_B}{2}.$$

Therefore,

$$\begin{aligned}
P(A) &= \frac{1}{2}\left(zA^{2\alpha} + \phi_T - \phi_B\right), \\
\Pi_T(A) &= \frac{1}{2}\left(zA^{2\alpha} - \phi_M\right), \\
\Pi_B(X) &= \frac{1}{2}\left(zX^{2\alpha} - \phi_M\right), \\
\Pi_{TB}(A, X) &= \frac{z}{2}\left(A^{2\alpha} + X^{2\alpha}\right) - \phi_M.
\end{aligned}$$

These payoffs imply

$$\Pi_T(A) > \Pi_B(X) \quad \Leftrightarrow \quad A > X, \tag{3}$$

$$\Pi_{TB}(A,X) > \Pi_T(A) \Leftrightarrow \Pi_B(X) > 0 \Leftrightarrow X > \left(\frac{\phi_M}{z}\right)^{\frac{1}{2\alpha}} \equiv \widehat{X}, \tag{4}$$
$$\Pi_{TB}(A,X) > \Pi_B(X) \Leftrightarrow \Pi_T(A) > 0 \Leftrightarrow A > \widehat{X}.$$

First, consider firms with $A > X$. By (3), choosing $\Pi_B(X)$ is never optimal for these firms. If $\widehat{X} < X$, then (4) implies that $\Pi_T(A)$ is neither optimal. So the relevant problem is

$$\max\{\Pi_{TB}(A,X), \Pi_P(A,X)\}.$$

Therefore, $\Pi_{TB}(A,X) = \Pi_P(A,X)$ defines an indifference line for the firms with $\widehat{X} < X < A$. If $X < \min\left\{\widehat{X}, A\right\}$, then (4) implies that $\Pi_{TB}(A,X)$ is dominated by $\Pi_T(A)$. So the relevant problem is

$$\max\{\Pi_T(A), \Pi_P(A,X)\}.$$

Therefore, $\Pi_T(A) = \Pi_P(A,X)$ defines an indifference line for the firms with $X < \min\left\{\widehat{X}, A\right\}$.

A symmetric argument applies for firms with $A < X$: if $\widehat{X} < A < X$, then $\Pi_{TB}(A,X) = \Pi_P(A,X)$ defines an indifference line, while if $A < \min\left\{X, \widehat{X}\right\}$, then $\Pi_B(A) = \Pi_P(A,X)$ defines an indifference line.

For the further characterization, suppose that distributions of A and X have a continuous support with an upper bound 1. We first set $\phi_M = 0$. Because this makes $\widehat{X} \equiv \left(\frac{\phi_M}{z}\right)^{\frac{1}{2\alpha}} = 0$, an immediate implication is that all firms, if they ever choose to participate in takeovers, will be on both sides of the market. While this is clearly counter-factual, it is a useful benchmark to study the property of the first best allocation, and to disentangle the role of (ϕ, z).

Proposition 1 *Assume $\phi_M > 0$ and define $\widehat{\phi}(\phi_M, z) \equiv -\frac{z-\phi_M}{2} + \sqrt{\frac{\phi_M}{z}}$.*

(a) *Some, but not all, firms engage in takeovers, and no firm is active on both sides of the market if and only if $\phi \in \left(-\frac{z-\phi_M}{2}, \widehat{\phi}(\phi_M, z)\right)$.*

(b) *For $z \leq 1$, all four options $\{\Pi_T(A), \Pi_B(X), \Pi_{TB}(A,X), \Pi_P(A,X)\}$ are chosen by* *positive measure of firms if and only if* $\phi \in \left(\widehat{\phi}\left(\phi_M, z\right), 1 - z + \phi_M\right)$.

(c) *For* $z > 1$*, all four options* $\left\{\Pi_T(A), \Pi_B(X), \Pi_{TB}(A,X), \Pi_P(A,X)\right\}$ *are chosen by positive measure of firms if and only if* $\phi \in \left(\widehat{\phi}\left(\phi_M, z\right), \frac{\phi_M}{z}\right)$.

The following figures show the selection pattern for $z < 1$ and $z > 1$. Red lines represent the indifference line defined by $\Pi_T(A) = \Pi_P(A,X)$, while blue lines represent the indifference line defined by $\Pi_{TB}(A,X) = \Pi_P(A,X)$. As ϕ increases, the area of non-participating firms shrinks for both cases, but their locations are quite different.

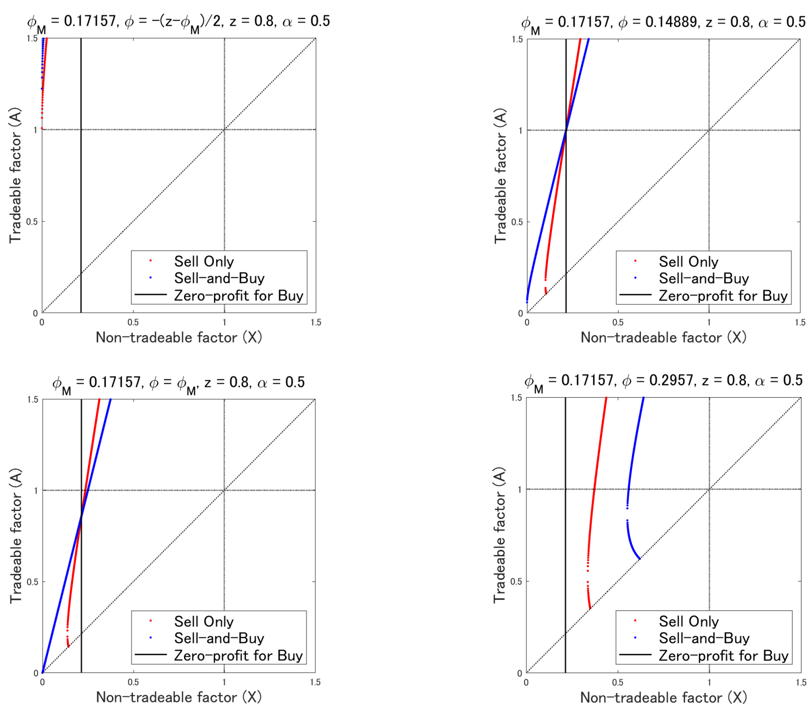

Figure 1. Low merger productivity $z < 1$.

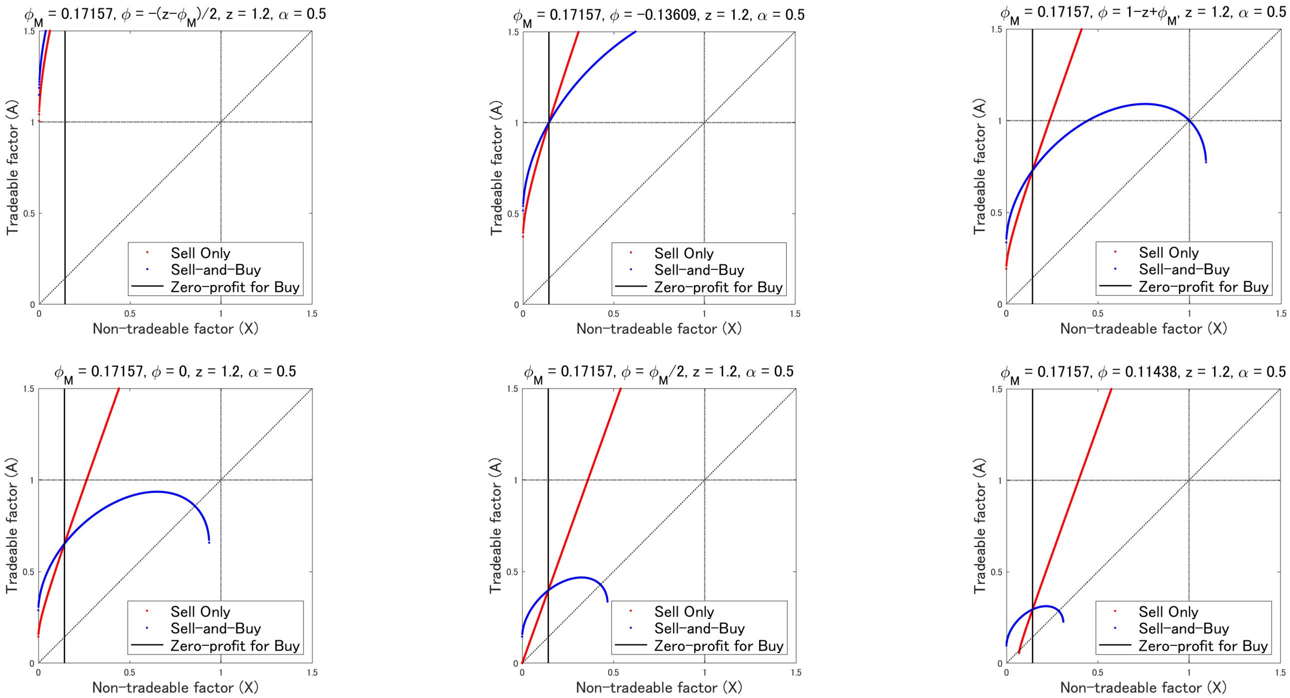

Figure 2. High merger productivity $z > 1$.

4 Asymmetric case

In this section we introduce trading costs proportional to transaction prices, $\tau_T > 0$ and $\tau_B > 0$. We also allow for asymmetry in technology $\alpha \neq \beta$. To keep the model tractable, we assume that each firm can be active on only one side of the market.[2] Accordingly, a firm with (A, X) solves

$$\max\left\{\Pi_T(A), \Pi_B(X), \Pi_P(A, X)\right\},$$

where each payoff is given by

$$\Pi_T(A) = (1 - \tau_T) P(A) - \phi_T,$$

$$\Pi_B(X) = \max_A \left\{ z A^\alpha X^\beta - (1 + \tau_B) P(A) \right\} - \phi_B, \tag{5}$$

$$\Pi_P(A, X) = A^\alpha X^\beta - \phi.$$

[2]For example, engaging in both types of activities can be too costly for small firms.

From the first order condition for (5),

$$\alpha z A^{\alpha-1} X^{\beta} = (1+\tau_B) P'(A).$$

If this has a unique solution, we denote it as

$$a^*(X) = \arg\max_A \left\{ z A^{\alpha} X^{\beta} - (1+\tau_B) P(A) \right\}.$$

Using $a^*(X)$,

$$\Pi_B(X) = z (a^*(X))^{\alpha} X^{\beta} - (1+\tau_B) P(a^*(X)) - \phi_B.$$

Given that $a^*(X)$ is monotonic (to be verified in equilibrium), its inverse function is

$$x^*(A) \equiv \left\{ \frac{1+\tau_B}{\alpha z} P'(A) A^{1-\alpha} \right\}^{\frac{1}{\beta}}.$$

This is a skill level of firms which demand projects of quality A.

To derive an equilibrium price function $\{P(A)\}_{A\in[A_{\min},A_{\max}]}$ in this general environment, we need to explicitly formulate a market-clearing condition and find a price schedule that clears all markets. Fix project quality level $a \in [A_{\min}, A_{\max}]$ and consider supply of, and demand for, projects of quality a. For the supply, target firms with (a, X) must satisfy $\Pi_T(A) \geq \Pi_P(A, X)$, which is equivalane to

$$X \leq \left\{ \frac{(1-\tau_T) P(a) + \phi - \phi_T}{a^{\alpha}} \right\}^{\frac{1}{\beta}} \equiv \overline{X}(a). \tag{6}$$

Similarly, bidder firms with $(a, x^*(a))$ (these are the firms who may demand projects of quality a) must satisfy $\Pi_B(x^*(a)) \geq \Pi_P(A, x^*(a))$, which is equivalane to

$$A \leq \left\{ 1 - \alpha \frac{P(a) + \frac{\phi_B - \phi}{1+\tau_B}}{P'(a)\, a} \right\}^{\frac{1}{\alpha}} z^{\frac{1}{\alpha}} a \equiv \overline{A}(a). \tag{7}$$

To formulate a market-clearing condition, let the CDF and PDF of project quality be G_A and g_A, and those of skill be G_X and g_X, where G_A and G_X are assumed to be independent. Then, at a particular value of $A = a$, a measure (density) of firms with project quality a is $g_A(a)$. Among these firms, only those who satisfy (6) are willing to be targets. Therefore, the supply *density* of projects of quality a is

$$S(a) \equiv g_A(a)\, G_X\left(\overline{X}(a)\right). \tag{8}$$

Similarly, a measure of firms who (potentially) demand project quality a is $g_X(x^*(a))$. Among these firms, only those who satisfy (7) are willing to be bidders. Therefore, the demand density of projects of quality a is

$$D(a) \equiv g(x^*(a))\, G_A\left(\overline{A}(a)\right). \tag{9}$$

Recall that targets with A and bidders with X can form a match only if $x^*(A) = X$. The market-clearing condition is

$$\int_{A_{\min}}^{a} S(A)\, dA = \int_{X_{\min}}^{x^*(a)} D(a^*(X))\, dX \qquad \text{for any } a \in [A_{\min}, A_{\max}]. \tag{10}$$

To use the change of variable,

$$\begin{aligned}
\frac{dX}{dA} &= \frac{dx^*(A)}{dA} = \left(\frac{1+\tau_B}{\alpha z}\right)^{\frac{1}{\beta}} \frac{1}{\beta} \left\{P'(A)\, A^{1-\alpha}\right\}^{\frac{1}{\beta}-1} \left\{P''(A)\, A^{1-\alpha} + P'(A)(1-\alpha)\, A^{-\alpha}\right\} \\
&= \frac{1}{\beta}\left(\frac{1+\tau_B}{\alpha z}\right)^{\frac{1}{\beta}} \left\{P'(A)\right\}^{\frac{1-\beta}{\beta}} A^{\frac{1-\alpha}{\beta}} \left\{P''(A) + (1-\alpha)\frac{P'(A)}{A}\right\}.
\end{aligned}$$

Therefore, differentiating (10) with respect to a yields

$$S(a) = D(a)\frac{1}{\beta}\left(\frac{1+\tau_B}{\alpha z}\right)^{\frac{1}{\beta}} \left\{P'(a)\right\}^{\frac{1-\beta}{\beta}} a^{\frac{1-\alpha}{\beta}} \left\{P''(a) + (1-\alpha)\frac{P'(a)}{a}\right\}.$$

All in all, the price function $P(A)$ must satisfy, for any $A \in [A_{\min}, A_{\max}]$,

$$\begin{aligned} g_A(A)\, G_X\left(\overline{X}(A)\right) &= g_X(x^*(A))\, G_A\left(\overline{A}(A)\right) \\ &\quad \times \frac{1}{\beta}\left(\frac{1+\tau_B}{\alpha z}\right)^{\frac{1}{\beta}} \{P'(A)\}^{\frac{1-\beta}{\beta}} A^{\frac{1-\alpha}{\beta}} \left\{P''(A) + (1-\alpha)\frac{P'(A)}{A}\right\}. \end{aligned} \tag{11}$$

Because $S(A)$ depends on $P(A)$ while $D(A)$ depends on $P(A)$ and $P'(A)$, (11) defines a second-order differential equation in $P(A)$.

Imposing uniform distributions $G_A(A) = A$ and $G_X(X) = X$ on $[0,1]$, (11) can be simplified. Let $\frac{P'(A)A}{P(A)} \equiv \eta_P(A)$ denote the elasticity of $P(A)$ and $\frac{P''(A)A}{P'(A)} \equiv \eta_{P'}(A)$ denote the elasticity of $P'(A)$. Using η_P and $\eta_{P'}$ (suppressing their dependence on A), (11) becomes

$$\left\{(\eta_P - \alpha) P - \alpha\frac{\phi_B - \phi}{1+\tau_B}\right\}(\eta_{P'} + 1 - \alpha)^{\alpha} = \beta^{\alpha}\alpha^{\frac{\alpha}{\beta}}\left(\frac{1-\tau_T}{1+\tau_B}\right)^{\frac{\alpha}{\beta}}\left(\frac{\eta_P P}{z}\right)^{1-\frac{\alpha}{\beta}}\left(P - \frac{\phi_T - \phi}{1-\tau_T}\right)^{\frac{\alpha}{\beta}} \tag{12}$$

4.1 Two special cases

If $\phi_B = \phi_T = \phi$, then (12) depends on $P(A)$ only through η_P and $\eta_{P'}$. Alternatively, if $\alpha = \beta$, then (12) becomes linear in P. These two special cases admit the following characterization.

We conjecture a form of price function:

$$P(A) = c_1 A^{c_0} + c_2, \tag{13}$$

which implies

$$P'(A) = c_1 c_0 A^{c_0 - 1} \qquad \text{and} \qquad P''(A) = c_1 c_0 (c_0 - 1) A^{c_0 - 2}. \tag{14}$$

In particular, $\eta_{P'} = \frac{P''(A)A}{P'(A)} = c_0 - 1$ so that $\eta_{P'} + 1 - \alpha$ in (12) becomes $c_0 - \alpha$. The corner condition $x^*(1) = 1$ (i.e., the best types match together) implies

$$1 = \left\{ \frac{1+\tau_B}{\alpha z} P'(1) \right\}^{\frac{1}{\beta}} \quad \Leftrightarrow \quad P'(1) = \frac{\alpha z}{1+\tau_B} \quad \Leftrightarrow \; c_1 = \frac{z}{1+\tau_B} \frac{\alpha}{c_0}.$$

Thus, substituting (13) and (14) into (12) defines two equations in two unknowns (c_0, c_2). If there is a unique solution, then we found a unique equilibrium in the class of functions (13). Moreover, the matching function is

$$x^*(A) = A^{\frac{c_0-\alpha}{\beta}} \quad \Leftrightarrow \quad a^*(X) = X^{\frac{\beta}{c_0-\alpha}}.$$

This is monotonically increasing if and only if $c_0 > \alpha$. We obtain the following results.

Proposition 2 *Let* $K \equiv \left(\frac{1-\tau_T}{1+\tau_B}\right)^{\frac{1}{1+\alpha}} \in (0,1)$.

(a) *If* $\alpha = \beta$, *then the matching function is* $x^*(A) = A^K$ *and the price function is*

$$P(A) = \frac{1}{1+K} \left\{ \frac{zA^{c_0} - (\phi_B - \phi)}{1+\tau_B} + \frac{K(\phi_T - \phi)}{1-\tau_T} \right\},$$

$$\textit{where } c_0 = (1+K)\alpha \quad \in (\alpha, 2\alpha).$$

(b) *If* $\phi_B = \phi_T = \phi$, *then the matching function is* $x^*(A) = A^{\frac{c_0-\alpha}{\beta}}$ *and the price function is* $P(A) = \frac{z}{1+\tau_B}\frac{\alpha}{c_0}A^{c_0}$, *where* $c_0 > \alpha$ *is a unique solution to*

$$\left(\frac{c_0-\alpha}{c_0}\right)^{1+\alpha} = z^{\frac{\alpha-\beta}{\beta}} \left(\frac{\beta}{c_0}\right)^{\alpha} \left(\frac{\alpha}{c_0}\right)^{\frac{\alpha}{\beta}} K^{(1+\alpha)\frac{\alpha}{\beta}}.$$

Remark 1 *Setting* $\tau_T = \tau_B = 0$ *in the case (a) nests the benchmark case with* $P(A) = \frac{1}{2}(zA^{2\alpha} + \phi_T - \phi_B)$.

Proposition 3 *Assume* $\alpha = \beta$ *and consider a matched pair* $(A, x^*(A))$.

(a) *Equilibrium firm values* $\Pi_T(A)$ *and* $\Pi_B(x^*(A))$ *satisfy*

$$\Pi_T(A) = K^\alpha \Pi_B(x^*(A)) + (1 - K^\alpha)\frac{1-K}{1+K}\phi.$$

(b) *If* $\phi > 0$*, then* $\Pi_T(A) < \Pi_B(x^*(A)) \Leftrightarrow A > \underline{A}$*, where* $\underline{A}$ *is uniquely defined by* $\frac{1-K}{1+K}\phi = \Pi_T(A)$*. The ratio* $\frac{\Pi_T(A)}{\Pi_B(x^*(A))}$ *decreases in* A*.*

(c) *If* $\phi < 0$*, then* $\Pi_T(A) < \Pi_B(x^*(A))$ *for all* A*. The ratio* $\frac{\Pi_T(A)}{\Pi_B(x^*(A))}$ *increases in* A*.*

Proposition 3 shows that with $\tau_B + \tau_T > 0$, the size of target firm relative to its matched bidder firm becomes smaller as A increases. For sufficiently large A, target firms are smaller than matched bidder firms. If, additionally, $\phi \leq 0$ holds, i.e., there is a fixed cost common to targets and bidders, then target firms are always smaller than bidder firms.

5 Conclusion

We studied the effect of trading costs in a competitive matching model of takeovers. The model yields novel and testable predictions. The model can be extended in many directions. In particular, studying selection patterns in a general asymmetric environment, giving all four options to firms, should help us understand how multi-dimensional firm heterogeneity and trading costs interact each other.

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6 Appendix

6.1 Derivation of the market-clearing condition

By substituting $g_A(A) = g_X(X) = 1$, $G_X\left(\overline{X}(A)\right) = \overline{X}(A)$, and $G_A\left(\overline{A}(X)\right) = \overline{A}(X)$, (11) becomes

$$\begin{aligned}\left\{\frac{(1-\tau_T)P(A)+\phi-\phi_T}{A^\alpha}\right\}^{\frac{1}{\beta}} &= \left\{1-\alpha\frac{P(A)+\frac{\phi_B-\phi}{1+\tau_B}}{P'(A)A}\right\}^{\frac{1}{\alpha}} z^{\frac{1}{\alpha}}A \\ &\times\frac{1}{\beta}\left(\frac{1+\tau_B}{\alpha z}\right)^{\frac{1}{\beta}}\{P'(A)\}^{\frac{1-\beta}{\beta}}A^{\frac{1-\alpha}{\beta}}\left\{P''(A)+(1-\alpha)\frac{P'(A)}{A}\right\}.\end{aligned}$$

Rearranging yields

$$\begin{aligned}\left\{1-\alpha\frac{P(A)+\frac{\phi_B}{1+\tau_B}}{P'(A)A}\right\}zA^\alpha &= \left\{\frac{(1-\tau_T)P(A)+\phi-\phi_T}{A^\alpha}\right\}^{\frac{\alpha}{\beta}} \\ &\times\beta^\alpha\left(\frac{\alpha z}{1+\tau_B}\right)^{\frac{\alpha}{\beta}}\frac{\{P'(A)\}^{\alpha\frac{\beta-1}{\beta}}A^{\alpha\frac{\alpha-1}{\beta}}}{\left\{P''(A)+(1-\alpha)\frac{P'(A)}{A}\right\}^\alpha}.\end{aligned}$$

Manipulating this further,

$$1-\alpha\frac{P(A)+\frac{\phi_B}{1+\tau_B}}{P'(A)A} = \left\{\frac{(1-\tau_T)P(A)+\phi-\phi_T}{A^\alpha}\right\}^{\frac{\alpha}{\beta}}\frac{\beta^\alpha}{z}\left(\frac{\alpha z}{1+\tau_B}\right)^{\frac{\alpha}{\beta}}\frac{\{P'(A)\}^{\alpha\frac{\beta-1}{\beta}}A^{\alpha\frac{\alpha-1}{\beta}}}{\{P''(A)A+(1-\alpha)P'(A)\}^\alpha}$$

$$\alpha\frac{P(A)+\frac{\phi_B}{1+\tau_B}}{P'(A)A} = \frac{\left[\begin{array}{l}\{P''(A)A+(1-\alpha)P'(A)\}^\alpha \\ -\frac{\beta^\alpha}{z}\left(\frac{\alpha z}{1+\tau_B}\right)^{\frac{\alpha}{\beta}}\{(1-\tau_T)P(A)+\phi-\phi_T\}^{\frac{\alpha}{\beta}}\{P'(A)\}^{\alpha\frac{\beta-1}{\beta}}A^{\alpha\left(\frac{\alpha-1}{\beta}-\frac{\alpha}{\beta}\right)}\end{array}\right]}{\{P''(A)A+(1-\alpha)P'(A)\}^\alpha}$$

$$\frac{1}{\alpha}\frac{P'(A)A}{P(A)+\frac{\phi_B}{1+\tau_B}} = \frac{\{P''(A)A+(1-\alpha)P'(A)\}^{\alpha}}{\{P''(A)A+(1-\alpha)P'(A)\}^{\alpha}-\frac{\beta^{\alpha}}{z}\left(\frac{\alpha z}{1+\tau_B}\right)^{\frac{\alpha}{\beta}}\left\{\frac{(1-\tau_T)P(A)+\phi-\phi_T}{A}\right\}^{\frac{\alpha}{\beta}}\{P'(A)\}^{\alpha\frac{\beta-1}{\beta}}}$$

$$\begin{aligned}&\left(\frac{1}{\alpha}\frac{P'(A)A}{P(A)+\frac{\phi_B}{1+\tau_B}}-1\right)\{P''(A)A+(1-\alpha)P'(A)\}^{\alpha}\\ =\ &\frac{1}{\alpha}\frac{P'(A)A}{P(A)+\frac{\phi_B}{1+\tau_B}}\frac{\beta^{\alpha}}{z}\left(\frac{\alpha z}{1+\tau_B}\right)^{\frac{\alpha}{\beta}}\left\{\frac{(1-\tau_T)P(A)+\phi-\phi_T}{A}\right\}^{\frac{\alpha}{\beta}}\{P'(A)\}^{\alpha\frac{\beta-1}{\beta}}\end{aligned}$$

$$\begin{aligned}&\frac{\frac{P'(A)A}{P(A)+\frac{\phi_B}{1+\tau_B}}-\alpha}{\frac{P'(A)A}{P(A)+\frac{\phi_B}{1+\tau_B}}}\{P''(A)A+(1-\alpha)P'(A)\}^{\alpha}\\ =\ &\frac{\beta^{\alpha}}{z}\left(\frac{\alpha z}{1+\tau_B}\right)^{\frac{\alpha}{\beta}}\left\{\frac{(1-\tau_T)P(A)+\phi-\phi_T}{P'(A)A}\right\}^{\frac{\alpha}{\beta}}\{P'(A)\}^{\alpha}\end{aligned}$$

$$\frac{\frac{P'(A)A}{P(A)+\frac{\phi_B-\phi}{1+\tau_B}}-\alpha}{\frac{P'(A)A}{P(A)+\frac{\phi_B-\phi}{1+\tau_B}}}\left\{\frac{P''(A)A}{P'(A)}+1-\alpha\right\}^{\alpha}=\frac{\beta^{\alpha}}{z}\left(\frac{\alpha z}{1+\tau_B}\right)^{\frac{\alpha}{\beta}}\left\{\frac{(1-\tau_T)P(A)-(\phi_T-\phi)}{P'(A)A}\right\}^{\frac{\alpha}{\beta}}$$

$$\frac{\eta_P-\alpha\left(1+\frac{\phi_B-\phi}{1+\tau_B}\frac{1}{P}\right)}{\eta_P}(\eta_{P'}+1-\alpha)^{\alpha}=z^{\frac{\alpha}{\beta}-1}\beta^{\alpha}\alpha^{\frac{\alpha}{\beta}}\left(\frac{1-\tau_T}{1+\tau_B}\right)^{\frac{\alpha}{\beta}}\left(\frac{1-\frac{\phi_T-\phi}{1-\tau_T}\frac{1}{P}}{\eta_P}\right)^{\frac{\alpha}{\beta}}.$$

Multiply $\eta_P P$ to obtain

$$\left\{(\eta_P-\alpha)P-\alpha\frac{\phi_B-\phi}{1+\tau_B}\right\}(\eta_{P'}+1-\alpha)^{\alpha}=\beta^{\alpha}\alpha^{\frac{\alpha}{\beta}}\left(\frac{1-\tau_T}{1+\tau_B}\right)^{\frac{\alpha}{\beta}}\left(\frac{\eta_P P}{z}\right)^{1-\frac{\alpha}{\beta}}\left(P-\frac{\phi_T-\phi}{1-\tau_T}\right)^{\frac{\alpha}{\beta}}.$$

6.2 Proofs

To be written.