

# Monetary Policy and Asset Bubbles in a Small Open Economy

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## Abstract

This paper examines the effects of monetary policy on fluctuations in asset bubbles in a small open economy. The paper's model incorporates a basic New Keynesian setup to introduce monetary policy. A Taylor type rule is followed in response of asset bubbles (the "leaning against the wind" policy). In this environment, there is a possibility that when a policy maker reacts strongly to bubbles, the bubbles changes that are caused by an unexpected foreign interest shock become small and welfare is improved more than in a situation wherein the Taylor rule is not used with regard to asset bubbles.

Keywords: bubbles, monetary policy, leaning against the wind policy, financial frictions, small open economy model.

JEL codes: E44, E52, F41

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# 1 Introduction

After the 2007 - 2009 financial crisis, the debate on the role of monetary policy has become more heated. Some believe that monetary policy should be used to stabilize the inflation rate, which is its conventional target. However, others consider that policy makers not only achieve inflation stabilization but also attend to asset markets in stabilizing the financial system. For the latter argument, increasing the interest rate is a desirable method to prevent rapid asset price inflation even if doing so generates a temporal gap between the inflation rate and its target. This type of policy is often called a “leaning against the wind policy.”

One plausible interpretation of large movements in asset prices is that these movements are caused by the existence of bubbles. Galí (2014)<sup>\*1</sup> theoretically analyzes the relationship between bubbles and a leaning against the wind policy. He shows that a policy with a positive interest rate response to bubbles leads to an increase in the volatility of bubbles and that if the size of bubbles is sufficiently large, optimal response to bubbles is negative. However his analysis is restricted to a closed economy. In developing countries, policy makers suffer a burden of a large movement of capital inflows to stabilize economic fluctuations and often adopt a leaning against the wind policy (e.g., Terrier et al. (2011)). This movement occurs with a large decrease in asset price (e.g., Mendoza (2010)). However, in emerging markets, the main policy target is the exchange rate. Thus, the present paper examines the effect of a leaning against the wind policy, which targeting to bubbles on economic fluctuations in developing countries.

This paper provides an open economic model introducing asset bubbles within a New Keynesian setup <sup>\*2</sup> and examines the role of monetary policy. In this model,

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<sup>\*1</sup> In addition to Galí (2014), other papers have also investigated the relationship between bubbles and monetary policy in a closed economy, such as Asriyan et al. (2016), Hirano et al. (2018), Ikeda (2019), Caraianni et al. (2020), and Galí (2021).

<sup>\*2</sup> There are existing papers that use a New Keynesian small open economy model with financial frictions such as Gertler et al. (2007), Aoki et al. (2016), and Davis and Presno (2017).

the Taylor rule is followed to react to the inflation rate and bubbles. When this rule reacts only to the inflation rate, the resulting monetary policy is considered conventional. However, when the Taylor rule is used to react to bubbles, the policy is considered a leaning against the wind policy. The main results are as follow: (1) According to foreign interest rate shocks, a leaning against the wind policy stabilizes changes in bubbles but has a negligible effect on changes of net exports. (2) Increasing the coefficient of bubbles has a stronger stabilizing effect on bubbles and the inflation rate if price rigidity is strong. (3) With respect to the welfare effect of the consumption-equivalent measure, the leaning against the wind policy can improve welfare by imposing a larger weight on bubbles terms of the Taylor rule. In contrast, when the price rigidity is high, the leaning against the wind policy would suppress welfare.

Another feature of this model is the explicit introduction of a borrowing constraint. This constraint significantly affects on the economy. In this model, when the foreign interest rate increases, borrowing constraints become severe, and investment decreases. This leads to a decline in the domestic demand of outputs. This lead to decrease in the price of capital and net worth also decreases.<sup>\*3</sup>. Thus, the demand for bubble assets decreases, and the size of bubbles also decreases. When the real interest rate is increased because of increases in the foreign interest rate, if the Taylor rule reacts to bubbles, a decrease in bubbles will lead to suppressing increases in the real interest rate. This effect is enhanced by high coefficients of bubbles terms and relaxes the borrowing constraint. Thus, decreases in investment are weakened and movement of bubbles is stabilized. However, when price rigidity is high, bubbles increase, but the real interest rate decreases after the initial period that the foreign shock occur. Thus, this policy stabilizes the movement of bubbles in this case.

In this model, there two types of agents: entrepreneurs who can invest capital and hold bubble assets and workers who supply labor and can not hold bubble assets. The leaning against the wind policy stabilizes the movement of bubbles, and the decrease

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<sup>\*3</sup> This channel for bubbles the similar to that in Hirano et al. (2018).

in entrepreneurs' net worth is diminished. Thus, the decrease in their consumption is lowered, and their welfare can be improved. This policy weakens a decrease in wages and thus a decrease in workers' consumption is diminished and their welfare can also be improved. However, when price rigidity is high, bubbles and wages increase. Because this policy stabilizes these movements, entrepreneurs and workers can-not gain from this policy, and their welfare can be suppressed.

This paper is related to many previous papers pertaining to rational bubbles. Some early contributions include Samuelson (1958), Tirole (1985), and Weil (1987). The present paper follows the literature on rational bubbles in an economy with financial frictions and heterogeneous agents\*<sup>4</sup> such as Kocherlakota (2009), Farhi and Tirole (2012), Martin and Ventura (2012), Aoki et al. (2014), Hirano et al. (2015), Hirano and Yanagawa (2016), and Kunieda and Shibata (2016) . The aforementioned papers examine the conditions of the existence of bubbly equilibria in a closed economy and show that bursting bubbles can replicate the movements of the 2007 - 2009 financial crisis.

The paper also relates to the literature on rational bubbles in an open economy with financial frictions. Some typical papers are Caballero and Krishnamurthy (2006), Basco (2014), Martin and Ventura (2015), Motohashi (2016), Ikeda and Phan (2019), and Miao et al. (2021). These papers analyze the conditions of the existence of bubbly equilibria and the effects of bubbles on a macro-economy. However, The aforementioned papers do not examine the effects of monetary policy.\*<sup>5</sup>.

The present paper is structured as follows. Section 2 details the construction a small open economy model with a New Keynesian setup. Section 3 includes a characterization of a steady-state equilibrium with bubbles. Section 4 examines the effect of monetary policy on the dynamics of a bubble economy. Section 5 concludes the paper.

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\*<sup>4</sup> Mitsui and Watanabe (1989) introduce the same set-up and show the conditions for the existence of money.

\*<sup>5</sup> Miao et al. (2021) analyze the relationship between foreign interest shocks and bubbles. They show that an increase in a foreign interest rate can lead to fluctuations and that the existence of bubbles amplifies these fluctuations.

## 2 Model

This paper considers a small open economy. There are entrepreneurs, workers, intermediate goods producers and final goods producers.

Final goods are only consumed in the home country and are made from intermediate goods. Final goods producers face a perfectly competitive market and have such technology as follows:

$$Y_t^d = \left( \int_0^1 (y_t^i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}, \quad (1)$$

where  $\eta > 1$ ,  $y_t^i$  is intermediate goods  $i$  and  $Y_t^d$  is final goods. From their optimal choice, the demand function for intermediate goods  $i$  is as follows:

$$y_t^i = \left( \frac{p_t^i}{P_t} \right)^{-\eta} Y_t^d. \quad (2)$$

$P_t$  is the price of final goods, and  $p_t^i$  is the price of each intermediate good. Thus, the price of final goods is written as follows:

$$P_t = \left( \int_0^1 (p_t^i)^{1-\eta} di \right)^{\frac{1}{1-\eta}}. \quad (3)$$

Intermediate goods producers face a monopolistically competitive market and input a capital, imported goods and labor to produce intermediate goods. Their production technology is as follows:

$$y_t^i = A k_t^{i\alpha_K} m_t^{i\alpha_M} \ell_t^{i1-\alpha_K-\alpha_M}, \quad (4)$$

where  $0 < \alpha_K < 1$ ,  $0 < \alpha_M < 1$  and  $\alpha_K + \alpha_M < 1$ .  $A$  is total factor productivity.  $k_t^i$  is capital goods,  $\ell^i$  is labor, and  $m_t^i$  is imported goods, which is the foreign county's goods. From their optimal choice, their marginal cost is as follows:

$$mc_t = \frac{1}{A} \left( \frac{Q_t}{\alpha_K} \right)^{\alpha_K} \left( \frac{\epsilon_t}{\alpha_M} \right)^{\alpha_M} \left( \frac{w_t}{1 - \alpha_K - \alpha_M} \right)^{1-\alpha_K-\alpha_M}. \quad (5)$$

$Q_t$  is the real capital price, and  $w_t$  is the real wage.  $\epsilon_t$  is the relative price of final goods to foreign goods, in that ,  $\epsilon_t = e_t^n P^f / P_t$ .  $e_t^n$  is the nominal exchange rate.  $P^f$  is price of foreign goods and is constant.

Intermediate goods are demanded by the home country agent and the foreign countries. Thus, intermediate goods producers meet domestic and foreign demand. This paper assumes that their prices are set according to producer currency pricing and thus law of one price holds (i.e.,  $p_t^i = e_t^n p_t^{f,i}$  where  $p_t^{f,i}$  is the price of intermediate goods in foreign countries). They pay the adjustment costs when they change their price.\*<sup>6</sup> They choose the current price to maximize their objective function. This function is as follows:

$$\sum_{t=0}^{\infty} \Lambda_{0,t} [(\frac{p_t^i}{P_t} - mc_t)y_t^i + y_t^{f,i} - \frac{\kappa}{2}(\frac{p_t^i}{p_{t-1}^i} - 1)^2], \quad (6)$$

where  $\kappa > 0$  and  $\Lambda_{0,t}$  is the pricing kernel. The second term is an adjustment cost (e.g., menu costs).  $y_t^{f,i}$  is foreign demand and is assumed to be the following:

$$y_t^{f,i} = (\frac{p_t^i}{e_t^n P^f})^{\eta^f} Y^f, \quad (7)$$

$$\int_0^1 y_t^{f,i} di \equiv EX_t, \quad (8)$$

where  $Y^f$  is foreign income and constant.  $EX_t$  is aggregate exports.

The first order conditions is as follows:

$$\begin{aligned} \pi_t(\pi_t - 1) = \frac{1}{\kappa}(\eta mc_t + 1 - \eta)Y_t^d + \frac{1}{\kappa}(\eta^f mc_t + 1 - \eta^f)(\frac{1}{\epsilon_t})^{-\eta^f} Y^f \\ + \Lambda_{t,t+1}\pi_{t+1}(\pi_{t+1} - 1). \end{aligned} \quad (9)$$

This equation is evaluated at the symmetric equilibrium.  $\pi_t$  is the domestic inflation rate i.e.,  $\pi_t = P_t/P_{t-1}$ . At the symmetric equilibrium, aggregate capital, imports and labor are as follows:

$$K_t = \int_0^1 k_t^i di, \quad L_t = \int_0^1 \ell_t^i di, \quad M_t = \int_0^1 m_t^i di.$$

At the symmetric equilibrium, the optimal conditions for the aggregate level of demand for inputs are as follows:

$$\frac{\epsilon_t M_t}{Q_t K_t} = \frac{\alpha_M}{\alpha_K}, \quad (10)$$

$$\frac{w_t L_t}{Q_t K_t} = \frac{1 - \alpha_K - \alpha_M}{\alpha_K}. \quad (11)$$

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\*<sup>6</sup> This paper follows Rotemberg (1982).

Next, let us consider the behavior of entrepreneurs. Entrepreneurs continuously exist in  $(0, 1)$ . They invest in a project that, in turn, makes a capital. The capital fully depreciates over one period. This project is as follows:

$$k_{t+1}^j = \alpha_t^j z_t^j,$$

where  $z_t^j$  is the level of investment.  $j$  is the index of entrepreneurs, and  $\alpha^j$  is the marginal productivity of this project. Entrepreneurs meet randomly high ( $\alpha^H > 0$ ) or low ( $\alpha^L > 0$ ) productivity each period. The probability that they encounter high (low) productivity is  $p$  ( $1 - p$ ). H-entrepreneurs have high productivity and are potential borrowers. L-entrepreneurs have low productivity and are potential domestic lenders. This is because H-entrepreneurs have higher productivity, and L-entrepreneurs obtain higher returns from lending than investing in their own technology.

Each entrepreneur's utility function is as follows:

$$\sum_{t=0}^{\infty} \beta^t \log c_t^j, \quad (12)$$

where  $c_t^j$  is each entrepreneur's consumption of the domestic goods and  $\beta$  is the discount factor (less than one). Their flow constraint is as follows:

$$c_t^j + z_t^j + P_t^b x_t^j = Q_t \alpha_{t-1}^j z_{t-1}^j - R_t b_{t-1}^j - R^f \epsilon_t b_{t-1}^{fj} + P_t^b x_{t-1}^j + b_t^j + \epsilon b_t^{fj}, \quad (13)$$

$$R_t = \frac{1 + i_{t-1}}{\pi_t}, \quad (14)$$

$$R^f = \frac{1 + i^f}{\pi_t^f}, \quad (15)$$

where  $x_t$  is the level of purchased bubble assets which is useless assets;  $P_t^b$  is bubbles;  $R_t$  is a domestic gross real interest rate;  $b_t^j$  is a domestic bond;  $R_t^f$  is the foreign gross real interest rate;  $b_t^{fj}$  is a foreign bond;  $i_t$  is a domestic nominal interest rate;  $i_t^f$  is a foreign nominal interest rate; and  $\pi_t^f$  is the foreign inflation rate (i.e.,  $\pi_t^f = P_t^f / P_{t-1}^f$ ). This paper defines net worth as  $e_t^j = Q_t \alpha_{t-1}^j z_{t-1}^j - R_t b_{t-1}^j - R^f \epsilon b_{t-1}^{fj} + P_t^b x_{t-1}^j$ . Since the foreign goods price is constant,  $\pi^f$  is one at any time, and  $R^f$  is constant and equal to  $i^f$ .

This paper assumes that there are imperfect domestic and foreign credit markets and that these markets are segmented. Thus, entrepreneurs face different borrowing

constraints when they borrow from the domestic market and the foreign market. <sup>\*7</sup> Entrepreneurs can borrow from domestic (foreign) lenders the rate of, at most, a fraction  $\theta$  ( $\theta^f$ ) of future profit. Because this model is deterministic, their borrowing constraints<sup>\*8</sup> are as follows:

$$R_{t+1}b_t^j \leq \theta Q_{t+1}k_{t+1}^j, \quad (16)$$

$$R^f \epsilon_t b_t^{fj} \leq \theta^f Q_{t+1}k_{t+1}^j, \quad (17)$$

where  $0 < \theta < 1$ ,  $0 < \theta^f < 1$  and  $\theta + \theta^f < 1$ . Since each entrepreneur has log-utility and does not make any transfers, they only consume a constant fraction  $(1 - \beta)$  of their net worth and save the res. Thus, their consumption is as follows:

$$c_t^j = (1 - \beta)e_t^j. \quad (18)$$

This paper focuses on the situation in which H-entrepreneurs are bounded by both borrowing constraints, that is,  $Q_{t+1}\alpha^H > R_{t+1}$  and  $Q_{t+1}\alpha^H > \pi_{t+1}^*R^f$  where  $\pi_t^*$  is the gross growth rate of  $\epsilon_t$  (i.e.,  $\pi_t^* = \epsilon_t/\epsilon_{t-1}$ ). The level of H-entrepreneurs' aggregate investment is as follows:

$$Z_t^H = \frac{\beta p E_t}{1 - \alpha^H Q_{t+1} \left( \frac{\theta}{R_{t+1}} + \frac{\theta^f}{\pi_{t+1}^* R^f} \right)}, \quad (19)$$

where  $E_t$  is their aggregate net worth. If H-entrepreneurs hold bubble assets, returns on bubbles are equal to returns in terms of their investment. In this case, L-entrepreneurs spend all of their net worth to purchase bubble assets and do not lend to H-entrepreneurs. Thus, H-entrepreneurs do not hold bubble assets.

L-entrepreneurs lend to H-entrepreneurs. This paper assumes that  $R_{t+1} \geq \pi_{t+1}^*R^f$  because H-entrepreneurs borrow from domestic lenders. Thus, if  $R_{t+1} > \pi_{t+1}^*R^f$ , L-entrepreneurs only lend to domestic borrowers. When the degree of financial frictions

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<sup>\*7</sup> In Aoki et al. (2010), borrowers face different levels of constraint when they borrow from domestic and foreign creditors. There is only one lead creditor who is a domestic creditor and other outside creditors who include foreign creditors. The lead creditor and outside creditors face different levels of restriction on borrowing because they have different skills of monitoring production and bargaining.

<sup>\*8</sup> This formulation follows Kiyotaki and Moore (1997), Aoki et al. (2010), and Hirano and Yanagawa (2016).

is high, they do not lend all of their net worth but instead spend it on other assets. Bubbles offset this shortage, and only L-entrepreneurs hold bubble assets<sup>\*9</sup>. To hold bubble assets at equilibrium, the return on bubbles is equal to the domestic gross real interest rate (*i.e.*,  $P_{t+1}^b/P_t^b = R_{t+1}$ ). When the return on bubble assets is equal to the return on investment or lending to foreign countries, L-entrepreneurs will invest in their project or lend to the foreign country. However, their productivity is sufficiently low, and the return on investment is less than the foreign interest rate.<sup>\*10</sup> Therefore, L-entrepreneurs do not invest in their project at the equilibrium with bubbles. With this constraint, the aggregate level of purchased bubble assets is as follows:

$$P_t^b X_t = \beta E_t + B_t^{fH} + B_t^{fL} - Z_t^H, \quad (20)$$

where  $B_t^{fH}$  is the aggregate quantity of H-entrepreneurs' foreign bonds and  $B_t^{fL}$  is the aggregate quantity of L-entrepreneurs' foreign bonds. This paper defines the ratio of purchased bubble assets to total savings as  $\psi_t$  (*i.e.*,  $\psi_t = P_t^b X/\beta E_t$ ). From the

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<sup>\*9</sup> L-entrepreneurs cannot use the return on bubble assets for collateral. Thus, they cannot borrow from foreign countries to hold bubble assets.

<sup>\*10</sup> If the return on investment is greater than the foreign interest rate, L-entrepreneurs borrow from foreign countries until they reach the borrowing constraint. Thus, the return on investment is as follows:

$$\frac{(1 - \theta^f)Q_{t+1}\alpha^L}{1 - \frac{Q_{t+1}\alpha^L}{\pi_{t+1}^* R^f}}.$$

Since they have bubbles assets, the following conditions holds at the equilibrium:

$$\frac{(1 - \theta^f)Q_{t+1}\alpha^L}{1 - \frac{Q_{t+1}\alpha^L}{\pi_{t+1}^* R^f}} = \frac{P_{t+1}^b}{P_t^b}.$$

This case rewritten as follows:

$$Q_{t+1}\alpha^L = \frac{\pi_{t+1}^* R^f}{\frac{P_{t+1}^b}{P_t^b} + \pi_{t+1}^* R^f (1 - \theta^f)}.$$

The denominator of the right hand is greater than one except the case in which the domestic gross real interest rate lower than one and  $\theta^f$  is sufficiently high. Thus,  $Q_{t+1}\alpha - L$  is less than  $\pi_{t+1}^* R^f$ . However, in this situation, L-entrepreneurs do not borrow from foreign countries.

above equation,  $\psi_t$  is as follows:

$$\psi_t = 1 - \left(1 - \frac{\theta^f Q_{t+1} \alpha^H}{R^f}\right) \phi_t p + \frac{B_t^{fL}}{\beta E_t}, \quad (21)$$

$$\text{where } \phi_t = \frac{1}{1 - Q_{t+1} \alpha^H \left(\frac{\theta}{R_{t+1}} + \frac{\theta^f}{\pi_{t+1}^* R^f}\right)}. \quad (22)$$

$\phi$  is leverage.

Let us consider behavior of workers. This paper assumes that workers hold equities in intermediate goods firms because they are more impatient than entrepreneurs (i.e.,  $\beta < \beta^w$ ) and do not hold collateral when they borrow from entrepreneurs<sup>\*11</sup>. Since intermediate goods producers are symmetrical, prices of equity are also symmetrical. Workers supply inelastic labor ( $L = 1$ ) to intermediate goods producers. Their utility is as follows:

$$\sum_{t=0}^{\infty} (\beta^w)^t \log C_t^w, \quad (23)$$

where  $C_t^w$  is workers' consumption. Their budget constraints are as follows:

$$c_t^w + q_t^n s_t^n = w_t + (q_t^n + \xi_t) s_{t-1}^n, \quad (24)$$

$$\xi_t = \int_0^1 [(1 - mc_t) y_t^i + (1 - mc_t) y_t^{f^i} - \frac{\kappa}{2} (\pi_t - 1)^2] di, \quad (25)$$

$q^n$  is the price of equity in the intermediate goods firm.  $s^n$  represents shares of the equity in the intermediate goods firm.  $\xi_t$  is profit from the production of intermediate goods. The first-order conditions for workers are as follows:

$$\frac{c_{t+1}^w}{\beta c_t^w} = \frac{\xi_t + q_{t+1}^n}{q_t^n},$$

$$\Lambda_{t,t+1} = \frac{c_{t+1}^w}{\beta c_t^w}.$$

Because workers hold equity, the pricing kernel is derived from their optimal conditions.

In this paper, monetary policy follows a Taylor type rule as follows:

$$1 + i_t = R \left(\frac{\pi_t}{\pi}\right)^{\omega_\pi} \left(\frac{Y_t^G}{Y^G}\right)^{\omega_Y} \left(\frac{P_t^b}{P^b}\right)^{\omega_{P^b}}, \quad (26)$$

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<sup>\*11</sup> This paper assumes that entrepreneurs can-not hold equity.

where  $Y^G$  is the gross domestic product (GDP) (i.e.,  $Y^G \equiv Y_t^d + EX_t - \epsilon_t M_t$ );  $\omega_\pi$  is the coefficient of the inflation rate;  $\omega_{Y^G}$  is it of GDP, and  $\omega_{P^b}$  is it of bubbles. When  $\omega_{P^b}$  is zero, the policy maker chooses the inflation rate and domestic goods as targets. This can be considered a conventional policy. When  $\omega_{P^b}$  is not zero, the policy maker adds bubbles as a policy target and applies a leaning against the wind policy to this model.

### 3 Equilibrium

This section summarizes the equilibrium conditions. Each capital letter denotes an aggregate variable. The domestic goods market clearing conditions is as follow:

$$Y_t^d = C_t^H + C_t^L + C_t^w + \frac{\kappa}{2}(\pi_t - 1)^2 + Z_t^H + Z_t^L. \quad (27)$$

The domestic credit market, the bubble asset market, and a stock market clearing conditions are as follows:

$$B_t^H + B_t^L = 0, \quad (28)$$

$$P_t^b X_t = P_t^b X, \quad (29)$$

$$s_t^n = 1, \quad (30)$$

$$(31)$$

where  $X$  is the aggregate supply of bubble assets and is constant. Net foreign debt evolves from the following dynamics;

$$B_t^{fH} + B_t^{fL} = R^f(B_{t-1}^{fH} + B_{t-1}^{fL}) + M_t - \epsilon_t^{\eta^f - 1} Y^f. \quad (32)$$

The capital dynamic is as follows:

$$K_{t+1} + K_{t+1}^n = \alpha^H Z_t^H + \alpha^L Z_t^L. \quad (33)$$

The competitive equilibrium is given by price variables ( $w_t, Q_t, \epsilon_t, \pi_t, \pi_t^*, R_t, R^f, i_t, P_t^b, q_t^n$ ) and endogenous variables ( $Y_t, K_t, M_t, L_t, C_t^H, C_t^L, C_t^w, Z_t^H, Z_t^L, B_t^H, B_t^L, B_t^{fH}, B_t^{fL}, E_t, EX_t, c_t^j, z_t^j, x_t^j, b_t^j, b_t^{fj}, s_t^n$ ), which are composed of agents' optimal choice and market clearing conditions. In particular, if  $P_t^b$  is positive at any  $t$ , this equilibrium is called a "bubbly equilibrium." This paper focuses on the bubbly equilibrium.

### 3.1 Steady State

Let us consider a bubbly equilibrium at a steady state.<sup>\*12</sup> Recall that this paper focuses on the case in which H-entrepreneurs borrow to hit both borrowing constraints, and the L-entrepreneurs lend to domestic H-entrepreneurs, hold bubble assets, and do not invest. If the return on bubbles is equal to the return on lending to foreign countries, the sum of the level of purchased bubble assets and lending to foreign countries is determined. Thus, the paper assumes that L-entrepreneurs do not lend to foreign countries at the steady state. Therefore, with regard to a steady-state equilibrium with bubbles, the following conditions must hold:

$$\begin{aligned} 0 &< \psi < 1, \\ R^f &> Q\alpha^L \\ R &< Q\alpha^H. \end{aligned}$$

At the steady state, the gross growth rate of net worth is one. This is represented as follows:

$$1 = Q\alpha^H \phi \beta (1 - \theta^f) + \beta \psi. \quad (34)$$

In addition,  $\psi$  at the steady state is as follows:

$$\psi = 1 - \phi p \left(1 - \frac{\theta^f Q \alpha^H}{R^f}\right). \quad (35)$$

By combining these two equations.  $Q$  at the steady state is obtained by following:

$$Q = \frac{R^f \mathcal{A}}{\alpha^H \mathcal{B}}, \quad (36)$$

$$\mathcal{A} \equiv 1 - \beta + \beta p, \quad (37)$$

$$\mathcal{B} \equiv (1 - \beta) R^f \theta + (1 - \beta + \beta p (1 - R^f)) \theta^f + R^f \beta p. \quad (38)$$

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<sup>\*12</sup> Of course, a steady-state equilibrium without bubbles exists. In the steady state without bubbles, the domestic real interest rate is equal to the foreign interest rate; thus, the gross inflation rate is greater than one. Because the real interest rate is less than one if the foreign interest rate is less than, the growth rate is higher than the real interest rate in the steady state.

From the above equations, the parameters of monetary policy are not included in the equations so that the monetary policy does not affect the ratio of bubbles to total savings in the steady-state equilibrium. Substitute  $Q$  and  $\psi$  into the conditions of existence of bubbles at the steady state. The conditions are rewritten as follows:

$$\mathcal{B} < R^f \mathcal{A}, \quad (39)$$

$$\mathcal{B} > \mathcal{A} \left( \frac{R^f}{1-p} \theta + \theta^f \right), \quad (40)$$

$$\mathcal{B} > \mathcal{A} \frac{\alpha^L}{\alpha^H}. \quad (41)$$

From this, the following proposition is provided\*<sup>13</sup>:

**Proposition 1.** *If  $\alpha^L/\alpha^H > R^f\theta/(1-p) + \theta^f$  and the following conditions holds, bubbles exist at the steady state as follows:*

$$\theta > \max \left( 0, \frac{\frac{\alpha^L}{\alpha^H} \mathcal{A} - (1-\beta + \beta p(1-R^f))\theta^f - R^f \beta p}{(1-\beta)R^f} \right),$$

$$\theta < \min \left( \left( \frac{\alpha^L}{\alpha^H} - \theta^f \right) \frac{1-p}{R^f}, 1 - \frac{(1-\beta + \beta p(1-R^f))\theta^f}{(1-\beta)R^f} \right).$$

*If  $\alpha^L/\alpha^H \leq R^f\theta/(1-p) + \theta^f$  and the following conditions holds, bubbles exist at the steady state as follows:*

$$\theta > \max \left( 0, \left( \frac{\alpha^L}{\alpha^H} - \theta^f \right) \frac{1-p}{R^f} \right),$$

$$\theta < \min \left( (1-p)\beta(1-\theta^f), 1 - \frac{(1-\beta + \beta p(1-R^f))\theta^f}{(1-\beta)R^f} \right).$$

As in previous research, bubbles exist in a range in which  $\theta$  is neither sufficiently high nor low. This reason for this is as follows. When  $\theta$  is sufficiently high, L-entrepreneurs can lend all their savings to H-entrepreneurs and do not hold bubbles. However, when  $\theta$  is sufficiently low, financial frictions are so strong that H-entrepreneurs cannot invest enough in their project and, thus, the supply of capital tends to be small. Therefore, the price of capital is relatively high and the returns on L-entrepreneurs' investments are higher than that of bubble assets. From the foregoing, L-entrepreneurs do not hold bubble assets at equilibrium.

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\*<sup>13</sup> The proof is provided in Appendix.

## 4 Policies

This section examines the influence of monetary policy. The present paper focuses on the equilibrium at which the return from domestic lending is higher than foreign lending (i.e.,  $R_{t+1} > \pi_{t+1}^* R_t^f$ )<sup>\*14</sup>. The monetary policy affects the dynamics of the model herein. In emerging markets, foreign interest rate shocks are one of the main drivers of the business cycle (e.g., Canova (2005) and Uribe and Yue (2006)). Thus, let us consider the situation in which the economy is at the steady state at first and an unexpected increase in the foreign interest rate occurs in Period 1. This increase is 1 % and the foreign interest rate returns to the original level according to the following AR(1) process:  $R_{t+1}^f = (1 - \rho^i)R^f + \rho^i R_t^f$ . Therefore, the economy returns to its original steady state<sup>\*15</sup>.

This paper numerically demonstrates the dynamics caused by foreign positive interest rate shocks and examines the policy effect. Some parameters follow Aoki et al. (2016) ( $\beta = 0.985, \zeta = 0.2, \zeta_0 = 5.89, \eta = 9, \alpha_K = 0.3, \alpha_M = 0.15$ ); Hirano and Yanagawa (2016) ( $\alpha^H = 1.1, \alpha^L = 1.05, p = 0.1$ ); Simonovska and Waugh (2014) ( $\eta^f = 5$ )<sup>\*16</sup>. Other parameters are fixed in terms of  $\kappa = 10, \theta = 0.3, \theta^f = 0.25, \beta^w = 0.97, R^f = 0.99, A = 1, \rho^i = 0.95, \omega^\pi = 1.5, \omega^{Y^G} = 0.5$ , and  $Y^f = 1$ . These parameters ensure the bubbly equilibrium.  $R^f$  is below one, and the gross domestic real interest rate is greater than it is at the equilibrium.

### 4.1 Leaning Against the Wind Policy

In this model, the leaning against the wind policy is represented by the Taylor rule can react to bubbles, as with Galí (2014). Let us consider a foreign interest rate

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<sup>\*14</sup> I consider that the equilibrium where the return from domestic lending is equal to foreign lending (i.e.,  $R_{t+1} > \pi_{t+1}^* R_t^f$ ) in the Appendix

<sup>\*15</sup> In this model, there are two equilibria: a bubbly equilibrium and a non-bubbly equilibrium. Thus, the model does not ensure that bubbles remain at equilibrium when unexpected shocks occur. However, if all entrepreneurs believe that bubbles remain, bubbles exist in the equilibrium after the occurrence of shocks. This paper focuses on the latter case.

<sup>\*16</sup> They estimate the trade elasticity of substitution using macroeconomic data.

shock and characterize the effect of a leaning against the wind policy by  $\omega_{Pb}$ .

(Figures 1 and 2 around here)

Figure 1 shows the dynamics in this situation. The vertical line is the rate of change from the steady state. The red (dashed) line is the benchmark case (i.e.,  $\omega_{Pb} = 0$ ). Thus, monetary policy only reacts to the inflation rate and seemingly stands with the conventional view. The blue (solid) line is the case in which monetary policy responds to bubbles (i.e.,  $\omega_{Pb} = 0.5$ ). The black (dotted) line is the case in which monetary policy responds to bubbles (i.e.,  $\omega_{Pb} = 1.5$ ).

From Figure 1, the higher  $\omega_{Pb}$  is, the fewer the changes in bubbles and GDP are. In addition, bubbles gently recover to their original level because the  $R_t$  is equal to the growth rate of bubbles. However, changes in the inflation rate are amplified by increasing  $\omega_{Pb}$ . Thus, according to a positive foreign interest shock, a leaning against the wind policy stabilizes the movement of bubbles but does not stabilize the inflation rate. Figure 1 shows that net exports are rapidly increased by a foreign interest rate shock and lead to a rapid decrease in capital inflow. When  $\omega_{Pb}$  is high, the increases in the net exports decrease at the initial period but increase after the period. However, the effects are negligible.

From Figure 2, an increase in the foreign interest rate enhances the borrowing constraint from abroad and thus decreases borrowing from abroad. Because net exports increase and capital inflow consequently decreases, domestic currency depreciates and  $\epsilon$  increases. Thus, imports decrease and output decreases in the initial period. Because investment decreases, the level of capital decreases; thus, output decreases after the initial period.

In this model, bubbles are determined by the demand for bubble assets because the supply is constant. From (20) and definition of net worth, the demand for bubble assets is rewritten as follows:

$$P_t^b X_t = \frac{1}{1-\beta} (Q_t K_t - R_t^f \epsilon_t B_{t-1}^h + \epsilon_t B_t^h - Z_t). \quad (42)$$

Thus, the demand for bubble assets is mainly subject to  $Q_t$ ,  $\epsilon_t B_t$  and  $Z_t$ . The

demand for intermediate goods decrease because output and exports decrease and the demand for capital decreases. In the initial period, the supply of capital does not change owing to the foreign interest shock; thus,  $Q$  decreases. This change damages entrepreneurs' net worth. In addition, increases in the foreign interest rate prevent borrowing from abroad. These factors have a negative effect on the demand for bubble assets. Although the shock has a negative effect on demand for bubble assets, this shock leads to a decrease in investment and this has a positive effect on demand for it. Thus, there are negative and positive effects on the demand for bubble assets. However, the negative effect dominates, and the size of bubbles decreases in the initial period. After the initial period,  $Q$  increases because the decrease in investment lowers the supply of capital. However, the negative effect still dominates, and bubbles are low compared to its steady state level.

The positive  $\omega^{P^b}$  stabilizes changes in the domestic gross real interest rate. This effect becomes stronger when it is high. Thus, the domestic market borrowing constraints are relaxed in this situation. This change diminishes the decrease in investment and borrowing from abroad. Thus, the decrease in imports is stabilized, and decreases in output and capital are diminished. These decreases weaken the negative effect on the demand for bubble assets, and bubbles changes are diminished.

Figure 2 also shows that an increase in the foreign interest rate leads to an increase in  $\epsilon_t$ . However, wages also decrease. The marginal costs are determined by these prices and are increased because of this shock after the initial period. When  $\omega_{P^b}$  is high, the fall in wages is substantial, and the increase in  $\epsilon$  is larger. Thus, the increase in marginal costs is amplified by an increase in  $\omega_{P^b}$ . Therefore, from (9), the degree of change in the inflation rate becomes greater when  $\omega_{P^b}$  is high.

In contrast to Galí (2014), in this model, a leaning against the wind policy can achieve some stabilization of bubbles. The intuitive reason for this difference is that there are borrowing constraints in this model. The positive foreign interest rate shock enhances the constraint for foreign borrowing and increases the domestic gross real interest rate. The policy that introduces the term of bubbles to the Taylor rule stabilizes the increase in the domestic gross real interest rate and relaxes borrowing

constraints. This stabilizes changes in the demand for bubble assets and the bubbles themselves.

Next, let us consider a case with strong price rigidity, namely  $\kappa = 44$ <sup>\*17</sup>.

(Figures 3 and 4 around here)

Figure 3 shows the dynamics after a foreign interest rate shock. Figure 3 shows that as in the previous case, the higher  $\omega_{P^b}$  is, the fewer the changes in bubbles are and that this policy has negligible the effect on changes in net exports. Unlike the previous case, changes in the inflation rate are modified by increasing  $\omega_{P^b}$ . Thus, a leaning against the wind policy stabilizes the movement of bubbles and the inflation rate in this case. However, changes in GDP are amplified when  $\omega_{P^b}$  is high.

From Figure 4, this policy stabilizes changes in  $R_t$  as in the case in which  $\kappa$  is low. After the initial period,  $R_t$  is less than zero and this relaxes constraints. Thus, this policy weakens this relaxation and amplifies the decrease in borrowing from abroad. This decrease has significantly negative effect on the demand for bubbles. As a result, an increase in demand for bubble assets and bubbles are stabilized by this policy. In contrast to the previous case, changes in  $w_t$  and  $\epsilon_t$  are stabilized; consequently, the increase in  $mc_t$  decreases. Therefore, changes in the inflation rate are stabilized. However, decreases in  $Z_t$  and  $M_t$  increase as  $\omega^{P^b}$  increases. Thus, the decrease in GDP is amplified by this policy.

## 4.2 Welfare

This subsection examines the effect of policy maker attitudes toward welfare. Let us consider that conventional monetary policy is the benchmark case in which the Taylor rule's parameters are set to  $\omega_\pi = 1.5$ ,  $\omega_{Y^G} = 0.5$ , and  $\omega_{P^b} = 0$ . This paper measures the welfare effect of the monetary policy that constantly changes benchmark consumption to make an agent indifferent between the benchmark case and another Taylor rule parameter case. This measure is defined as  $\mu$ . Let us assume that at first,

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<sup>\*17</sup> Aoki et al. (2016) use this value.

the economy is in a steady state with bubbles, all entrepreneurs hold an average net worth, and a positive foreign interest rate shock occurs at  $t = 0$ . To compare the welfare effects of various monetary policies, the transition dynamics while approaching the steady state are calculated. Thus, some entrepreneur's  $\mu$  is satisfied by the following equation:

$$E \sum_{t=0}^{\infty} \beta^t \log c_t^i = E \sum_{t=0}^{\infty} \beta^t \log((1 + \mu^e)c_t^b),$$

where  $c_t^b$  is consumption in the benchmark case and  $\mu^e$  is the entrepreneur's  $\mu$ . Similarly, workers'  $\mu$  is satisfied by the following equation:

$$\sum_{t=0}^{\infty} (\beta^w)^t \log C_t^w = \sum_{t=0}^{\infty} (\beta^w)^t \log((1 + \mu^w)C_t^{wb}).$$

$C_t^{wb}$  is consumption in the steady state and  $\mu^w$  is workers'  $\mu$ . The numerical results<sup>\*18</sup> are summarized in Tables 1 and 2.

	$\kappa = 10 : R > R^f$	$\kappa = 44 : R > R^f$
$\omega_{P^b} = 0$	0	0
$\omega_{P^b} = 0.5$	0.000013	-0.000035
$\omega_{P^b} = 1.5$	0.000024	-0.000052

Table1  $\mu^e$

	$\kappa = 10 : R > R^f$	$\kappa = 44 : R > R^f$
$\omega_{P^b} = 0$	0	0
$\omega_{P^b} = 0.5$	0.0000001	-0.0000010
$\omega_{P^b} = 1.5$	0.0000002	-0.0000014

Table2  $\mu^w$

Table 1 shows the entrepreneur's welfare measure (i.e.,  $\mu^e$ ). Table 2 shows the workers' welfare measure (i.e.,  $\mu^w$ ). The case where  $\omega_\epsilon$  and  $\omega_{P^b}$  are zero is the

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<sup>\*18</sup> This paper considers small shocks. Thus, the measures are small. The bigger the shocks is, the greater the measures are.

benchmark case. Thus,  $\mu^e$  and  $\mu^w$  are zero in that case. These tables show that entrepreneur and worker gains become larger with increases in  $\omega_{P^b}$  when price rigidity is low but these gains become smaller when price rigidity is high. From Figure 1, when  $\omega_{P^b}$  is high, the decrease in bubbles is diminished. Thus, the decrease in net worth is similarly relaxed, and entrepreneurs can consume more than when  $\omega_{P^b}$  is low. From Figure 2, when  $\omega_{P^b}$  is high, changes in wages stabilize. This leads to a small decrease in workers consumption except the initial period. Thus, there is a possibility that as foreign interest rate shocks occur, policy makers improve the welfare of entrepreneurs and workers through the policy. However, when price rigidity is strong, from Figure 3 and 4, this policy diminishes increases in bubbles, and wages; thus, it decreases entrepreneurs and workers' consumption and welfare<sup>\*19</sup>.

### 4.3 Robustness

Previous subsections show that the leaning against policy weakens changes in bubbles under the specified parameters. However, the choice of values for some parameters is arbitrary. This subsection checks whether this policy has the same effect under other values of  $\theta$ ,  $\theta^f$  and  $\omega^{Y^G}$ <sup>\*20</sup>.

(Figure 5 and 6 around here)

The lines in the figures are the same as in the previous subsection. Figure 5 demonstrates the rate of change in bubbles after a foreign interest rate shock under other parameter values of  $\theta, \theta^f$  in the case in which  $\kappa$  is 10 and Figure 6 is for the case in which  $\kappa$  is 44<sup>\*21</sup>. Figures 5 and 6 show that the leaning against the wind policy weakens changes in bubbles under these parameter value. These have an important role in the existence conditions of bubbles in a steady state and affect the level of bub-

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<sup>\*19</sup> When workers are more impatient, this policy has opposite effects on workers' welfare. This case is examined in the Appendix.

<sup>\*20</sup> There are some typical examples. I check the policy effect under many other values.

<sup>\*21</sup> The values of parameter except for  $\theta, \theta^f, \omega^{Y^G}$  are same as those in the previous subsection. When the parameters have these values, the bubbly equilibrium exists when the range of the total value of  $\theta$  and  $\theta^f$  is in almost (0.4, 0.9).

bles. However, these values slightly affect the rate of the change in bubbles. When  $\theta^f$  is low, the amount of borrowing from abroad is small. Thus, the effect of a foreign interest rate shock on bubbles are weakened.

(Figure 7 and 8 around here)

Figure 7 demonstrates the rate of change in bubbles after a foreign interest rate shock under other parameter values of  $\omega^{YG}$  in the case in which  $\kappa$  is 10 and Figure 8 is the case in which  $\kappa$  is 44. These figures show that the leaning against the window policy weakens changes in bubbles in these cases.  $\omega_{YG}$  does not change the stabilizing effect of this policy on bubbles but affects the sign of changes in bubbles. Thus, there is the possibility that the effect of this policy on entrepreneurs' welfare is the opposite because their net worth includes bubble assets. In particular, when  $\kappa = 10$  and  $\omega^{YG}$  is greater than one, since this policy decreases the increase in bubbles, entrepreneurs' gain from the policy decreases. When  $\kappa = 44$  and  $\omega^{YG} = 0$ , the decrease in bubbles is weakened; thus, entrepreneurs' gain from the policy increases.

#### 4.4 Targeting Other Assets Price

This paper assumes that policy makers can observe bubbles but do not always distinguish between bubble assets and other assets. Thus, let us consider that the Taylor rule reacts to other asset price instead of bubbles. This subsection examines four types of Taylor rules, that is, a Taylor rule reacts to  $Q_t$ , reacts to the sum of  $Q_t$  and  $P_t^b$ , reacts to  $q_t^n$  and reacts to sum of  $q_t^n$  and  $P_t^b$ .  $\omega$  denotes each type of Taylor rule's coefficient term of asset prices. Therefore, the Taylor rule is rewritten as follow:

$$1 + i_t = R \left( \frac{\pi_t}{\pi} \right)^{\omega_\pi} \left( \frac{Y_t^G}{Y^G} \right)^{\omega_Y} \left( \frac{\mathcal{X}_t}{\mathcal{X}} \right)^\omega,$$

where  $\mathcal{X} = Q, Q + P^b, q^n$  or  $q^n + P^b$ .

(Figure 9 and 10 around here)

Figure 9 demonstrates the rate of change in bubbles after a foreign interest rate shock under each type of Taylor rule in the case when  $\kappa$  is 10; Figure 10 illustrates the same

when  $\kappa$  is 44. The red (dashed) line denotes when  $\omega = 0$ . The blue (solid) line shows when  $\omega = 0.1$ . The black (dotted) line represents when  $\omega = 0.2$ <sup>\*22</sup>. Figure 9 shows that when  $\kappa = 10$ , if the Taylor rule reacts to  $Q$  or  $Q + P^b$ , it amplifies changes in bubbles, but if the Taylor rule reacts to  $q^n$  or  $q^n + P^b$ , it stabilizes them. However, from Figure 10, when  $\kappa = 44$  this policy effects on changes in bubbles are the opposite. Therefore, there are possibility that when policy makers do not distinguish between bubble assets and other assets, bubbles will fluctuate.

## 5 Conclusion

The present paper constructed a small open economy model with nominal price rigidity, monetary policy, and bubbles. The paper examined the effects of monetary policy on bubble changes with an unexpected increase in the foreign interest rate. The paper showed that when a Taylor rule reacts to bubbles, (a leaning against the wind policy), it can stabilize changes in bubbles generated by a foreign interest shock. In addition, these changes decreases as the bubbles coefficient term increases. The intuitive reason for this is that increases in the real interest rate are stabilized by the Taylor rule. This relaxes enhancement of borrowing constraint. This effect is encouraged by a larger bubble coefficient and stabilized bubble changes.

This policy amplifies the increasing wages and the relative price of foreign goods to domestic goods, which is the cost of imported goods. This leads to large changes in the marginal cost of intermediate goods and the inflation rate. However, when price rigidity is high, this policy modifies increasing wages and the relative price and thus the inflation rate.

This paper also demonstrates that, with respect to the welfare effect of the consumption equivalent measure, a leaning against the wind policy can improve welfare on entrepreneurs and workers. However, when price rigidity is high, this relation became opposite. Thus, there are possibility that this policy suppresses welfare on

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<sup>\*22</sup> When  $\omega$  is high, the equilibrium at which the return from domestic lending is higher than that from foreign lending does not exist.

entrepreneurs and workers when price rigidity is high.

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## Appendix

### A.1 Proof of Proposition 1

This subsection provides the proof of Proposition 1. From the main part of the paper, the conditions of existence of bubbles at steady state and conditions of  $\theta$  are as follows:

$$\mathcal{B} < R^f \mathcal{A}, \quad (43)$$

$$\mathcal{B} > \mathcal{A} \left( \frac{R^f}{1-p} \theta + \theta^f \right), \quad (44)$$

$$\mathcal{B} > \mathcal{A} \frac{\alpha^L}{\alpha^H}, \quad (45)$$

$$0 < \theta < 1 - \theta^f. \quad (46)$$

From these equations, when  $\alpha^L/\alpha^H > R^f \theta / (1-p) + \theta^f$ , the conditions of existence of bubbles at steady state are rewritten as follows:

$$\mathcal{B} < R^f \mathcal{A}, \quad (47)$$

$$\mathcal{B} > \mathcal{A} \frac{\alpha^L}{\alpha^H}, \quad (48)$$

$$\frac{\alpha^L}{\alpha^H} > \frac{R^f \theta}{1-p} - \theta^f \quad (49)$$

$$0 < \theta < 1 - \theta^f. \quad (50)$$

These conditions are characterized by  $\theta$ :

$$\theta < 1 - \frac{(1 - \beta + \beta p(1 - R^f))\theta^f}{(1 - \beta)R^f}, \quad (51)$$

$$\theta > \frac{\frac{\alpha^L}{\alpha^H} \mathcal{A} - (1 - \beta + \beta p(1 - R^f))\theta^f - R^f \beta p}{(1 - \beta)R^f}, \quad (52)$$

$$\theta < \left( \frac{\alpha^L}{\alpha^H} - \theta^f \right) \frac{1-p}{R^f}, \quad (53)$$

$$0 < \theta < 1 - \theta^f. \quad (54)$$

In the steady state, the gross growth rate of bubbles i.e.  $P_{t+1}^b/P_t^b$  is one and thus the gross domestic interest rate is one. This paper paper assumes that the gross interest

rate is higher than the gross foreign interest rate. Thus,  $R^f \leq 1$ . Therefore, the right hand side of (52) is smaller than the right hand side of (55). From (52),(53), (54) and (55), the conditions of existence of bubbles at steady state, which is characterized by  $\theta$  is as follows:

$$\theta > \max \left( 0, \frac{\frac{\alpha^L}{\alpha^H} \mathcal{A} - (1 - \beta + \beta p(1 - R^f))\theta^f - R^f \beta p}{(1 - \beta)R^f} \right),$$

$$\theta < \min \left( \left( \frac{\alpha^L}{\alpha^H} - \theta^f \right) \frac{1 - p}{R^f}, 1 - \frac{(1 - \beta + \beta p(1 - R^f))\theta^f}{(1 - \beta)R^f} \right).$$

Next, let us consider the case in which  $\alpha^L/\alpha^H < R^f\theta/(1 - p) + \theta^f$ , the conditions of existence of bubbles at steady state are rewritten as follows:

$$\mathcal{B} < R^f \mathcal{A}, \quad (55)$$

$$\mathcal{B} > \mathcal{A} \left( \frac{R^f}{1 - p} \theta + \theta^f \right), \quad (56)$$

$$\frac{\alpha^L}{\alpha^H} < \frac{R^f \theta}{1 - p} - \theta^f \quad (57)$$

$$0 < \theta < 1 - \theta^f. \quad (58)$$

These conditions are characterized by  $\theta$ :

$$\theta < 1 - \frac{(1 - \beta + \beta p(1 - R^f))\theta^f}{(1 - \beta)R^f}, \quad (59)$$

$$\theta < (1 - p)\beta(1 - \theta^f), \quad (60)$$

$$\theta > \left( \frac{\alpha^L}{\alpha^H} - \theta^f \right) \frac{1 - p}{R^f}, \quad (61)$$

$$0 < \theta < 1 - \theta^f. \quad (62)$$

From these equations, the conditions of existence of bubbles at steady state, which is characterized by  $\theta$  is as follows

$$\theta > \max \left( 0, \left( \frac{\alpha^L}{\alpha^H} - \theta^f \right) \frac{1 - p}{R^f} \right),$$

$$\theta < \min \left( (1 - p)\beta(1 - \theta^f), 1 - \frac{(1 - \beta + \beta p(1 - R^f))\theta^f}{(1 - \beta)R^f} \right).$$

## A.2 The Equilibrium That Returns from Domestic Lending and from Foreign Lending Are Equal

The main text of this paper focuses on the equilibrium in which the returns from domestic lending and foreign lending have a differential (i.e.,  $R_{t+1} > \pi_{t+1}^* R^f$ ). However, when a foreign interest rate is sufficiently high, domestic lenders lend to foreign countries. Thus, no-arbitrage conditions hold and returns from domestic lending and foreign lending are equal (i.e.,  $R_{t+1} = \pi_{t+1}^* R^f$ ). The value of  $R^f$  is one in this subsection. In this case, L-entrepreneurs lend abroad, and this lending affects the demand for bubbles assets. Let us consider the equilibrium in this case.

(Figures 11 and 12 around here)

Figures 11 and 12 show the dynamics after a foreign interest rate shock at the equilibrium such that the domestic and foreign gross real interest rates are equal. Figure 11 illustrates when price rigidity is low ( $\kappa = 10$ ), and Figure 12 illustrates when price rigidity is high ( $\kappa = 44$ ). Note that the panels of  $\epsilon B^{fL}$  demonstrate the level of the change from steady state. These figures show that as in the situation in which  $R_{t+1} > \pi_{t+1}^* R^f$  the leaning against the wind policy stabilizes the changes in bubbles but amplifies the changes in the inflation rate even if price rigidities are high. In addition, the degree of the movements is larger than the previous equilibrium. When the foreign interest rate increases, lending abroad is attractive for L-entrepreneurs. They spent net worth to the lending to the abroad and decrease holding bubbles assets. This leads to decreases in demand for bubbles assets and decreases bubbles. Thus, bubbles decrease in cases in high price rigidities and low price rigidities. In this situation this policy decreases the increase in the domestic gross real interest rate and stabilizes changes in bubbles.

### A.3 Welfare of Impatient Workers

In the main text, the leaning against the wind policy amplifies the decrease in  $C^w$  in the initial period when  $\kappa = 10$  and stabilizes it in the initial period when  $\kappa = 44$ . Thus, if workers are sufficiently impatient, they evaluate the effects of this policy highly in the period, and their welfare is subject to these evaluations. First, let us examine the effect of this policy on the dynamics of  $C^w$ ,  $w$ ,  $\xi$ ,  $P^b$ ,  $E_t$  in the situation in which workers are sufficiently impatient in the same manner of the main text.

(Figures 13 and 14 around here)

Figures 13 and 14 show the dynamics after a foreign interest rate shock in the situation in which  $\beta^w = 0.9$ . Figure 13 illustrates when price rigidity is low ( $\kappa = 10$ ), and Figure 14 illustrates when price rigidity is high ( $\kappa = 44$ ). Figures 13 and 14 show that the effects of this policy on these variables is the same as its when  $\beta^w = 0.97$ . Thus, this policy has the same effect on the welfare of entrepreneurs. Next, let us measure the welfare of workers. Table 3 shows the workers' welfare level (i.e.,  $\mu^w$ ).

	$\kappa = 10 : R > R^f$	$\kappa = 44 : R > R^f$
$\omega_{P^b} = 0$	0	0
$\omega_{P^b} = 0.5$	-0.000002	0.000002
$\omega_{P^b} = 1.5$	-0.000003	0.000003

Table3  $\mu^w$

The cases in which  $\omega_{P^b}$  are zero is the benchmark case. Thus,  $\mu^w$  is zero in that case. The table shows that workers' gains become small with increases in  $\omega_{P^b}$  in the case in which price rigidity are low. The income of workers includes wages and the profits from the production of intermediate goods. When  $\omega_{P^b}$  is high, the decrease in wages is stabilized but changes in the profit are amplified. Thus, the decrease in workers' consumption after the initial period is stabilized but in the initial period, it is amplified. Since workers are impatient, they evaluate this decrease in consumption

highly in the period; thus,  $\mu^w$  decreases with the increase in  $\omega_{Pb}$ . However, this table shows that workers' gains become larger with increases in  $\omega_{Pb}$  when price rigidity is high. In contrast to the low price rigidity case, when  $\omega_{Pb}$  is high, the increase in wages and decrease in the profit are stabilized. Workers evaluate increase in consumption highly in the period; thus,  $\mu^w$  increases with an increase in  $\omega_{Pb}$ . Therefore, there is a possibility that this policy has an opposite effect on entrepreneurs' and workers' welfare if workers are sufficiently impatient.

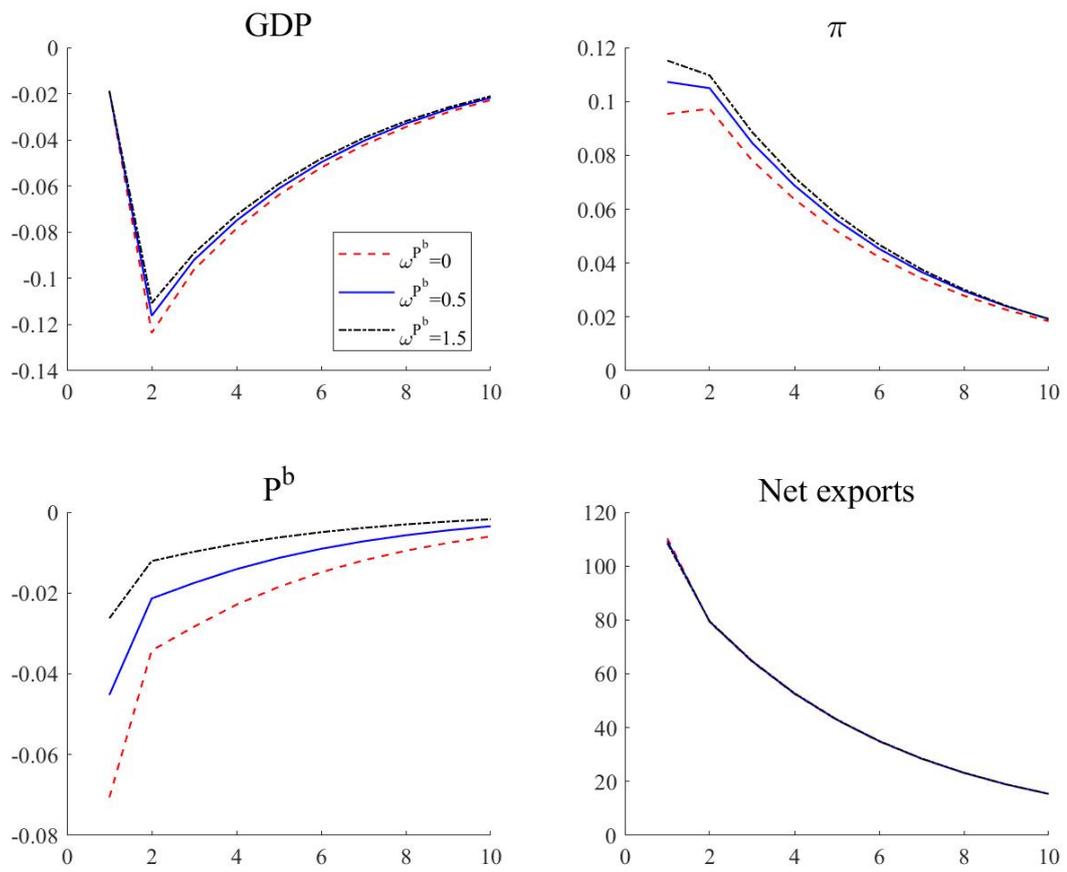


Figure1 Impulse responses to a foreign interest rate shock with a leaning against the wind policy when price rigidity is low

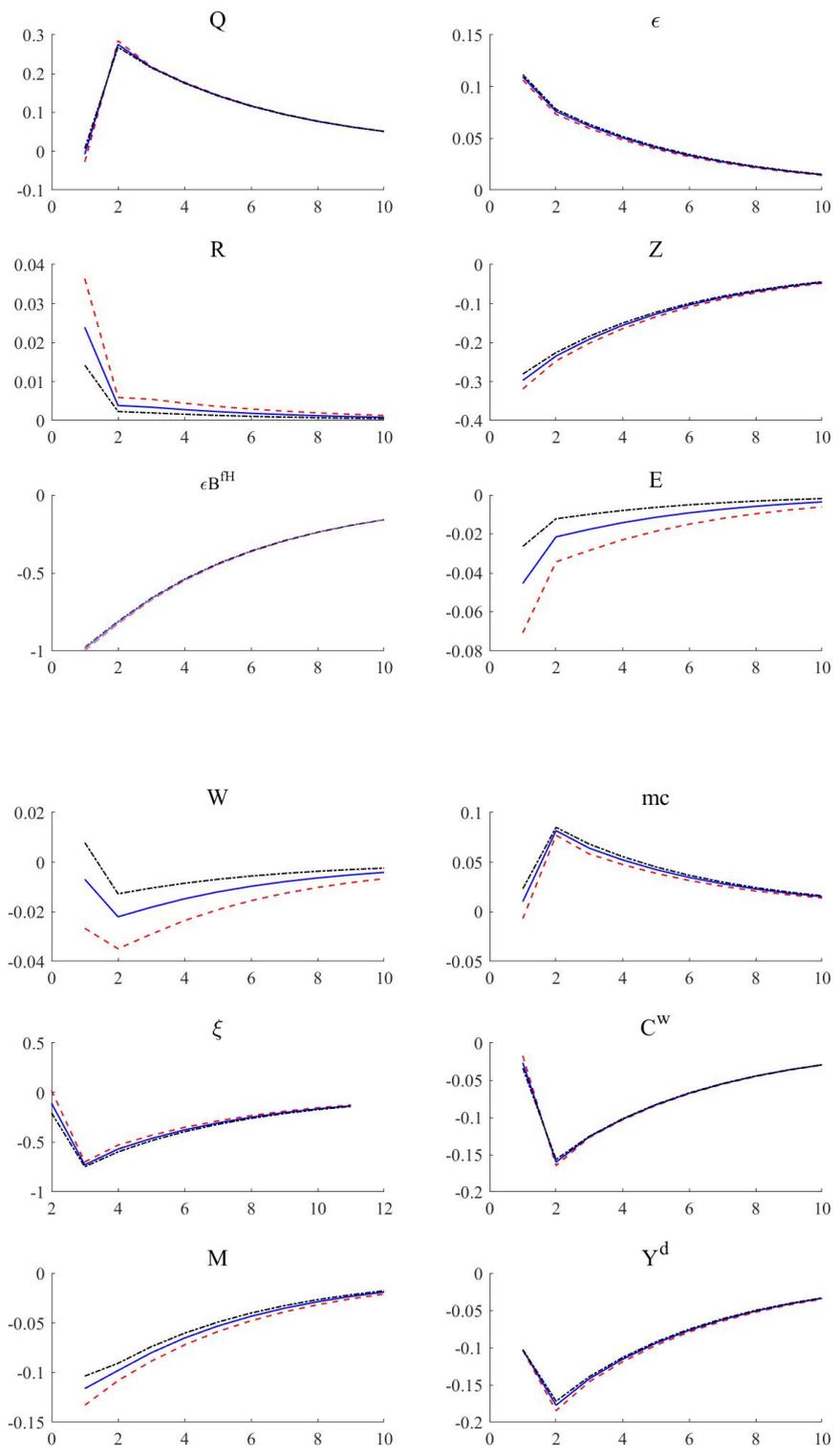


Figure2 Impulse responses to a foreign interest rate shock with a leaning against the wind policy when price rigidity is low

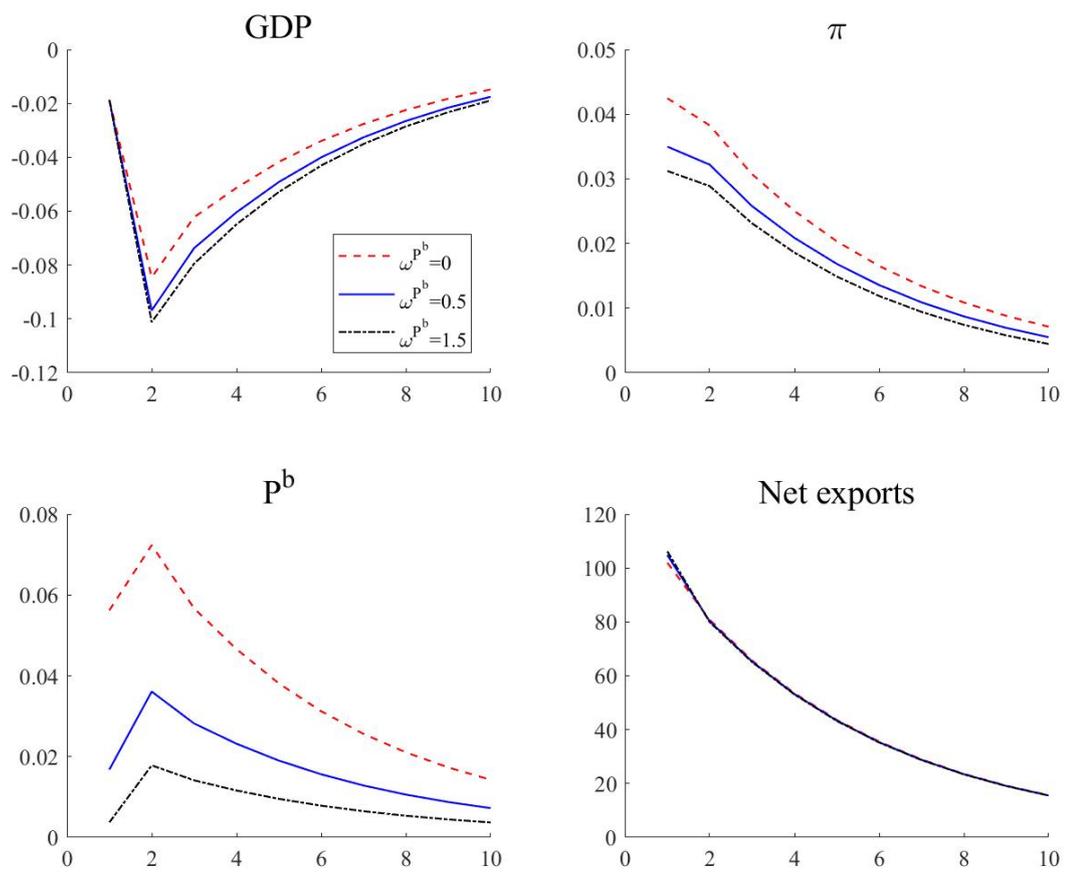


Figure3 Impulse responses to a foreign interest rate shock with a leaning against the wind policy when price rigidity is high

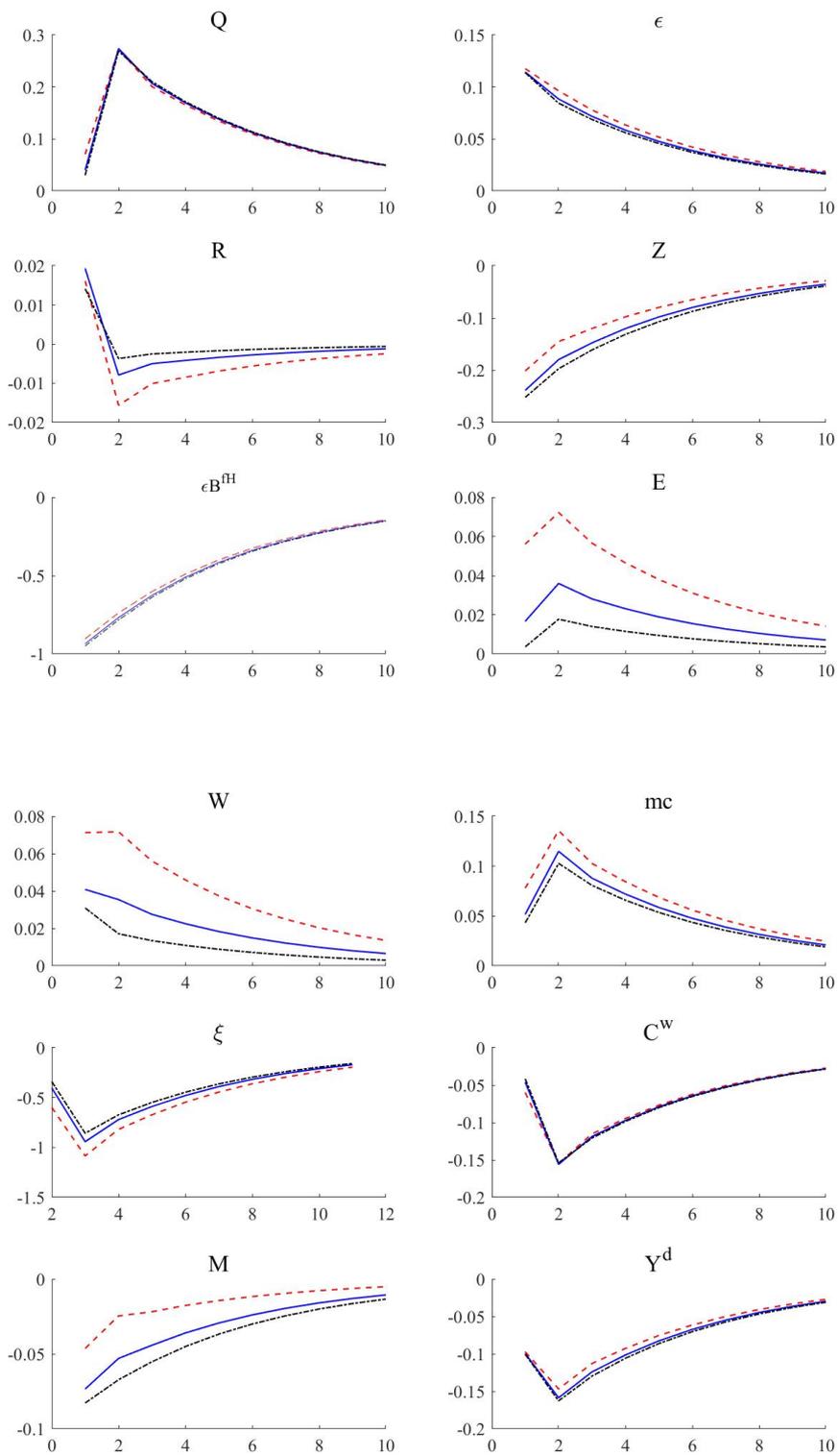


Figure4 Impulse responses to a foreign interest rate shock with a leaning against the wind policy when price rigidity is high

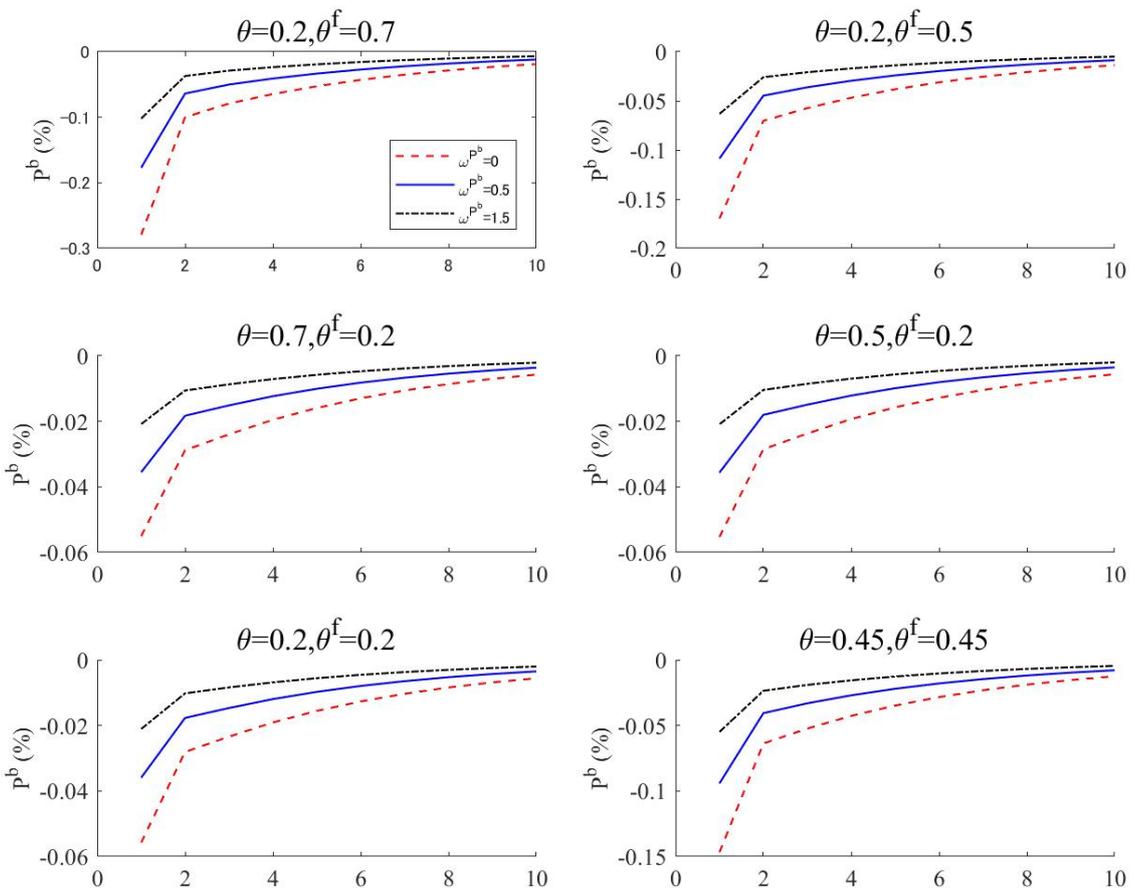


Figure5 Impulse responses to a foreign interest rate shock with another values of  $\theta$  and  $\theta^f$  when price rigidity is low

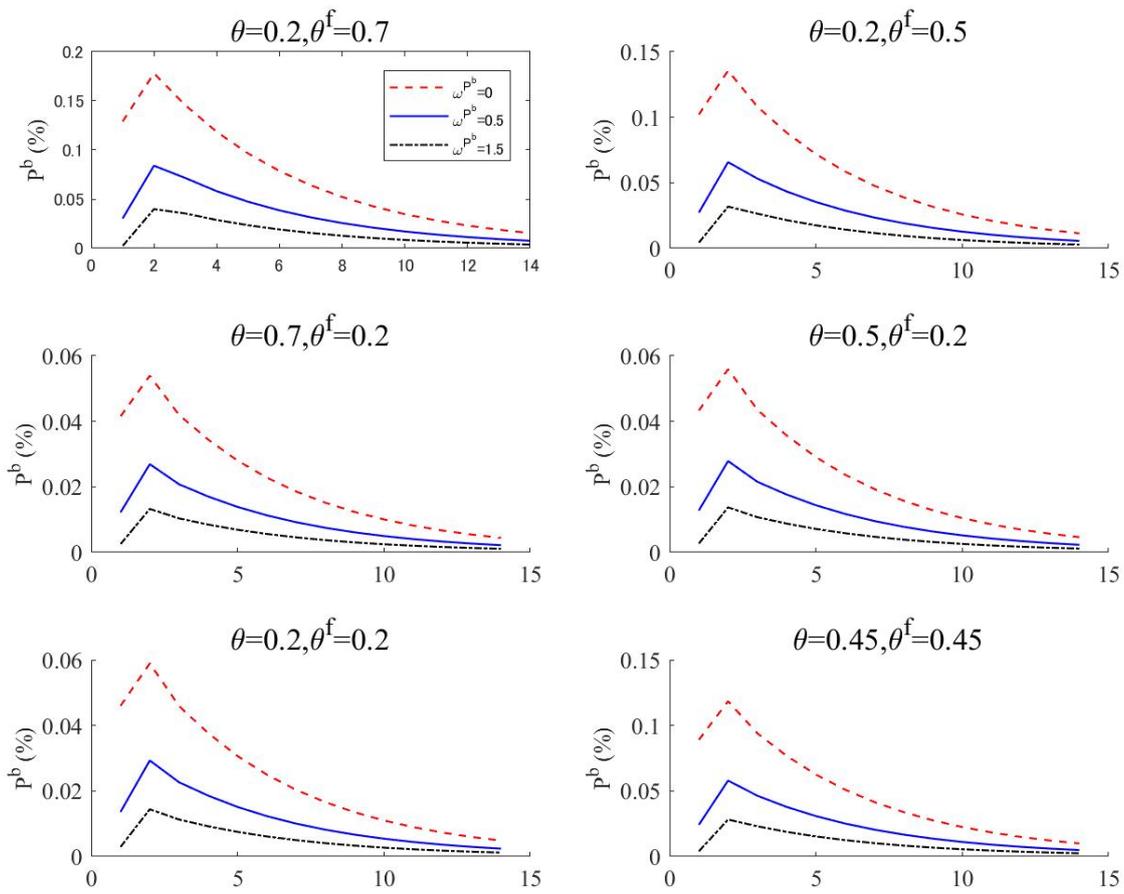


Figure6 Impulse responses to a foreign interest rate shock with with another values of  $\theta$  and  $\theta^f$  when price rigidity is high

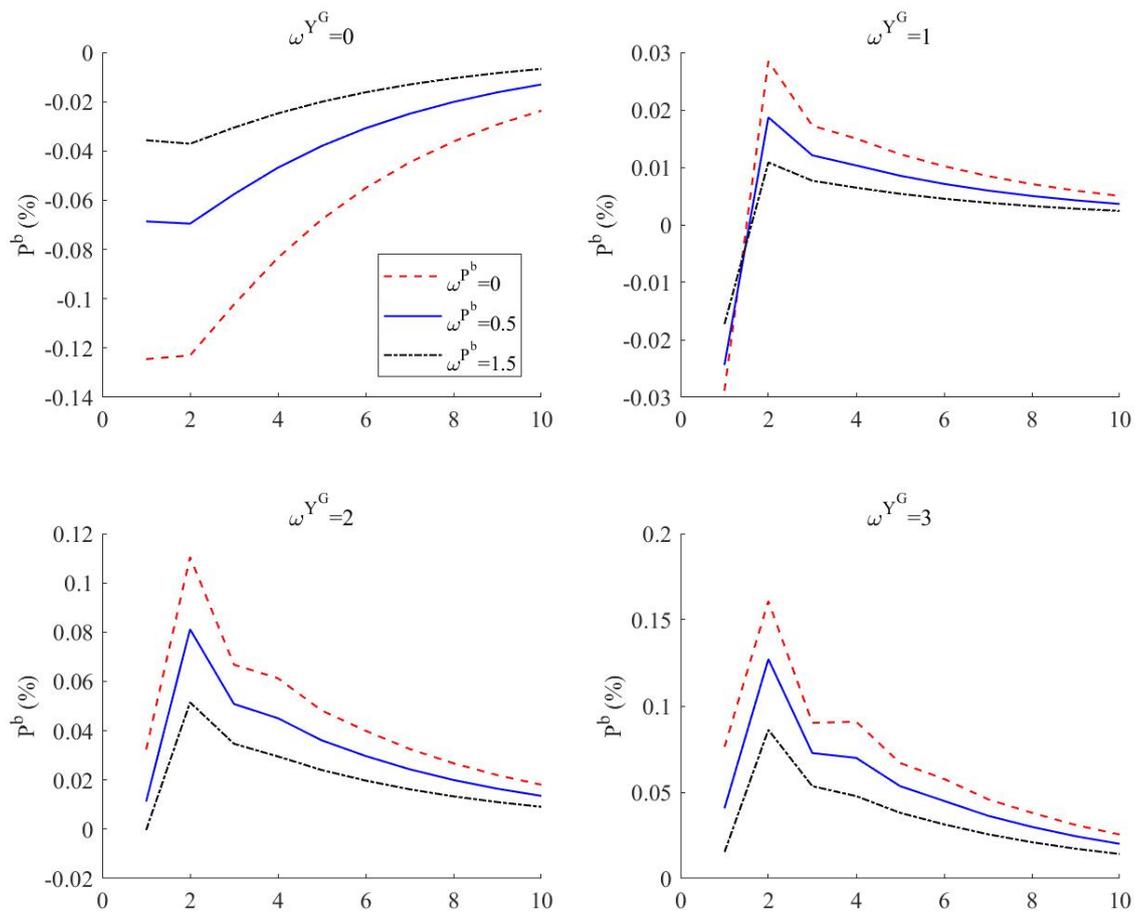


Figure7 Impulse responses to a foreign interest rate shock with another value of  $\omega^{Y^G}$  when price rigidity is low

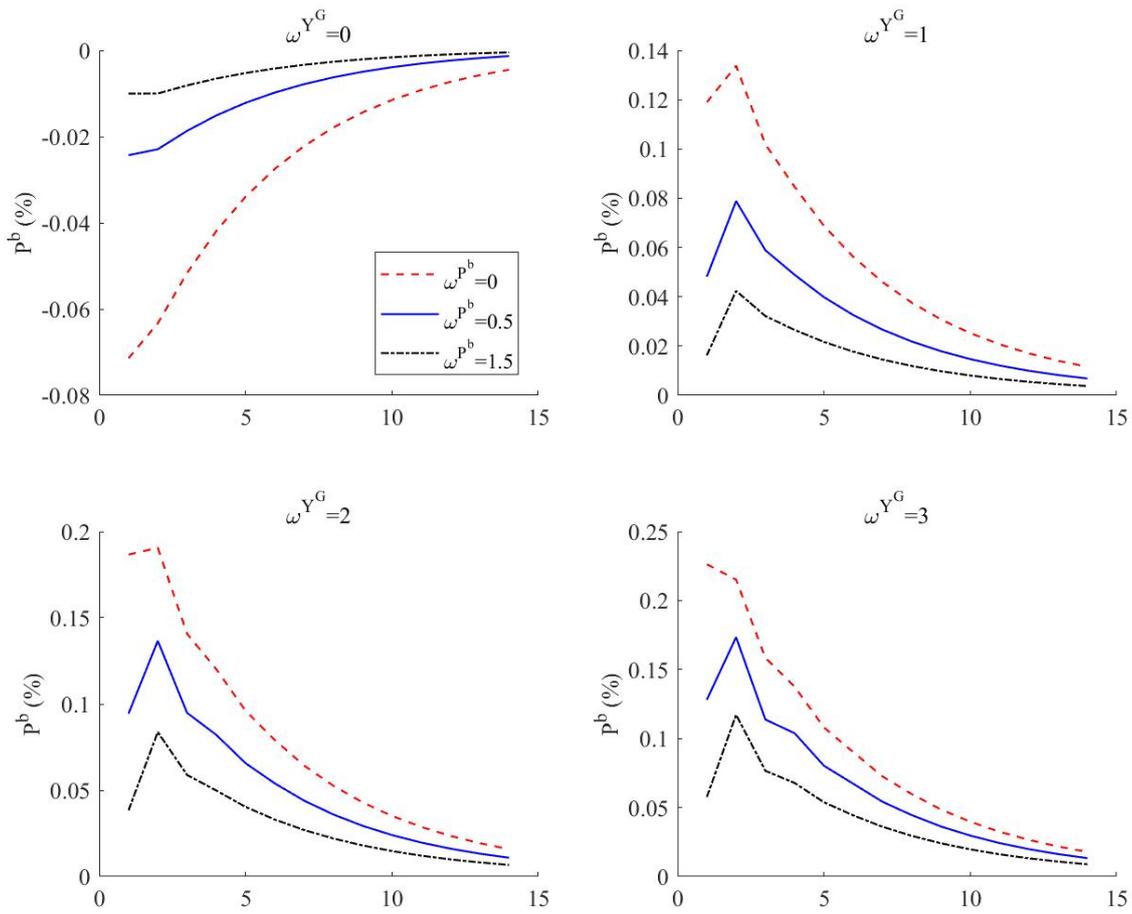


Figure8 Impulse responses to the foreign interest rate shock with another value of  $\omega^{Y^G}$  when price rigidity is high

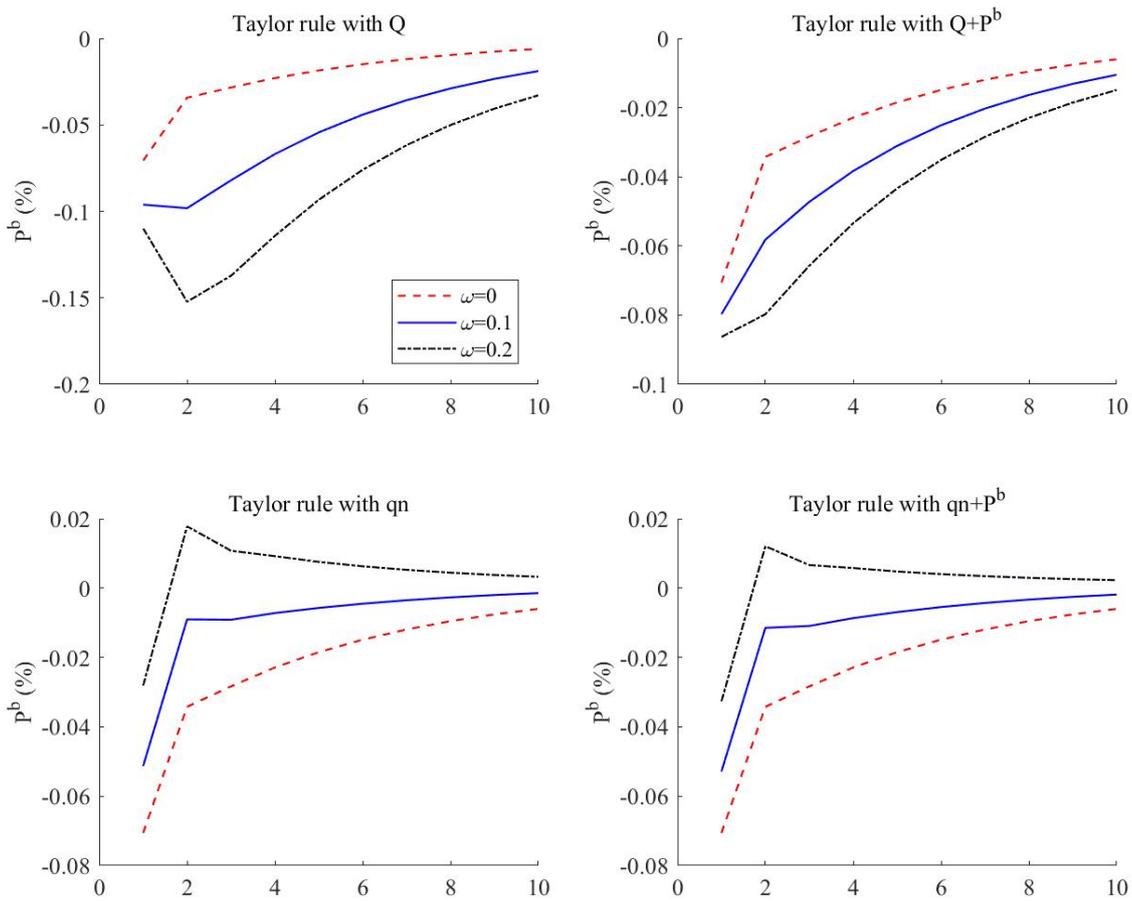


Figure9 Impulse responses to a foreign interest rate shock when the Taylor rule reacts to  $Q$ ,  $Q + P^b$ ,  $qn$  or  $qn + P^b$  instead of  $P^b$  and price rigidity is low

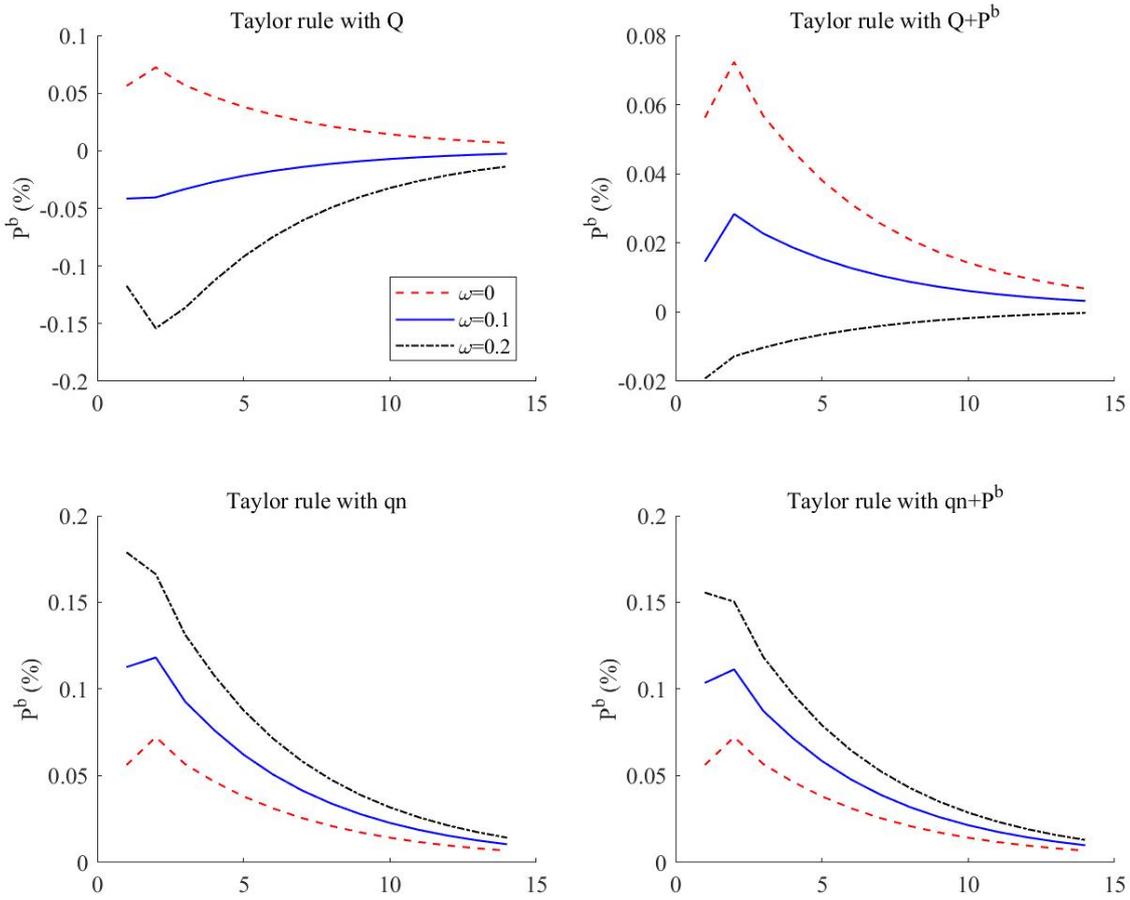


Figure10 Impulse responses to a foreign interest rate shock when the Taylor rule reacts to  $Q$ ,  $Q + P^b$ ,  $qn$  or  $qn + P^b$  instead of  $P^b$  and price rigidity is high

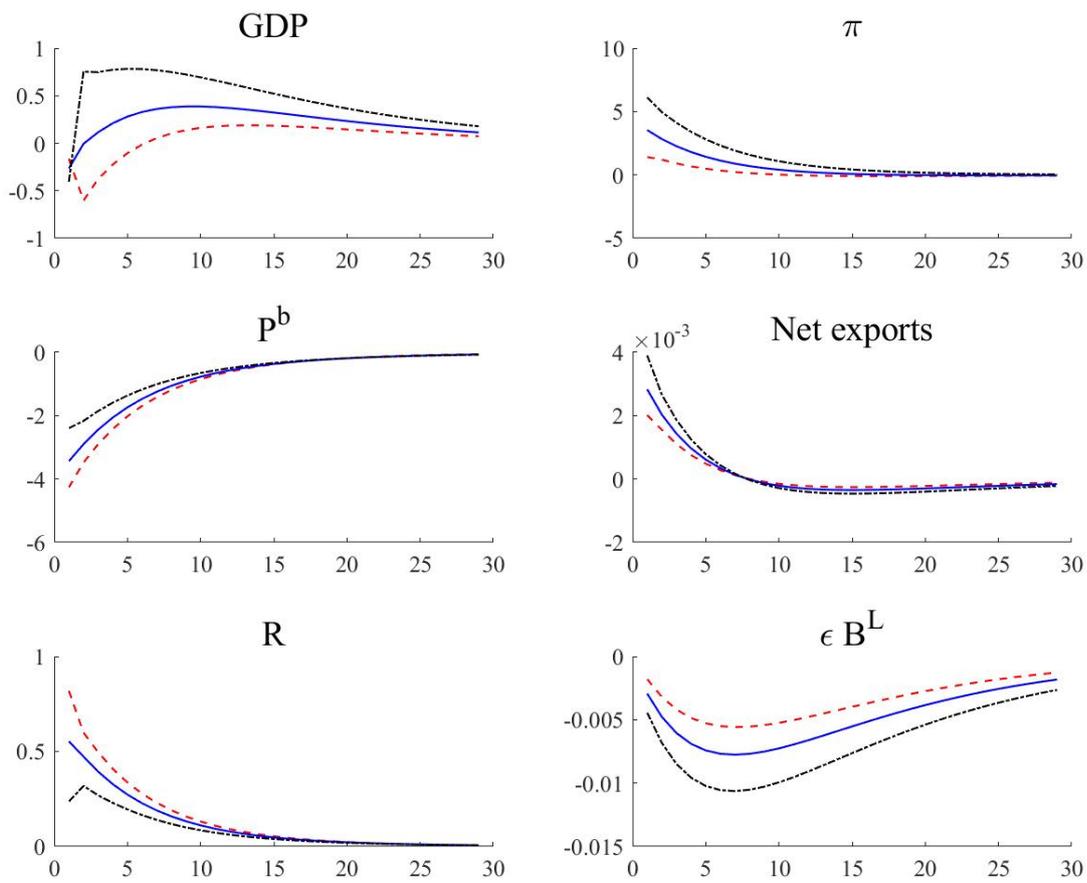


Figure11 Impulse responses to a foreign interest rate shock when price rigidity is low and  $R_{t+1} = \pi_{t+1}^* R^f$

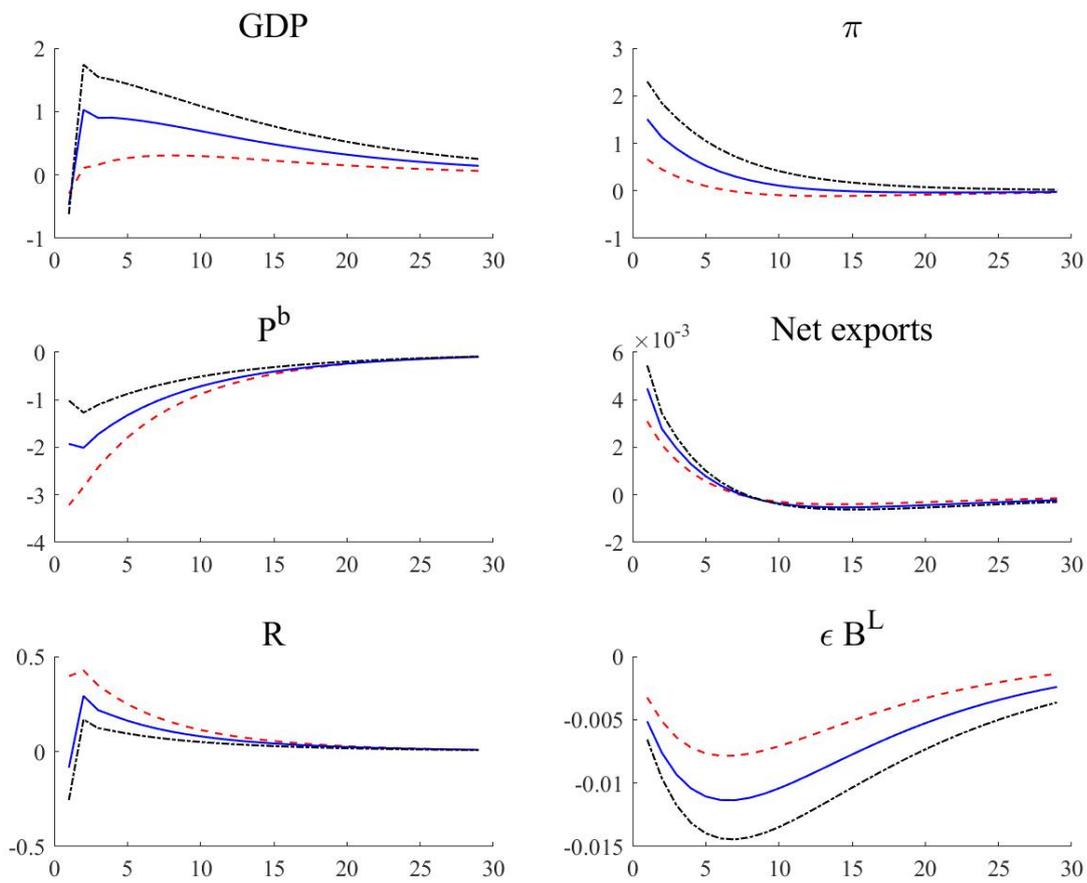


Figure12 Impulse responses to a foreign interest rate shock when price rigidity is high and  $R_{t+1} = \pi_{t+1}^* R^f$

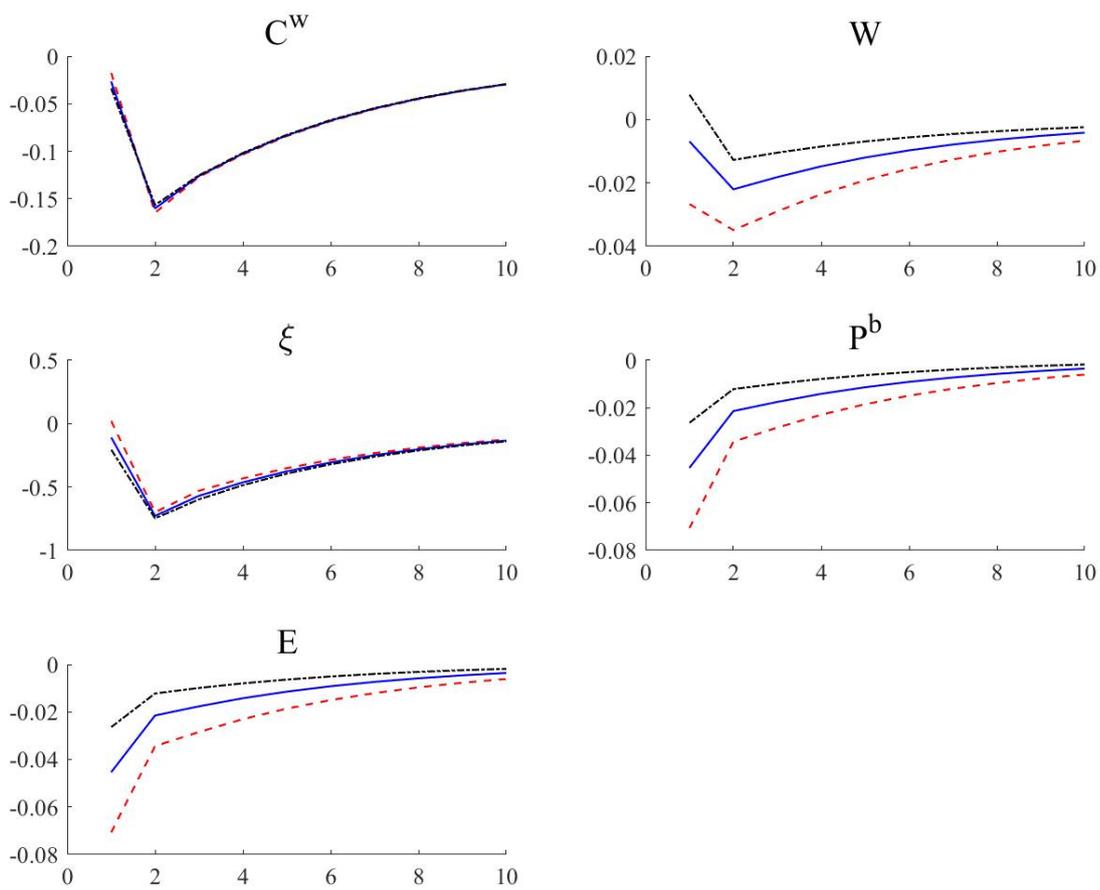


Figure13 Impulse responses to a foreign interest rate shock when worker are sufficiently impatient and price rigidity is low

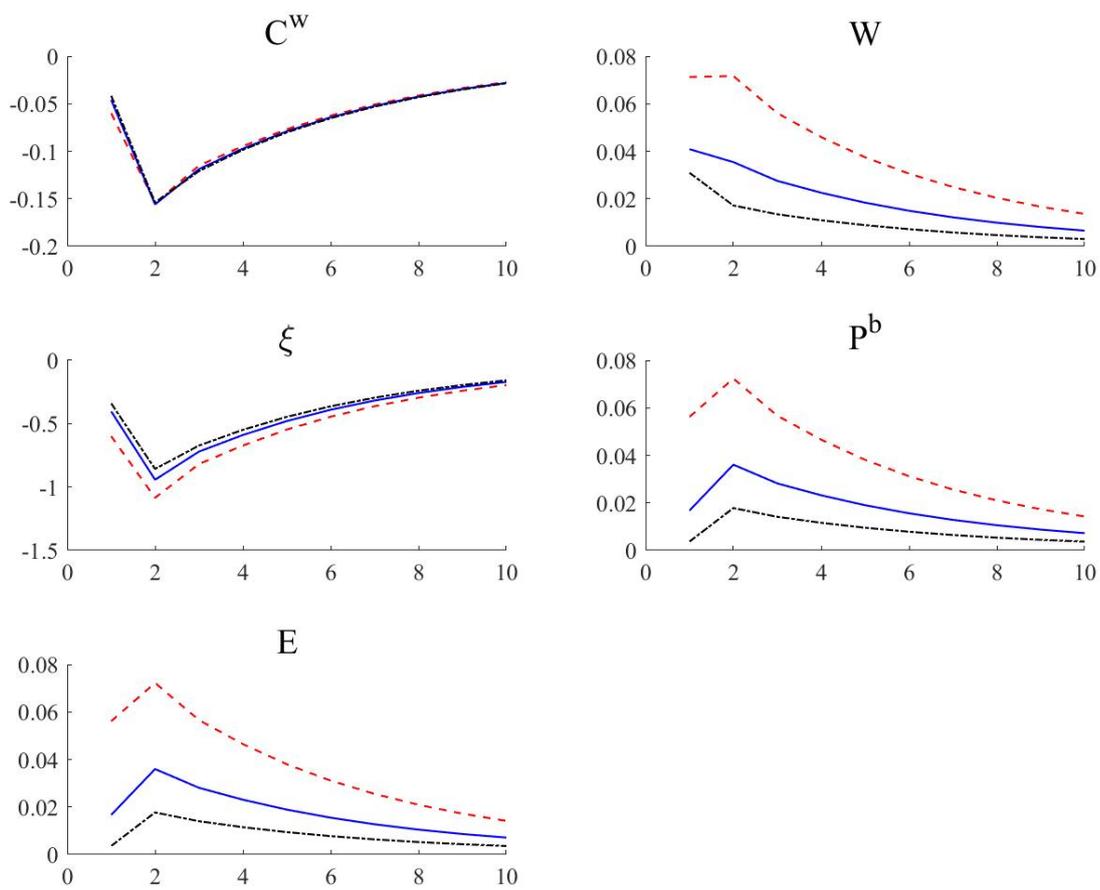


Figure14 Impulse responses to a foreign interest rate shock when worker are sufficiently impatient and price rigidity is high