

Reallocation of the Intermediation Risk: The Role of Interbank Markets and Interest on Reserves in the Macroeconomy*

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Abstract

We present a theory of the interbank market, where banks are risk averse and subject to idiosyncratic intermediation risk. Because banks face heterogeneous risk-return trade-offs, there are gains from reallocating loans among banks. In a partial equilibrium setup, the model admits closed form solutions for bank variables, allowing for their comparative statics with respect to the level of bank risk aversion and interest on reserves (IOR). We embed the interbank market in a simple general equilibrium model to measure its contribution to welfare. The interbank market significantly increases banks' risk-taking, thereby increasing production and the household consumption. The average household consumption in a stochastic steady state increases by 13.56% – 23.53%, depending on the treatment of the intermediation loss. At the same time, the interbank market raises volatility of production and consumption. This real impact of the interbank market becomes smaller when IOR is increased.

Keywords: Banks, Interbank market, Interest on reserves, Spreads.

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1 Introduction

A large body of empirical work has established the importance of the risk premium component for the cost of capital.¹ In a context of the banking sector, this corresponds to *the loan spread* – the corporate loan rate minus the deposit rate. More recently, interest rate on reserves (IOR), positive or negative, has attracted significant attention as a new policy tool.² IOR introduces another bank spread, which we call *the deposit spread* – the deposit rate minus IOR. While the loan spread has been studied both empirically and theoretically, our understanding of the deposit spread is limited. How are these two spreads related to each other? How do banks and the central bank affect the spreads? Can these spreads tell us anything about the macroeconomy? We propose a simple general equilibrium model with a banking sector to answer these questions.

In our model banks intermediate between households and firms, subject to idiosyncratic intermediation risk. Banks start with the same amount of resources, and choose a fraction of that resource to be lent out to firms with a risky production technology. The remaining fraction stays in the banking sector as reserves, earning IOR set by the central bank. After the loan is extended to firms, the banks become heterogeneous in the success probability of their loan. A key friction in the model is that the loan term offered to firms cannot be made contingent on the idiosyncratic intermediation risk. Because banks are risk averse and each bank faces a different risk-return trade-off, there are gains from trading loan among themselves. In this environment, an interbank market is used for trading exposures to the idiosyncratic intermediation risk.

In the interbank market, banks with high success probability would like to buy loans from those with low success probability. This portfolio rebalancing in the interbank market affects the aggregate loan supply to firms. Furthermore, the interbank loan trading creates a need for interbank borrowing/lending of reserves, because banks with sufficiently high success probability may want to make their reserve position temporarily negative, meaning that they borrow reserves from other banks to purchase risky loans. We assume that the central bank facilitates borrowing/lending of reserves among banks at IOR. The net payment of IOR to banks is funded by lump sum tax on households. Finally, the deposit rate is determined by banks' free entry condition. Banks behave competitively, yet they earn strictly positive profits because they must be compensated for taking risk. Banks' exclusive access to IOR, combined with their risk aversion, allows the central bank to affect the aggregate loan supply

¹See Campbell and Cochrane (1999), Bernanke and Kuttner (2005), Bekaert et al. (2013), Gertler and Karadi (2015), di Giovanni et al. (2020).

²See Berentsen and Monnet (2008), Bech and Klee (2011), Kashyap and Stein (2012), Ireland (2014), Cochrane (2014), Hendrickson (2017), Ulate (2021).

to firms by changing IOR.

We study a benchmark case where individual banks' ex post profit is allowed to be negative (i.e., in the absence of the limited liability in the interbank market). In a partial equilibrium setup, the model admits a closed form expression for the deposit rate and other bank variables. We show that the loan spread is always greater than the deposit spread. We also numerically verify that this result extends to a general equilibrium setup where the corporate loan rate and aggregate saving (resources that banks start with in each period) are endogenized.

To investigate the role of the interbank market, we also study the case without the interbank market. When banks cannot trade loans after the realization of their success probabilities, the risk-return trade-off facing the entire banking sector deteriorates. As a result, banks reduce the corporate loan supply, and the bank profit decreases. In addition, the loan spread widens, while the deposit spread narrows, relative to the benchmark case with the interbank market. The model has two implications for the interbank market. First, the interbank market increases welfare and banks' net profit. Thus, residual claimants of the bank profit (households in our model) have a private incentive to create the interbank market. Second, the model predicts that when the interbank market stops functioning for some reason, the loan spread will go up while the deposit spread as well as the bank profit will go down. Thus, in our model, the failure of the interbank market would appear as if a representative bank became more risk averse.

To quantify the welfare contribution of the interbank market, we compute a percentage increase in the household average consumption in a stochastic steady state, relative to the case without the interbank market. As a first scenario, we add the intermediation loss back to the household income to isolate the wealth effect directly imposed by failing banks from the effect of changes in bank behaviors. In our benchmark calibration, the welfare contribution of the interbank market is 13.56%. As a second scenario, we treat the intermediation loss as a social loss. In this case, the welfare contribution of the interbank market increases to 23.53%. Intuitively, when intermediation loss is a social loss, reallocating loans to better banks who are less likely to fail becomes more important. Viewing these two scenarios as the two polar cases, the welfare contribution of the interbank market lies somewhere between 13.56% and 23.53% in our model.

The model has implications for the IOR policy. First, low IOR is stimulative, even in the negative range. Suppose that individual banks become more risk averse. Then the deposit rate goes down, while the corporate loan rate goes up. As a result, the deposit spread goes down, while the loan spread goes up. This is because an increase in bank risk aversion reduces the aggregate supply of capital. This pushes up the return on capital. At the same

time, the banks require higher compensation for the same level of expected profit. This reduces the payout to households in the form of the return on the deposit. To counteract these changes induced by the elevated bank risk aversion, the central bank should lower IOR.

The rest of the paper is organized as follows. After the review of the related literature, we explain a model environment in Section 2. Section 3 studies the interbank market equilibrium, and compare it to the equilibrium without the interbank market, assuming that banks are not protected by the limited liability. Section 4 investigates a general equilibrium effect. Section 5 concludes.

Related literature.

There is a large banking literature on the bank risk-taking channel of monetary policy (Dell’Ariccia and Marquez (2013), Jimenez et al. (2014), Paligorova and Santos (2017)). Most models in this literature feature a single bank in partial equilibrium, which precludes the analysis of the interbank market. We construct a general equilibrium model with heterogeneous banks, allowing us to study whether and how the interbank market affects risk-taking by banks.

A number of works have pointed out that booms are time of low risk aversion (Campbell and Cochrane (1999), Bernanke and Kuttner (2005), Bekaert et al (2013), di Giovanni et al. (2020)). Yet, the majority of general equilibrium macro models treat banks as risk neutral.³ This makes the investigation of the risk premium channel difficult, if not impossible.⁴ A few exceptions that directly model risk averse banks include Malamud and Schrimpf (2017), and Bianchi and Bigio (2020). Relative to these works, our focus is on the role of the interbank market as a way to reallocate idiosyncratic risk. In this regard, our model is closest to Boissay et al. (2016), which is also a competitive real model with an explicit interbank market. In their model, however, banks are risk neutral and portfolio rebalancing plays no role. Also, they do not model IOR, which is a key policy tool in our model.

2 Model

We present a model of a competitive economy with a representative household, a representative firm, and a mass one of heterogeneous banks. We describe a non-bank sector in the next two subsections. The banking sector is described in subsection 2.3.

³For example, Challe et al. (2013), Boissay et al. (2016), Corbae and D’Erasmus (2019), Arce et al. (2020), Coimbra and Rey (2020).

⁴In the finance literature, modeling banks as risk averse agents is a common practice (Ho and Saunders (1985), Drechsler et al (2018), Dugast et al. (2019), Di Tella and Kurlat (2021)). We follow this tradition in modeling heterogeneous banks. Relative to this literature, we keep the banking sector relatively simple, but embed it in a standard general equilibrium model.

2.1 Firms

The representative firm produces using a Cobb-Douglas production function subject to a technology shock z :

$$\pi = \max_{K,L} \{zK^\alpha L^{1-\alpha} + (1-\delta)K - RK - wL\}.$$

Technology shock z is the only aggregate shock in this economy. The household supplies fixed labor $L = 1$ inelastically at the wage rate w , while the capital K must be borrowed from the banks at a corporate loan rate R . The firm's first order conditions are

$$R = z\alpha K^{\alpha-1} + 1 - \delta, \tag{1}$$

$$w = z(1-\alpha)K^\alpha.$$

By solving (1) for K , a corporate loan demand is

$$K(R; z) = \left(\frac{z\alpha}{R - (1-\delta)} \right)^{\frac{1}{1-\alpha}}. \tag{2}$$

This loan demand and a loan supply from the banking sector together determine the corporate loan rate R in a general equilibrium.

2.2 Households

The representative household solves the following problem.

$$\mathcal{V}(A, z) = \max_{A'} \left\{ u(c) + \beta E \left[\mathcal{V}(A', z') \mid A, z \right] \right\}, \quad z' \sim G(z), \tag{3}$$

$$\text{s.t.} \quad c + A' = (rA + \pi_B + \chi - \tau) + (w + \pi). \tag{4}$$

The term G is a distribution from which the next period technology shock z' is drawn. In the right hand side of the budget constraint (4), (w, π) are the wage income and the firm profit determined in the production sector. The other terms are bank-related sources of the household income. Technology shock z affects the household income through w as well as through R , because bank-related sources of the household income depend on R . Following Boissay et al. (2016), we call A *deposit* and accordingly, r is a *deposit rate*. The term π_B is the bank profit. The remaining term $\chi - \tau$ is a lump-sum transfer. First, χ is the intermediation loss caused by banks. We add this loss back to the household income to isolate the welfare loss caused by exogenous bank failures. Second, τ is a tax used to pay IOR to

banks. The determination of bank-related income is explained in the following subsection.

2.3 Banks

There is a continuum of banks that live for one period, with an identical mean-variance preference over their risky payoff $\tilde{\pi}_B$,

$$U(\tilde{\pi}_B) = E[\tilde{\pi}_B] - \frac{\gamma}{2} \text{Var}[\tilde{\pi}_B]$$

where $\gamma > 0$ is a risk-aversion parameter. This preference structure admits tractable aggregation of banks facing idiosyncratic intermediation risk. There are three stages within each period: ex ante, interim and ex post. Banks act at the ex ante stage and at the ex post stage. Banks' heterogeneous types are realized at the interim stage.

At the ex ante stage, all banks are homogeneous. They start with the same amount of deposit $A > 0$. Banks choose the amount of risky loan $V \in [0, A]$, which makes their reserve position $W = A - V \in [0, A]$.⁵ Reserves W earn a safe return R_0 per unit. We call R_0 an *interest on reserves*, IOR. The net interest payment $(R_0 - 1)W$ is funded by lump-sum tax τ on households. The risky loan has a stochastic return \tilde{R} per unit. This choice of V will affect loan trading among banks in the interbank market (ex post stage). Banks rationally anticipate the impact of V on their subsequent behavior at the ex post stage.

At the interim stage, banks become heterogeneous in their intermediation risk. We assume that each bank independently draws a success probability p of the risky loan from a probability distribution f defined on $[p_L, p_H]$. For type p banks, the stochastic return is

$$\tilde{R} = \begin{cases} R & \text{with probability } p, \\ 0 & \text{with probability } 1 - p. \end{cases}$$

The mean of \tilde{R} for type p bank is pR , while its variance is $\omega(p)R^2$ with $\omega(p) \equiv p(1-p)$. We assume that the outcome of the risky loan (success or failure) is independent *across banks of the same type*, such that a measure $pf(p)$ of type p banks succeed. This means that intermediation risk is not the aggregate risk in that a fraction of failing banks $E[1-p] = \int_{p_L}^{p_H} (1-p)f(p)dp$ is known with certainty.⁶ We impose the following restriction on (p_L, p_H) .

Assumption 1 $\frac{1}{2} < p_L < p_H < 1$.

⁵If there is a mandatory reserve requirement to hold θA , $\theta \in (0, 1)$, banks choose $V \in [0, (1-\theta)A]$ and the excess reserve is $\tilde{W} = (1-\theta)A - V \in [0, (1-\theta)A]$. We set $\theta = 0$ for simplicity, but the results generalize with minor modifications.

⁶Throughout the paper, we use the notation $E[X(p)] = \int_{p_L}^{p_H} X(p)f(p)dp$ for any function of p , $X(p)$.

First, $\frac{1}{2} < p_L$ implies that $\omega(p) \in (0, \frac{1}{4})$ is decreasing in p so that the higher type p is associated with the higher mean and the lower variance. Second, $p_H < 1$ implies that all banks face intermediation risk.⁷

At the ex post stage, banks have an identical portfolio (V, W) but they are heterogenous in success probability p . This heterogeneity creates gains from loan trading among banks. Banks can trade a unit of risky loan for Q units of reserves. Each type p bank submits an order $k(p; Q)$. A market-clearing condition $\int_{p_L}^{p_H} k(p; Q) f(p) dp = 0$ determines a market-clearing price Q^* . After trading in the interbank market, banks of type p hold a portfolio of loans and reserves $(V + k(p; Q^*), W - k(p; Q^*) Q^*)$. After returns on this position as well as intermediation risk are realized, banks pay rA to households. The remaining profit becomes π_B . As we show below, the banking sector as a whole always remains solvent. Hence, households face no uncertainty regarding the payment of rA .

The timing for banks within a period is summarized in **Table 1**.

Table 1. Within-period timing for banks.

Ex ante	: Choose (V, W) subject to $V + W = A$.
Interim	: Type $p \in [p_L, p_H]$ is drawn from $f(p)$.
Ex post	: Adjust (V, W) through interbank market trading.

2.4 General Equilibrium

In the aggregate, $rA + \pi_B$ equals the banking sector's gross profit Π . Because the intermediation loss due to failed banks are not included in Π , $\Pi + \chi$ is the sum of returns on reserves and returns on risky loan before the intermediation loss is subtracted, i.e., $\Pi + \chi = R_0W + RV$. Finally, the net IOR payment to banks is $(R_0 - 1)W$, which is funded by the lump-sum tax $\tau = (R_0 - 1)W$ on households. Accordingly, gathering all the bank-related sources of income in the household's budget constraint (4),

$$rA + \pi_B + \chi - \tau = \Pi + \chi - \tau = R_0W + RV - (R_0 - 1)W = W + RV.$$

It may appear that R_0 is irrelevant for households, but it affects welfare through banks' choice of (V, W) . Within each period, the timing of the model is as follows.

1. A technology shock z realizes for firms. New banks enter to receive A from households. Free entry condition determines r .
2. Banks choose supply of loans V and reserves $W = A - V$ taking R and R_0 as given. Firms choose demand for loans K taking R as given. $V = K$ determines R .

⁷Without any other friction, banks of type $p = 1$ would dominate all other banks.

3. Banks draw success probabilities. Reallocation of V occurs in the interbank market.
4. Firms undertake production. Intermediation risk for banks' risky loans is realized.
5. Households collect incomes from banks and firms: the return on bank deposit (rA), the profit of banks (π_B), the loss from bank failures (χ), tax ($-\tau$), wage (w), and the profit of firms (π).
6. Banks exit. Households consume and choose saving A' .

With this timing assumption, there is no within-period uncertainty for households. In particular, the deposit rate r is contingent upon the current period technology shock z . We define the *deposit spread* by $r - R_0$, and the *loan spread* by $R - r$. In the next section, we study how the two spreads are determined in the banking sector taking (A, R) as given.

3 Interbank Markets and Bank Spreads

In this section, we characterize equilibrium allocation in the banking sector taking (A, R) as given. In subsections 3.1 through 3.3, we study equilibrium with the interbank market. In subsection 3.4, we study equilibrium without the interbank market for comparison.

First, we make two assumptions for (A, R) . In Section 4, we verify that these assumptions can be satisfied in a general equilibrium.

Assumption 2 $2R_0 \leq R$.

Together with **Assumption 1**, **Assumption 2** implies $R_0 < p_L R$. This means that even the banks with the worst intermediation risk finds the risky loan profitable on average. In general equilibrium, R will be determined as a function of technology shocks. Therefore, **Assumption 2** imposes a lower bound for technology shocks for a given level of R_0 . We discuss this in greater details in Section 4.

Assumption 3 $\frac{1}{4\gamma R_0} \frac{(E[\frac{1}{1-p}])^2}{E[\frac{1}{\omega(p)}]} < A$.

Assumption 3 ensures that the interior optimum choice of V is feasible for *any* value of R , whether or not the interbank market exists. **Assumption 3** makes it clear that our results are not driven by the constrained choice of V .

In subsection 3.1, we first describe the interbank market, and derive the optimal demand for loan trading for a given V . In subsection 3.2, we study the ex ante loan choice V^* . Combining these two, we obtain the net demand for loan trading $k^*(p)$ for each bank type

p . Finally in subsection 3.3, we derive bank-related variables as a function of $k^*(p)$ and V^* , and characterize the two spreads $r^* - R_0$ and $R - r^*$.

3.1 Interbank Loan Trading

In the inter-bank market, banks can trade a unit of loan for Q units of reserves. Each type p bank submits an order $k(p; Q)$. To derive the optimal order, suppose that banks of type p choose net demand k for a given price Q . Their risky payoff is given by

$$\tilde{\pi}_B(p; k) = \begin{cases} R(V + k) + R_0(W - Qk) & \text{with probability } p, \\ R_0(W - Qk) & \text{with probability } 1 - p. \end{cases} \quad (5)$$

The risky payoff (5) depends on V and Q , but we suppress the notation to focus on the choice of k . Importantly, $W - Qk$ may take a negative value, i.e., some banks may borrow additional reserves in excess of W to fund interbank loan purchase $k > \frac{W}{Q}$.

Remark. Borrowing reserves creates two issues for failing banks. First, $R_0(W - Qk)$ in the second line of (5) implicitly assumes that interbank borrowing is possible at R_0 . More generally, lenders may demand a higher “interbank rate” above R_0 . Second, borrowing *and failing* make the ex post profit negative. In our setup, a measure $(1 - p)f(p)$ of type p banks fails. If these banks were protected by the limited liability, their behavior would change.

For the first issue, we assume that the central bank acts as a public intermediary for banks: it borrows reserves from banks with $W > Qk$, and lends them to banks with $W < Qk$, both at the same rate R_0 . This is always feasible because aggregating $W - Qk$ across banks yields $W > 0$ in any market-clearing equilibrium. For the second issue, we simply assume that banks are not protected by the limited liability, and allow the realized profit to be negative. This means that the central bank collects $R_0(Qk - W) > 0$ from all borrowing banks (including failed ones) to pay back to lending banks.⁸

Because banks are risk-averse, type p banks’ expected payoff from the risky payoff (5) is

$$U(\tilde{\pi}_B(p; k)) = R_0A + (pR - R_0)V + (pR - R_0Q)k - \frac{\gamma}{2}\omega(p)(R(V + k))^2. \quad (6)$$

Type p banks choose k to maximize (6) by taking Q as given. Formally, they solve

$$\max_k \left\{ (pR - R_0Q)k - \frac{\gamma}{2}\omega(p)(R(V + k))^2 \right\}.$$

⁸If banks were protected by the limited liability, the central bank would need to set the lending rate above R_0 to break even after paying back $R_0(W - Qk) > 0$ to lending banks. We investigate this issue in a separate paper.

Because the second order condition is satisfied, solving the first order condition $pR - R_0Q = \gamma\omega(p)R^2(V+k)$ for k yields the optimal demand

$$k(p; Q) \equiv \frac{1}{\gamma R} \frac{p - \frac{R_0}{R}Q}{\omega(p)} - V. \quad (7)$$

The market-clearing price Q^* is a solution to $\int_{p_L}^{p_H} k(p; Q) f(p) dp = 0$. We later verify that $p - \frac{R_0}{R}Q^* > 0$ holds for any $p \in [p_L, p_H]$. This means that every bank retains positive risky loan position, i.e., $k(p; Q^*) + V > 0$ for any $p \in [p_L, p_H]$. Substituting (7) into (6) yields

$$u^*(p, V) \equiv U(\tilde{\pi}_B(p; k(p; Q^*))) = R_0 \{A + (Q^* - 1)V\} + \frac{1}{2\gamma} \frac{(p - \frac{R_0}{R}Q^*)^2}{\omega(p)}.$$

The ex ante payoff is obtained by aggregating $u^*(p)$ over p :

$$U^*(V) \equiv E[u^*(p, V)] = R_0 \{A + (Q^* - 1)V\} + \frac{1}{2\gamma} E \left[\frac{(p - \frac{R_0}{R}Q^*)^2}{\omega(p)} \right]. \quad (8)$$

3.2 Loan Supply by Banks

To choose optimal V at the ex ante stage, banks must consider how Q^* depends on V . From the market-clearing condition,

$$\int_{p_L}^{p_H} \left(\frac{1}{\gamma R} \frac{p - \frac{R_0}{R}Q}{\omega(p)} - V \right) f(p) dp = 0 \Leftrightarrow E \left[\frac{1}{1-p} \right] - E \left[\frac{1}{\omega(p)} \right] \frac{R_0}{R} Q = \gamma R V.$$

We denote a harmonic mean of $X(p)$ by $E^h[X(p)] \equiv \left(E \left[\frac{1}{X(p)} \right] \right)^{-1}$. With this notation, the market-clearing price is

$$Q^*(V) \equiv \frac{R}{R_0} E^h[\omega(p)] \left(\frac{1}{E^h[1-p]} - \gamma R V \right). \quad (9)$$

Substituting (9) into (8), banks' ex ante problem is

$$\max_{V \in [0, A]} \left\{ R_0 \{A + (Q^*(V) - 1)V\} + \frac{1}{2\gamma} E \left[\frac{(p - \frac{R_0}{R}Q^*(V))^2}{\omega(p)} \right] \right\}.$$

Taking the first order condition,

$$Q^*(V) - 1 = \frac{\partial Q^*(V)}{\partial V} \left\{ \frac{1}{\gamma R} E \left[\frac{p - \frac{R_0}{R}Q^*(V)}{\omega(p)} \right] - V \right\}.$$

Substituting the expression of $Q^*(V)$ given in (9), the first order condition becomes

$$Q^*(V) - 1 = \frac{\partial Q^*(V)}{\partial V} \left\{ \frac{1}{\gamma R} E \left[\frac{p - E^h[\omega(p)] \left(\frac{1}{E^h[1-p]} - \gamma R V \right)}{\omega(p)} \right] - V \right\} = 0.$$

Thus, the first order condition is simply $Q^*(V) = 1$.⁹ Therefore, the optimal choice of V is obtained by solving $Q^*(V) = 1$.¹⁰ Thus, we have the following result.

Lemma 1 *The corporate loan supply is*

$$V^*(R) = \frac{1}{\gamma R} E \left[\frac{p - \frac{R_0}{R}}{\omega(p)} \right], \quad (10)$$

and **Assumption 3** implies $V^*(R) < A$ for any R .

In subsection 3.4, we show that the optimal choice of V in the absence of the interbank market is smaller than $V^*(R)$. Hence, **Assumption 3** is also sufficient for the interiority of the ex ante investment choice regardless of whether the interbank market exists or not.

We denote the optimal net demand by $k^*(p) \equiv k(p; Q^*)$. Substituting $Q^*(V^*) = 1$ and $V = V^*$ into the optimal order (7),

$$k^*(p) + V^* = \frac{1}{\gamma R} \frac{p - \frac{R_0}{R}}{\omega(p)}.$$

Because this is positive for any $p \in [p_L, p_H]$, all banks retain some amount of risky loan after trading.¹¹ From (10), the loan trading demand from type p banks is

$$k^*(p) = \frac{1}{\gamma R} \left\{ \frac{p - \frac{R_0}{R}}{\omega(p)} - E \left[\frac{p - \frac{R_0}{R}}{\omega(p)} \right] \right\}, \quad (11)$$

which sums up to zero when aggregated across all types.

⁹The second order condition is satisfied because $\frac{\partial Q^*(V)}{\partial V} = -\frac{R}{R_0} E^h[\omega(p)] \gamma R < 0$.

¹⁰If we assume instead that banks take Q^* as given at the ex ante stage by ignoring its dependence on V , $\frac{\partial Q^*}{\partial V} = 0$ and again we obtain $Q^* = 1$ as the first order condition.

¹¹If $p_L < \frac{R_0}{R}$, some banks submit a non-price contingent sell order $-V$ in equilibrium. While this case is also tractable, we focus on the case with interior solutions.

3.3 Bank Profit and Loss, Deposit Rate, and Spreads

By substituting $Q^* = 1$, V^* , and $k^*(p)$ into (5), type p banks' risky payoff is

$$\tilde{\pi}_B^*(p) \equiv \tilde{\pi}_B(p; k^*(p)) = \begin{cases} R(V^* + k^*(p)) + R_0(W - k^*(p)) & \text{with probability } p, \\ R_0(W - k^*(p)) & \text{with probability } 1 - p. \end{cases}$$

Interbank borrowing occurs if and only if $k^*(p_H) > W$.¹² When this condition is satisfied, a fraction $(1 - p)$ of banks of type p above the threshold level \hat{p} (defined by $k^*(\hat{p}) = W$) will end up with the negative profit. However, we show below that the aggregate bank profit remains positive. This means that the banking sector as a whole remains solvent.

The gross profit Π^* (i.e., before paying back the deposit to households) is obtained by aggregating $\tilde{\pi}_B^*(p)$ over p :

$$\Pi^* \equiv \int_{p_L}^{p_H} (R_0W + pR(V^* + k^*(p))) f(p) dp = R_0A + (E[p]R - R_0)V^* + RE[pk^*(p)]. \quad (12)$$

The intermediation loss from bank failures is $\chi^* \equiv E[(1 - p)R(k^*(p) + V^*)]$. At the ex ante stage, the banks' optimal payoff is

$$U^* \equiv U^*(V^*) = \Pi^* - \frac{\gamma}{2}E[\omega(p)(R(V^* + k^*(p)))^2].$$

In general equilibrium, A is saving by households. We assume that banks behave competitively vis-a-vis households, subject to the free entry condition. Accordingly, the deposit rate r^* is determined by $r^*A = U^*$.¹³ Because of risk aversion, the banking sector earns a strictly positive net profit given by

$$\pi_B^* = \Pi^* - r^*A = \Pi^* - U^* = \frac{\gamma}{2}E[\omega(p)(R(V^* + k^*(p)))^2] > 0.$$

Remark. Our preferred interpretation is that the deposit rate is determined ex ante, and hence common across all bank types. This is consistent with the idea that bank type p is banks' private information and the deposit rate cannot depend on p . Alternatively, we can let the heterogenous deposit rates be determined by $r^*(p)A = U(\tilde{\pi}_B^*(p))$, and define $r^* \equiv \int_{p_L}^{p_H} r^*(p) f(p) dp$. Both setups yield the same results. The key assumption is that the

¹²If $k^*(p_H) \leq W$, then all banks maintain non-negative reserve positions and hence receive $R_0(W - k^*(p))$ from the central bank.

¹³Our preferred interpretation is that the deposit rate is determined ex ante, and hence common across all bank types. This is consistent with the idea that bank type p is banks' private information and the deposit rate cannot depend on p . However, we can let the heterogenous deposit rates be determined by $r^*(p)A = U(\tilde{\pi}_B^*(p))$, and define $r^* \equiv \int_{p_L}^{p_H} r^*(p) f(p) dp$. Both setups yield the same results.

intermediation risk is pooled *within the same bank type*. This ensures that a measure $f(p)$ of banks of type p can collectively pay back $r^*(p)A$ to the households.

The expressions of $(\Pi^*, \chi^*, U^*, \pi_B^*, r^*)$ depend on V^* and $k^*(p)$. With (10) and (11), we obtain the expressions of $(\Pi^*, \chi^*, U^*, r^*\pi_B^*,)$ as a function of (R, A) and other parameters.

Lemma 2 (bank variables)

- (a) The gross profit is $\Pi^* = AR_0 + \frac{1}{\gamma}E \left[\frac{(p - \frac{R_0}{R})^2}{\omega(p)} \right]$.
- (b) The intermediation loss is $\chi^* = \frac{1}{\gamma}E \left[\frac{(1-p)(p - \frac{R_0}{R})}{\omega(p)} \right]$.
- (c) The ex ante bank payoff is $U^* = R_0A + \frac{1}{2\gamma}E \left[\frac{(p - \frac{R_0}{R})^2}{\omega(p)} \right]$.
- (d) The deposit rate is

$$r^* = R_0 + \frac{1}{2A\gamma}E \left[\frac{(p - \frac{R_0}{R})^2}{\omega(p)} \right]. \quad (13)$$

- (e) The net profit is

$$\pi_B^* = \frac{1}{2\gamma}E \left[\frac{(p - \frac{R_0}{R})^2}{\omega(p)} \right]. \quad (14)$$

Deriving these expressions is straightforward, so we omit the proof. Properties of the bank spreads $r^* - R_0$ and $R - r^*$ are gathered below.

Proposition 1 (bank spreads)

Suppose that **Assumptions 1,2,3** hold.

- (a) $0 < r^* - R_0 < R - r^*$.
- (b) For a given (A, R) , the bank spreads satisfy

$$\begin{aligned} \frac{\partial (r^* - R_0)}{\partial R_0} &< 0 & \text{and} & \frac{\partial (R - r^*)}{\partial R_0} < 0. \\ \frac{\partial (r^* - R_0)}{\partial \gamma} &< 0 & \text{and} & \frac{\partial (R - r^*)}{\partial \gamma} > 0. \\ \frac{\partial (r^* - R_0)}{\partial R} &> 0 & \text{and} & \frac{\partial (R - r^*)}{\partial R} > 0. \end{aligned}$$

- (c) $\frac{\partial}{\partial \gamma} \frac{\partial r^*}{\partial R} < 0 < \frac{\partial}{\partial \gamma} \frac{\partial r^*}{\partial R_0}$.

Part (a) shows that banks' optimal choice ensures that the loan spread is greater than the deposit spread. This can serve as an empirical test of the way we model the banking

sector. While the deposit rate higher than IOR may seem counter-factual, it is natural in our model for the following reasons. First, A in our model has a property of equity, as its return depends on the profitability of banks. Second, the deposit in our model does not play any transaction role. The transaction value of the deposit should reduce its return relative to our model. Third, in our model, banks are competitive. Considering non-competitive banks in the deposit market should further reduce the deposit rate.¹⁴

Part (b) is comparative statics of the bank spreads with respect to (R_0, γ, R) . An increase in R_0 leads to a decrease in both spreads, while an increase in R leads to an increase in both spreads. An increase in γ raises the loan spread and lowers the deposit spread, resulting in a greater gap between the two spreads.¹⁵

Part (c) shows that bank risk aversion γ has opposite effects on the sensitivity of r^* to R_0 and that to R . While r^* is increasing in both R_0 and R , as banks become more risk averse, the deposit rate becomes relatively more sensitive to R_0 than to R . This is due to banks' endogenous choice of V^* . In the proof of **Proposition 2**, we show

$$\frac{\partial r^*}{\partial R_0} = \frac{A - V^*}{A}, \quad \frac{\partial r^*}{\partial R} = \frac{R_0 V^*}{R A}, \quad \frac{\partial r^*}{\partial \gamma} = -\frac{r^* - R_0}{\gamma}. \quad (15)$$

From the expression of V^* in (10), higher risk aversion leads to lower V^* and higher reserves. This makes banks more protected from shocks coming from R but less so from shocks coming from R_0 .

Remark. In this section we treat (A, R) as parameters, just as banks in the model do. While part (a) continues to hold in general equilibrium as long as (A, R) satisfies **Assumptions 1** through **3**, part (b) and part (c) need to qualify the general equilibrium effect: the corporate loan rate R would respond to changes in R_0 and γ , even in a short-run where A is fixed. For the four comparative statics of the two spreads with respect to R_0 and γ , we will show that the result $0 < \frac{\partial}{\partial \gamma}(R - r^*)$ is *strengthened*, while the other three results will be weakened due to an endogenous reaction of R . However, in Section 4 we show numerically that the only result that changes its sign in general equilibrium is $\frac{\partial(R - r^*)}{\partial R_0}$, i.e., in general equilibrium, an increase in R_0 *raises* the loan spread. This indicates the difficulty of separately identifying the effect of R_0 and that of γ in general equilibrium. On the other hand, the comparative statics results with respect to R remain valid in general equilibrium. More discussion is given in Section 4.

¹⁴See Ulate (2021) for such a model.

¹⁵From the expression of (13), the effect of A and γ cannot be separately identified by looking at the bank spreads alone. However, from the expression of (14), changes in γ should simultaneously affect the bank profit, while changes in A should leave the bank profit unaffected.

3.4 A Role of the Interbank Market

To study a role of the interbank market, we characterize the banking sector allocation without the interbank market. If there is no interbank market, type p banks face the risky payoff

$$\tilde{\pi}_B^{nt}(p; V) = \begin{cases} RV + R_0W & \text{with probability } p, \\ R_0W & \text{with probability } 1 - p. \end{cases}$$

The superscript “ nt ” stands for no-trade. Because a measure $pf(p)$ of type p banks succeed, the gross profit in the banking sector as a function of V is

$$\Pi^{nt}(V) \equiv \int_{p_L}^{p_H} (R_0W + pRV) f(p) dp = R_0A + (E[p]R - R_0)V. \quad (16)$$

Type p banks’ expected payoff is $U(\tilde{\pi}_B^{nt}(p; V)) = R_0A + (pR - R_0)V - \frac{\gamma}{2}\omega(p)(RV)^2$. At the ex ante stage, the banks solve

$$U^{nt} = \max_{V \in [0, A]} \left\{ \Pi^{nt}(V) - \frac{\gamma}{2}R^2E[\omega(p)]V^2 \right\}, \quad (17)$$

where $\Pi^{nt}(V)$ is given in (16). Let V^{nt} be the optimal choice and

$$\Pi^{nt} \equiv \Pi^{nt}(V^{nt}) = R_0A + (E[p]R - R_0)V^{nt}$$

be the optimal gross profit. The intermediation loss from bank failures is $\chi^{nt} \equiv E[(1 - p)RV^{nt}]$.

The ex ante payoff is

$$U^{nt} = \Pi^{nt} - \frac{\gamma}{2}E[\omega(p)](RV^{nt})^2.$$

Banks pay out the gross return to A determined by $r^{nt}A = U^{nt}$. Because of risk aversion, the banking sector earns a strictly positive net profit given by

$$\pi_B^{nt} = \Pi^{nt} - r^{nt}A = \Pi^{nt} - U^{nt} = \frac{\gamma}{2}E[\omega(p)](RV^{nt})^2 > 0.$$

The ex ante problem (17) is well-behaved, and solving the first order condition $E[p]R - R_0 = \gamma E[\omega(p)]R^2V$ for V yields

$$V^{nt}(R) \equiv \frac{1}{\gamma R} \frac{E[p] - \frac{R_0}{R}}{E[\omega(p)]}. \quad (18)$$

Assumptions 1 and 2 imply $V^{nt} > 0$. Below we show that $V^{nt} < V^*$, so **Assumption 3** implies $V^{nt} < A$.

To facilitate the comparison of the two allocations with and without the interbank market,

the following result is useful.

Lemma 3 *With the interbank market, the deposit spread is*

$$r^* - R_0 = \frac{V^*}{2A} \left(E \left[g \left(p; \frac{R_0}{R} \right) p \right] R - R_0 \right), \quad (19)$$

where $g \left(p; \frac{R_0}{R} \right) \equiv \frac{\frac{p - \frac{R_0}{R}}{\omega(p)}}{E \left[\frac{p - \frac{R_0}{R}}{\omega(p)} \right]}$ is increasing in p and satisfies $E \left[g \left(p; \frac{R_0}{R} \right) \right] = 1$ and $E \left[g \left(p; \frac{R_0}{R} \right) p \right] \in (E[p], 1)$. Without the interbank market, the deposit spread is

$$r^{nt} - R_0 = \frac{V^{nt}}{2A} (E[p] R - R_0). \quad (20)$$

Importantly, from the expression of V^* and $k^*(p)$ given in (10) and (11), $g \left(p; \frac{R_0}{R} \right) = \frac{V^* + k^*(p)}{V^*}$. Comparison of (19) and (20) shows that loan trading in the interbank market raises the marginal type that determines the excess return on the risky loan. As we show below, this increases the ex ante choice of loan V^* relative to V^{nt} . Thus, for a given R and A , the partial equilibrium impact of the interbank market on the deposit rate can be decomposed into the aggregation effect $E \left[g \left(p; \frac{R_0}{R} \right) p \right]$ (relative to $E[p]$) and the quantity effect V^* (relative to V^{nt}).

Table 2 lists expressions of bank variables with and without the interbank market.

Table 2. Bank variables with and without the interbank market.

Variables	With the Interbank market	Without the interbank market
V	$\frac{1}{\gamma R} E \left[\frac{p - \frac{R_0}{R}}{\omega(p)} \right] \equiv V^*$	$\frac{1}{\gamma R} \frac{E[p] - \frac{R_0}{R}}{E[\omega(p)]} \equiv V^{nt}$
Π	$R_0 A + V^* (E [g (p; \frac{R_0}{R}) p] R - R_0)$	$R_0 A + V^{nt} (E [p] R - R_0)$
χ	$RV^* (1 - E [g (p; \frac{R_0}{R}) p])$	$RV^{nt} (1 - E [p])$
U	$R_0 A + \frac{V^*}{2} (E [g (p; \frac{R_0}{R}) p] R - R_0)$	$R_0 A + \frac{V^{nt}}{2} (E [p] R - R_0)$
r	$R_0 + \frac{V^*}{2A} (E [g (p; \frac{R_0}{R}) p] R - R_0)$	$R_0 + \frac{V^{nt}}{2A} (E [p] R - R_0)$
π_B	$\frac{V^*}{2} (E [g (p; \frac{R_0}{R}) p] R - R_0)$	$\frac{V^{nt}}{2} (E [p] R - R_0)$
$r - R_0$	$\frac{V^*}{2A} (E [g (p; \frac{R_0}{R}) p] R - R_0)$	$\frac{V^{nt}}{2A} (E [p] R - R_0)$

Proposition 2 gathers results that help us understand the role of the interbank market.

Proposition 2 (the role of the interbank market)

Suppose that **Assumptions 1,2,3** hold.

- (a) $0 < V^{nt} < V^* < A$, $\pi_B^{nt} < \pi_B^*$, and $r^{nt} < r^*$.
- (b) $\frac{\partial r^*}{\partial \gamma} < \frac{\partial r^{nt}}{\partial \gamma} < 0 < \frac{\partial r^*}{\partial R_0} < \frac{\partial r^{nt}}{\partial R_0} < 1$ and $0 < \frac{\partial r^{nt}}{\partial R} < \frac{\partial r^*}{\partial R} < 1$.
- (c) $\frac{\partial}{\partial \gamma} \frac{\partial r^*}{\partial R} < \frac{\partial}{\partial \gamma} \frac{\partial r^{nt}}{\partial R} < 0 < \frac{\partial}{\partial \gamma} \frac{\partial r^{nt}}{\partial R_0} < \frac{\partial}{\partial \gamma} \frac{\partial r^*}{\partial R_0}$.

Part (a) shows that the interbank market promotes risk-taking by banks. This increases the level of bank profit and the deposit rate. Thus, in our model residual claimants of banks have an incentive to create the interbank market. To be precise, this is a partial equilibrium result. However, we show numerically that $\pi_B^{nt} < \pi_B^*$ also hold in general equilibrium where (A, R) takes different values with and without the interbank market. This rationalizes the existence of the interbank market. Part (a) also implies

$$r^{nt} - R_0 < r^* - R_0 < R - r^* < R - r^{nt},$$

i.e., the partial equilibrium effect of the interbank market on spreads is to close the gap between the loan spread and the deposit spread. Since we normally see an active interbank market, we can restate this implication in the negative: if the interbank market stops operating, we should expect the gap between the two bank spreads to go up. We later verify that this result also holds up in general equilibrium.

Part (b) shows that the positive effect of R_0 on the deposit rate is weakened, while that of R is strengthened by the interbank market. Also, the negative effect of γ on the deposit rate is strengthened by the interbank market. Part (c) shows that the sign of the comparative statics with respect to γ is same with or without the interbank market, but its magnitude becomes greater with the interbank market. Because the comparative statics results in (15) also hold without the interbank market (i.e., (15) with V^* replaced with V^{nt} remains true), all the results in part (b) and part (c) follow from $V^{nt} < V^*$ and $r^{nt} < r^*$. Intuitively, as the interbank market encourages risk-taking by banks, banks' profit and hence the deposit rate become more sensitive to changes in R than to changes in R_0 . Because V^* itself is more sensitive to γ than V^{nt} is, the comparative statics with respect to γ becomes greater in magnitude with the interbank market than without it.

In this section, we treated (R, A) as parameters. In the next section, we close the model in general equilibrium by endogenizing (R, A) .

4 General Equilibrium

In this section, we embed the model of interbank market in a simple general equilibrium environment in order to investigate the role of the interbank market and IOR in the macro-economy. We first discuss the determination of the endogenous variables that turn out to be independent of A .

Determination of R and w .

For a given technology shock z , the corporate loan market-clearing condition

$$V^*(R) = K(R; z)$$

determines the market-clearing loan rate $R^*(z)$. We denote the equilibrium risky loan by $V^*(z) \equiv V^*(R^*(z)) = K(R^*(z); z)$. In the absence of the interbank market, $(V^{nt}(z), R^{nt}(z))$ are defined similarly. Importantly, neither (V^*, R^*) nor (V^{nt}, R^{nt}) depends on A . Because the banking sector with the mean-variance preference chooses V optimally independent of A , the loan supply to firms is separated from the household saving decision. This is in contrast to standard general equilibrium models where the capital supply is tightly connected to the household saving decision. Because $V^{nt}(R) < V^*(R)$ for a given R , $R^*(z) < R^{nt}(z)$ hold whenever both exist.

The equilibrium wage is given by

$$z^{\frac{1}{1-\alpha}} (1-\alpha) \left(\frac{\alpha}{R^*(z) - (1-\delta)} \right)^{\frac{\alpha}{1-\alpha}} = w^*(z). \quad (21)$$

Because $w^{nt}(z)$ is obtained by replacing $R^*(z)$ with $R^{nt}(z)$ in (21), $R^*(z) < R^{nt}(z)$ implies $w^*(z) > w^{nt}(z)$. Finally, an introduction of the interbank market induces a movement along the corporate loan demand curve (2). Therefore, as long as the loan demand curve is more convex than $KR = \text{constant}$, capital income is higher with the interbank market than without it (i.e., $R^*(z)V^*(z) > R^{nt}(z)V^{nt}(z)$). The determination of $R^*(z)$ and $R^{nt}(z)$ is shown in **Figure 1**. In the figure, IBM stands for the interbank market.

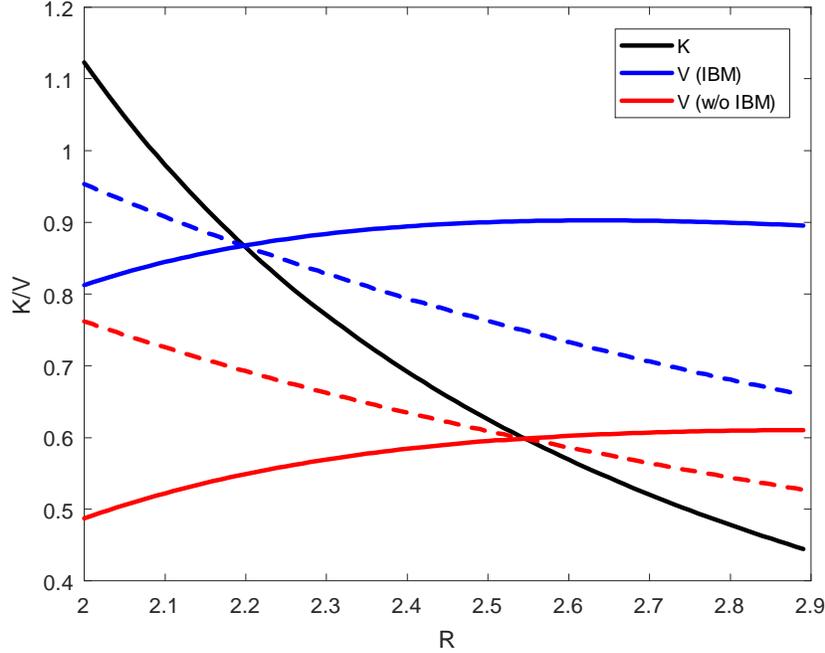


Figure 1. Corporate loan market clearing.
Note. The dashed lines are $KR = \text{constant}$.

From **Figure 1**, it should be clear that any parameter change that shifts up the corporate loan supply curve results in a decrease in $R^*(z)$ and an increase in $V^*(z)$. Also, the position of the dashed lines indicates that in response to the shift up of the loan supply curve, an increase in the loan quantity dominates a decrease in return such that capital income increases.

Comparative statics of the bank spreads. Now that we can analyze how $R^*(z)$ changes in response to changes in R_0 and γ , let us revisit the partial equilibrium comparative statics results in **Proposition 1**. Because the partial equilibrium effects of R on the two spreads were positive ($\frac{\partial(r^* - R_0)}{\partial R} > 0$ and $\frac{\partial(R - r^*)}{\partial R} > 0$), the general equilibrium movement of R may overturn some of the comparative statics results in partial equilibrium. Namely, because an increase in either R_0 or γ leads to an increase in R in general equilibrium, the two comparative statics results

$$\frac{\partial(r^* - R_0)}{\partial R_0} < 0 \quad \text{and} \quad \frac{\partial(r^* - R_0)}{\partial \gamma} < 0$$

may change their signs through $\frac{\partial(r^* - R_0)}{\partial R} > 0$. However, it turns out that this general equilibrium effect is sufficiently small that *the deposit spread is still decreasing in R_0 and γ* in

general equilibrium. On the other hand, for another comparative statics result

$$\frac{\partial (R - r^*)}{\partial R_0} < 0,$$

we find that the general equilibrium effect through R (i.e., $\frac{\partial (R - r^*)}{\partial R} > 0$) dominates. Therefore, *the loan spread is increasing in R_0 in general equilibrium*. Finally, the result $\frac{\partial (R - r^*)}{\partial \gamma} > 0$ is strengthened by a general equilibrium increase in R .

Bounds on technology shocks z .

To make sure that the corporate loan market equilibrium exists with or without the interbank market, we impose bounds on technology shocks $z \in [\underline{z}, \bar{z}]$. First, we define the upper bound \bar{z} as follows. Let $\bar{R}^{nt} \equiv \arg \max_R V^{nt}(R)$ and $\bar{V}^{nt} \equiv V^{nt}(\bar{R}^{nt})$. From the expression of $V^{nt}(R)$, we have $\bar{R}^{nt} = 2\frac{R_0}{E[p]}$. Because the corporate loan demand $K(R; z)$ given in (2) satisfies $K(R; 0) = 0$ and it is increasing in z without bound, $V^{nt}(R) = K(R; z)$ has a unique solution $z = z^{nt}(R)$ for any $R > 1 - \delta$. Then we define $\bar{z} = z^{nt}(\bar{R}^{nt})$ such that $\bar{V}^{nt} = K(\bar{R}^{nt}; \bar{z})$ holds. Because $V^{nt}(R)$ is hump-shaped with a peak attained at $R = \bar{R}^{nt}$, for any $z < \bar{z}$, the corporate loan demand curve $K(R; z)$ intersects with the corporate loan supply curve $V^{nt}(R)$ at the point where the latter is increasing in R . Recall that $V^*(R) > V^{nt}(R)$ for any R . Because $K(R; z)$ is strictly decreasing in R , the corporate loan market equilibrium exists for any $z \leq \bar{z}$, with or without the interbank market. Moreover, the equilibrium loan quantity is higher but the return is lower with the interbank market than without it. That is, $V^{nt}(z) < V^*(z)$ and $R^*(z) < R^{nt}(z)$ for any $z \leq \bar{z}$.

Next, to ensure $R \geq 2R_0$ (**Assumption 2**) in general equilibrium, we define the lower bound \underline{z} as follows. Let $\bar{R}^* \equiv \arg \max_R V^*(R)$ and $\bar{V}^* \equiv V^*(\bar{R}^*)$.¹⁶ First, $V^*(R) = K(R; z)$ has a unique solution $z = z^*(R)$ for any $R > 1 - \delta$. Because $2R_0 < \bar{R}^* < \bar{R}^{nt} \Leftrightarrow 1 < \frac{E^h[1-p]}{E^h[\omega]} < \frac{1}{E[p]}$ holds, as long as $1 - \delta < R_0$, we can define $\underline{z} \equiv z^*(2R_0)$ such that $V^*(2R_0) = K(2R_0; \underline{z})$ holds. Then we have $R^*(z) \in [2R_0, R^*(\bar{z})]$ for any $z \in [\underline{z}, \bar{z}]$.¹⁷

Determination of A .

From the household's problem (3), the Euler equation is

$$u'(c) = \beta E[u'(c') r'(A', z') | A, z].$$

¹⁶From the expression of $V^*(R)$, $\bar{R}^* = 2\frac{E^h[1-p]}{E^h[\omega]}R_0$ and $\bar{V}^* = \frac{1}{4\gamma R_0} \frac{E^h[\omega]}{(E^h[1-p])^2}$.

¹⁷A necessary and sufficient condition for $\underline{z} < \bar{z}$ is $2R_0 < R^*(\bar{z})$. Because \bar{z} depends on R_0 , this implicitly imposes a constraint on the value of R_0 . We could not prove whether this condition holds generally. However, we numerically verified that $\underline{z} < \bar{z}$ for a reasonable range of parameter values. In any of our numerical results, $R_0 \in \left(1 - \delta, \frac{R^*(\bar{z})}{2}\right)$ is satisfied.

In a non-stochastic steady state where $c = c' = c^*(z)$ and $A = A' = A^*(z)$, the Euler equation collapses to $r^* = \frac{1}{\beta}$. Clearly, the same characterization holds in the absence of the interbank market, so $r^{nt} = \frac{1}{\beta}$. Define $\pi_B^*(z)$ by substituting $R^*(z)$ into (14). From $\pi_B^*(z) = A(r^* - R_0)$, we have

$$A^*(z) = \frac{1}{\frac{1}{\beta} - R_0} \pi_B^*(z).$$

In the absence of the interbank market, $A^{nt}(z) = \frac{1}{\frac{1}{\beta} - R_0} \pi_B^{nt}(z)$ hold. Thus, in a non-stochastic steady state, the share of bank profit in the aggregate saving is independent of whether the interbank trading occurs or not:

$$\frac{\pi_B^*(z)}{A^*(z)} = \frac{\pi_B^{nt}(z)}{A^{nt}(z)} = \frac{1}{\beta} - R_0.$$

This shows that in a non-stochastic steady state, $A^*(z)$, and hence reserves, become sensitive to the value of R_0 near its upper bound $\frac{1}{\beta}$.

In a stochastic steady state, r must increase to compensate households for bearing the risk of technology shocks and banks. We will see how this affects the bank spreads as well as real variables. We gather some properties of general equilibrium in **Proposition 3**.

Proposition 3 (general equilibrium impact of the interbank market)

Suppose $\delta \in (1 - R_0, \frac{1}{2}]$ and $z \in [\underline{z}, \bar{z}]$.

- (a) $2R_0 \leq R^*(z) < R^{nt}(z) \leq \bar{R}^{nt}$.
- (b) $0 < V^{nt}(z) < V^*(z) \leq \bar{V}^*$.
- (c) $w^{nt}(z) < w^*(z)$.
- (d) $R^{nt}(z)V^{nt}(z) < R^*(z)V^*(z)$.

First, $\underline{z} \leq z \Leftrightarrow 2R_0 \leq R^*(z)$, while $z \leq \bar{z} \Leftrightarrow R^{nt}(z) \leq \bar{R}^{nt}$. The lower bound \underline{z} ensures **Assumption 2**, which, together with **Assumption 1**, implied $p_L R^*(z) > R_0$. For z below this bound, the risky return $R^*(z)$ is so low that some low type banks do not want to keep a positive amount of the risky loan in the interbank market. The upper bound \bar{z} ensures that $R^{nt}(z)$ is increasing in z . For z above the upper bound, the existence of $R^{nt}(z)$ depends on the precise shape of the loan demand function (2), and when it exists, $V^{nt}(z)$ is decreasing in z .¹⁸ Second, the results $R^*(z) < R^{nt}(z)$ and $V^{nt}(z) < V^*(z)$ follow from the simple demand-supply analysis, where the interbank market shifts up the loan supply curve. The

¹⁸This occurs because too high success return makes the loan profit too risky and the risk-averse banks cannot accommodate too much risk. Equilibrium corporate loan may decrease in response to an increase in z , or there may not be a market-clearing price (i.e., excess demand for loan at any level of R).

wage increases because an increase in loan supply increases the marginal product of labor. Finally, capital income increases in response to an upward shift in the corporate loan supply curve because the loan demand is sufficiently price elastic.

4.1 Numerical Evaluation

We evaluate a stochastic steady state numerically, and compare it with a non-stochastic steady state. We use $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ for the household utility function. For the productivity shock, we assume the following AR(1) process:

$$\log(z_t) = (1 - \rho) \log(z_{ss}) + \rho \log(z_{t-1}) + \varepsilon_t, \quad \varepsilon_t \sim IID N(0, \sigma_\varepsilon^2),$$

where σ_ε and z_{ss} are chosen such that the two bounds (\underline{z}, \bar{z}) obtain with the discretization of the process with a 15-state Markov chain. For the bank type distribution, we use a two type distribution such that $p = p_H$ with probability λ . Parameters used in the numerical evaluation are summarized below.

Table 3. Baseline parameter values.

(p_L, p_H)	(0.6, 0.9)	γ	1
λ	0.3	R_0	1
α	0.36	ρ	0.89
δ	0.1	σ_ε	0.0244
β	0.97	z_{ss}	3.2907
σ	4.5		

We first summarize a common feature that runs through all the tables presented below. All the listed variables, *except the three variables R , χ , and $R - r$* , are greater with the interbank market than without it. This shows the robustness of the role of the interbank market highlighted in our model: it promotes risk-taking and hence increases production and welfare. At the same time, the corporate loan rate R and the loan spread $R - r$ decrease because of the larger supply of loan. The intermediation loss decreases because loan trading among banks shifts the effective type distribution upward. These effects of the interbank market are weakened by increasing R_0 , and strengthened by increasing γ and z_{ss} .

Non-stochastic steady state. **Table 4** through **6** present the results for a non-stochastic steady state where technology shock is fixed at z_{ss} . Because $r = \frac{1}{\beta} = 1.0309$ is fixed, it is not presented in the tables. In the table, IBM stands for interbank market. In each table, we present different comparative statics results. For example, in **Table 4**, panel

(a) through (c) corresponds to different values of R_0 . In each panel, the bottom row contains a percentage change of each variable with the interbank market relative to the same variable without the interbank market.

Table 4. Non-stochastic steady state for different values of R_0 .

(a) $R_0 = 0.99$										
	R	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.187	8.924	0.878	0.365	0.321	3.140	3.052	2.009	1.156	0.040
w/o IBM	2.532	5.606	0.605	0.229	0.475	2.747	2.686	1.758	1.502	0.040
% change	-13.65	59.18	45.02	59.18	-32.49	14.32	13.63	14.32	-23.02	0

(b) $R_0 = 1$ (baseline)										
	R	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.197	11.660	0.867	0.360	0.317	3.126	3.039	2.000	1.166	0.030
w/o IBM	2.545	7.318	0.598	0.226	0.472	2.735	2.675	1.750	1.514	0.030
% change	-13.65	59.34	44.89	59.34	-32.79	14.28	13.60	14.28	-22.95	0

(c) $R_0 = 1.01$										
	R	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.208	17.017	0.856	0.356	0.313	3.112	3.026	1.991	1.177	0.020
w/o IBM	2.557	10.668	0.591	0.223	0.469	2.724	2.665	1.743	1.526	0.020
% change	-13.66	59.52	44.75	59.52	-33.09	14.24	13.56	14.24	-22.89	0

Table 4 shows that as R_0 increases, V decreases and the loan spread increases. Note that the latter result is the opposite of the partial equilibrium result where R was taken as given. In general equilibrium, the higher IOR raises the loan spread by reducing the loan supply by banks. The contribution of the interbank market to welfare, measured by the percentage change in steady state consumption, becomes smaller as R_0 increases.

Table 5. Non-stochastic steady state for different values of γ .(a) $\gamma = 0.9$

	R	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.127	11.949	0.946	0.369	0.327	3.226	3.131	2.064	1.096	0.030
w/o IBM	2.452	7.334	0.655	0.226	0.498	2.826	2.761	1.809	1.421	0.030
% change	-13.25	62.92	44.33	62.92	-34.27	14.12	13.40	14.12	-22.86	0

(b) $\gamma = 1$ (baseline)

	R	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.197	11.660	0.867	0.360	0.317	3.126	3.039	2.000	1.166	0.030
w/o IBM	2.545	7.318	0.598	0.226	0.472	2.735	2.675	1.750	1.514	0.030
% change	-13.65	59.34	44.89	59.34	-32.79	14.28	13.60	14.28	-22.95	0

(c) $\gamma = 1.1$

	R	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.267	11.401	0.799	0.352	0.307	3.035	2.955	1.942	1.236	0.030
w/o IBM	2.637	7.285	0.549	0.225	0.449	2.652	2.597	1.697	1.607	0.030
% change	-14.05	56.50	45.46	56.50	-31.53	14.44	13.79	14.44	-23.06	0

Table 5 shows that as γ increases, V decreases and the loan spread increases. This is qualitatively same with an increase in R_0 . On the contrary, the contribution of the interbank market to welfare becomes *larger* as γ increases. This suggests that in time of high risk aversion, the welfare benefit of keeping the interbank market active becomes significant.

Table 6. Non-stochastic steady state for different values of z .(a) $z = 2.9616$

	R	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.090	10.275	0.842	0.317	0.282	2.783	2.699	1.781	1.059	0.030
w/o IBM	2.403	6.221	0.584	0.192	0.435	2.441	2.382	1.562	1.372	0.030
% change	-13.03	65.16	44.05	65.16	-35.16	14.04	13.31	14.04	-22.82	0

(b) $z = 3.2907$ (baseline)

	R	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.197	11.660	0.867	0.360	0.317	3.126	3.039	2.000	1.166	0.030
w/o IBM	2.545	7.318	0.598	0.226	0.472	2.735	2.675	1.750	1.514	0.030
% change	-13.65	59.34	44.89	59.34	-32.79	14.28	13.60	14.28	-22.95	0

(c) $z = 3.6198$

	R	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.309	13.064	0.884	0.404	0.350	3.463	3.375	2.216	1.278	0.030
w/o IBM	2.693	8.425	0.606	0.260	0.506	3.024	2.963	1.935	1.663	0.030
% change	-14.28	55.05	45.82	55.05	-30.86	14.54	13.90	14.54	-23.14	0

Table 6 shows that as z increases, V increases and the loan spread increases. The welfare benefit of the interbank market also increases in z .

Stochastic steady state. To characterize a stochastic steady state, we solve the household’s problem by value function iteration. After computing the stationary distribution over the state variables (A, z) , we calculate a mean for each variable. In the tables below, the means for each variable are reported.

Table 7. Stochastic steady state for different values of R_0 (mean).

(a) $R_0 = 0.99$

	R	r	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.189	1.376	0.954	0.877	0.365	0.321	3.144	3.056	2.012	0.812	0.386
w/o IBM	2.536	1.232	0.953	0.605	0.230	0.475	2.750	2.691	1.760	1.303	0.242
% change	-13.67	11.67	0.14	45.05	59.11	-32.48	14.33	13.59	14.33	-37.63	59.22

(b) $R_0 = 1$ (baseline)

	R	r	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.199	1.381	0.954	0.866	0.361	0.317	3.130	3.043	2.003	0.818	0.381
w/o IBM	2.548	1.239	0.953	0.597	0.226	0.472	2.738	2.680	1.752	1.308	0.239
% change	-13.67	11.48	0.13	44.92	59.27	-32.77	14.30	13.56	14.30	-37.50	59.39

(c) $R_0 = 1.01$

	R	r	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.210	1.386	0.955	0.855	0.356	0.314	3.116	3.030	1.994	0.823	0.376
w/o IBM	2.560	1.246	0.953	0.591	0.223	0.469	2.727	2.669	1.745	1.314	0.236
% change	-13.68	11.29	0.14	44.79	59.44	-33.07	14.26	13.53	14.26	-37.36	59.56

Note. Reported numbers are mean values in the stochastic steady state.

First, compared with the non-stochastic steady state in **Table 4**, the loan spread becomes much smaller while the deposit spread becomes much larger. Therefore, the gap between the two spreads narrow significantly. This is mostly due to an increase in the deposit rate in a stochastic environment. **Table 7** also shows that as R_0 increases, V decreases and the loan spread increases. This is qualitatively same with the non-stochastic case in **Table 4**. The major difference from the non-stochastic case is that the deposit spread becomes stochastic, and decreases (on average) as R_0 increases. Therefore, the gap between the two spreads increases in R_0 . The welfare contribution of the interbank market decreases in R_0 , but this effect is rather minor. **Table 7** shows that reserves increase in R_0 . This is consistent with

the finding in Hendrickson (2017). He argues that by paying IOR, the Fed hindered the portfolio reallocation channel of monetary policy.

Table 8. Stochastic steady state for different values of γ (mean).

(a) $\gamma = 0.9$											
	R	r	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.129	1.388	0.962	0.945	0.370	0.327	3.230	3.136	2.067	0.740	0.388
w/o IBM	2.454	1.238	0.960	0.655	0.227	0.498	2.830	2.767	1.811	1.216	0.238
% change	-13.27	12.13	0.23	44.36	62.84	-34.26	14.14	13.33	14.14	-39.12	62.99

(b) $\gamma = 1$ (baseline)											
	R	r	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.199	1.381	0.954	0.866	0.361	0.317	3.130	3.043	2.003	0.818	0.381
w/o IBM	2.548	1.239	0.953	0.597	0.226	0.472	2.738	2.680	1.752	1.308	0.239
% change	-13.67	11.48	0.13	44.92	59.27	-32.77	14.30	13.56	14.30	-37.50	59.39

(c) $\gamma = 1.1$											
	R	r	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	2.269	1.374	0.951	0.798	0.353	0.307	3.039	2.959	1.945	0.895	0.374
w/o IBM	2.641	1.238	0.950	0.548	0.225	0.449	2.655	2.602	1.699	1.402	0.238
% change	-14.07	10.90	0.14	45.49	56.44	-31.52	14.46	13.75	14.46	-36.13	56.54

Note. Reported numbers are mean values in the stochastic steady state.

Table 8 shows that as γ increases, V decreases. The loan spread increases while the deposit spread decreases. The welfare contribution of the interbank market increases in γ , similarly to the non-stochastic case.

In a stochastic steady state, we can compute the coefficient of variation (the standard deviation divided by the mean) for each variable. **Table 9** reports the coefficient of variation for each variable in the same environment as in **Table 7**.

Table 9. Stochastic steady state for different values of R_0 (coefficient of variation).(a) $R_0 = 0.99$

	R	r	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	0.026	0.023	0.134	0.012	0.063	0.056	0.058	0.057	0.058	0.075	0.083
w/o IBM	0.030	0.017	0.131	0.009	0.080	0.040	0.056	0.056	0.056	0.056	0.089
% change	-12.75	32.85	1.88	33.59	-20.41	41.29	2.08	1.39	2.08	34.42	-6.83

(b) $R_0 = 1$ (baseline)

	R	r	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	0.026	0.023	0.134	0.013	0.064	0.057	0.058	0.057	0.058	0.075	0.083
w/o IBM	0.030	0.017	0.132	0.009	0.080	0.040	0.057	0.056	0.057	0.056	0.090
% change	-12.74	32.99	1.85	32.36	-20.56	41.94	2.09	1.40	2.09	33.79	-6.70

(c) $R_0 = 1.01$

	R	r	A	V	π_B	χ	Y	C	w	$R - r$	$r - R_0$
IBM	0.026	0.022	0.135	0.013	0.064	0.058	0.058	0.057	0.058	0.074	0.084
w/o IBM	0.030	0.017	0.132	0.010	0.081	0.040	0.057	0.057	0.057	0.056	0.090
% change	-12.74	33.24	1.84	31.24	-20.71	42.58	2.10	1.34	2.10	33.32	-7.07

Note. Reported numbers are the standard deviation divided by the mean.

Comparing **Table 9** with **Table 7** reveals that for the most variables, the interbank market moves the mean and the volatility in the same direction. For example, the mean of the loan is significantly increased, but so is its volatility. The exception to this pattern is the bank profit π_B . The impact of the interbank market on the bank profit is the higher mean and the lower volatility. This again shows the existence of a strong incentive for those receiving π_B to create the interbank market.

Finally, we also study the case where the intermediation loss χ is not paid back to households. In this case, the intermediation loss becomes a social loss. This naturally lowers consumption, but increases the welfare contribution of the interbank market. Intuitively, when intermediation loss is a social loss, reallocating loans to better banks becomes more important. In the stochastic steady state with the benchmark parameterization, the percentage increase in consumption due to the interbank market goes up from 13.56% (when households receive χ) to 23.53% (when they don't). If we view the case where households receive χ and the case where χ is completely lost as the two polar cases, the welfare contribution of the interbank market lies somewhere between 13.56% and 23.53%.

From the numerical investigation, we draw the following policy implication. In the period with high risk aversion in the banking sector, loan supply is significantly reduced, which leads to lower real economic activity. It leads to the higher loan spread and the lower deposit spread. Keeping the interbank market active becomes more important, because shutting it down means larger drop in the amount of corporate loan and the level of real activity. To counteract these effects induced by high bank risk aversion, the central bank should reduce R_0 . In our model, the IOR policy is effective in the negative range (i.e., $R_0 < 1$) as well.

5 Conclusion

We presented a model of interbank markets and IOR, embedded in a simple general equilibrium setup. The model is tractable, rationalizes the two bank spreads, and offers a number of testable implications for bank variables. In this paper, we focus on the benchmark case where banks, individually or as a whole, can achieve the interior solution to their optimization problem. In particular, the assumed timing ensured that even though some banks fail, it does not affect the aggregate banking sector. In a companion paper, we investigate two extensions of the current model. First, we introduce limited liability in the interbank market. This significantly changes the behavior of banks, especially the banks with higher type. Second, we consider an alternative timing assumption where the deposit rate cannot be contingent on technology shocks. This introduces the possibility that the banking sector as a whole cannot pay out the promised deposit rate. We expect that both extensions generate a trade-off between efficiency and stability of the banking sector.

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6 Appendix: For Online Publication

Proof of **Lemma 1**.

Solving $Q^*(V) = 1$ for V yields (10). From the expression of (10), $V^*(R)$ is non-monotonic in R and there is unique R that maximizes $V^*(R)$. Hence, $V^*(R) < A$ for any R is implied by $\max_R V^*(R) < A$, which is **Assumption 3**. ■

Proof of **Lemma 3**.

From (13) and (14), $r^* - R_0 = \frac{\pi_B^*}{A}$. We show that $\frac{1}{\gamma} E \left[\frac{(p - \frac{R_0}{R})^2}{\omega(p)} \right] = V^* (E [g(p; \frac{R_0}{R}) p] R - R_0)$.

$$\begin{aligned}
 \frac{1}{\gamma} E \left[\frac{(p - \frac{R_0}{R})^2}{\omega(p)} \right] &= \frac{1}{\gamma R} E \left[R \frac{p - \frac{R_0}{R}}{\omega(p)} \left(p - \frac{R_0}{R} \right) \right] \\
 &= \frac{1}{\gamma R} E \left[R \frac{p - \frac{R_0}{R}}{\omega(p)} p \right] - \frac{1}{\gamma R} E \left[\frac{p - \frac{R_0}{R}}{\omega(p)} \right] R_0 \\
 &= V^* \left(\frac{\frac{1}{\gamma R} E \left[\frac{p - \frac{R_0}{R}}{\omega(p)} p \right] R}{V^*} - R_0 \right) \\
 &= V^* \left(E \left[\frac{\frac{p - \frac{R_0}{R}}{\omega(p)}}{E \left[\frac{p - \frac{R_0}{R}}{\omega(p)} \right]} p \right] R - R_0 \right) \\
 &= V^* \left(E \left[g \left(p; \frac{R_0}{R} \right) p \right] R - R_0 \right).
 \end{aligned}$$

The proof for $r^{nt} - R_0$ is similar using $r^{nt} - R_0 = \frac{\pi_B^{nt}}{A}$ and $\pi_B^{nt} = \frac{1}{2\gamma} \frac{(E[p] - \frac{R_0}{R})^2}{E[\omega(p)]}$. ■

Proof of **Proposition 1**.

(a) That $0 < r^* - R_0$ is obvious from the expression of r^* . From (??), we have $E [g(p; \frac{R_0}{R}) p] R - r^* = (1 - \frac{V^*}{2A}) (E [g(p; \frac{R_0}{R}) p] R - R_0)$. Hence, $V^* < A \Leftrightarrow \frac{V^*}{2A} < 1 - \frac{V^*}{2A}$ implies $r^* - R_0 < E [g(p; \frac{R_0}{R}) p] R - r^* < R - r^*$.

(b) We show $\frac{\partial r^*}{\partial R_0} = \frac{A - V^*}{A}$, $\frac{\partial r^*}{\partial R} = \frac{R_0}{R} \frac{V^*}{A}$, and $\frac{\partial r^*}{\partial \gamma} = -\frac{r^* - R_0}{\gamma}$, which imply all the results. Recall

$$r^* = R_0 + \frac{1}{2A\gamma} E \left[\frac{(p - \frac{R_0}{R})^2}{\omega(p)} \right].$$

Because $\frac{(p - \frac{R_0}{R})^2}{\omega(p)}$ is continuous in all the variables, we can exchange the order of partial

derivative and integration. Hence,

$$\frac{\partial r^*}{\partial R_0} = 1 - \frac{1}{A\gamma R} E \left[\frac{p - \frac{R_0}{R}}{\omega(p)} \right] = 1 - \frac{V^*}{A}.$$

Similarly,

$$\frac{\partial r^*}{\partial R} = \frac{1}{A\gamma} \frac{R_0}{R^2} E \left[\frac{p - \frac{R_0}{R}}{\omega(p)} \right] = \frac{R_0}{R} \frac{V^*}{A}.$$

Finally,

$$\frac{\partial r^*}{\partial \gamma} = -\frac{1}{\gamma^2} \frac{1}{2A} E \left[\frac{\left(p - \frac{R_0}{R}\right)^2}{\omega(p)} \right] = -\frac{r^* - R_0}{\gamma}.$$

(c) This follows from $\frac{\partial r^*}{\partial R_0} = \frac{A-V^*}{A}$, $\frac{\partial r^*}{\partial R} = \frac{R_0}{R} \frac{V^*}{A}$, and the fact that V^* is decreasing in γ .

■

Proof of Proposition 2.

(a) First, $V^* > V^{nt}$ is equivalent to $E \left[\frac{p - \frac{R_0}{R}}{\omega(p)} \right] > \frac{E[p] - \frac{R_0}{R}}{E[\omega(p)]}$. Let $\omega(p) \equiv X$ and $p - \frac{R_0}{R} \equiv Y$. Then the inequality to prove is $E \left[\frac{Y}{X} \right] > \frac{E[Y]}{E[X]} \Leftrightarrow E[Y] < E[X] E \left[\frac{Y}{X} \right]$. Because $E[X] E \left[\frac{Y}{X} \right] = E \left[X \frac{Y}{X} \right] - Cov \left[X, \frac{Y}{X} \right]$, this is equivalent to

$$E[Y] < E[Y] - Cov \left[X, \frac{Y}{X} \right] \Leftrightarrow Cov \left[X, \frac{Y}{X} \right] < 0.$$

Because $X = \omega(p)$ is strictly decreasing in $p \in [p_L, p_H] \subset (\frac{1}{2}, 1)$ while $\frac{Y}{X} = \frac{p - \frac{R_0}{R}}{\omega(p)}$ is strictly increasing in p , the result holds for any non-degenerate distribution f .¹⁹ **Assumptions 1** and **2** imply $V^{nt} > 0$, while **Assumption 3** implies $V^* < A$.

Next, $U^* > U^{nt}$ immediately implies $r^* > r^{nt}$ and $\pi_B^* > \pi_B^{nt}$. $U^* > U^{nt}$ is equivalent to

$$E \left[\frac{\left(p - \frac{R_0}{R}\right)^2}{\omega(p)} \right] > \frac{\left(E[p] - \frac{R_0}{R}\right)^2}{E[\omega(p)]}.$$

By the convexity, $\left(E[p] - \frac{R_0}{R}\right)^2 < E \left[\left(p - \frac{R_0}{R}\right)^2 \right]$ for any non-degenerate distribution f . Hence it suffices to show

$$E \left[\frac{\left(p - \frac{R_0}{R}\right)^2}{\omega(p)} \right] \geq \frac{E \left[\left(p - \frac{R_0}{R}\right)^2 \right]}{E[\omega(p)]}.$$

¹⁹For a formal proof that $Cov[f(X), g(X)] > 0$ for a random variable X and two increasing functions f and g , see for example Schmidt (2003).

Let $\omega(p) \equiv X$ and $(p - \frac{R_0}{R})^2 \equiv Y$. Then the above inequality is $E[\frac{Y}{X}] \geq \frac{E[Y]}{E[X]} \Leftrightarrow E[X] E[\frac{Y}{X}] \geq E[Y]$. Because $E[X] E[\frac{Y}{X}] = E[X \frac{Y}{X}] - Cov[X, \frac{Y}{X}] = E[Y] - Cov[X, \frac{Y}{X}]$,

$$E\left[\frac{Y}{X}\right] \geq \frac{E[Y]}{E[X]} \Leftrightarrow 0 \geq Cov\left[X, \frac{Y}{X}\right].$$

Because $\omega(p) \equiv X$ is strictly decreasing in $p > \frac{1}{2}$, it suffices to show that $\frac{Y}{X} = \frac{(p - \frac{R_0}{R})^2}{\omega(p)}$ is increasing in $p > \frac{1}{2}$.

$$\frac{\partial Y}{\partial p X} = \frac{2(p - \frac{R_0}{R})\omega(p) - (p - \frac{R_0}{R})^2(1 - 2p)}{\{\omega(p)\}^2} = \frac{p - \frac{R_0}{R}}{\{\omega(p)\}^2} \left\{ 2\omega(p) - \left(p - \frac{R_0}{R}\right)(1 - 2p) \right\}$$

is positive because $p > \frac{1}{2} \geq \frac{R_0}{R}$ by **Assumptions 1** and **2**.

(b) $\frac{\partial r^*}{\partial \gamma} < \frac{\partial r^{nt}}{\partial \gamma} < 0 < \frac{\partial r^*}{\partial R_0} < \frac{\partial r^{nt}}{\partial R_0} < 1$ and $0 < \frac{\partial r^{nt}}{\partial R} < \frac{\partial r^*}{\partial R} < 1$.

(c) $\frac{\partial}{\partial \gamma} \frac{\partial r^*}{\partial R} < \frac{\partial}{\partial \gamma} \frac{\partial r^{nt}}{\partial R} < 0 < \frac{\partial}{\partial \gamma} \frac{\partial r^{nt}}{\partial R_0} < \frac{\partial}{\partial \gamma} \frac{\partial r^*}{\partial R_0}$.

(b) Following the proof of **Proposition 1(b)**, we have $\frac{\partial r^{nt}}{\partial R_0} = \frac{A - V^{nt}}{A}$, $\frac{\partial r^{nt}}{\partial R} = \frac{R_0}{R} \frac{V^{nt}}{A}$, and $\frac{\partial r^{nt}}{\partial \gamma} = -\frac{r^{nt} - R_0}{\gamma}$. Combining these with $V^* > V^{nt}$ and $r^* > r^{nt}$ yields the results.

(c) It suffices to show $\frac{\partial V^*}{\partial \gamma} < \frac{\partial V^{nt}}{\partial \gamma} < 0$. This holds from $E\left[\frac{p - \frac{R_0}{R}}{\omega(p)}\right] > \frac{E[p] - \frac{R_0}{R}}{E[\omega(p)]}$ as proved in

Proposition 1(a). ■

Proof of **Proposition 3**.

(a) through (c) are explained in the main text.

(d) First, from the corporate loan demand (2), $K(1; z) = \left(\frac{z\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}$. Then for any z , the two curves $K(R; z)$ and $X(R; z) \equiv \frac{\left(\frac{z\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}}{R}$ intersect at a point $(R, K) = \left(1, \left(\frac{z\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}\right)$. Because $\delta \in \left(1 - R_0, \frac{1}{2}\right] \Leftrightarrow 1 \leq 2(1 - \delta) < 2R_0$, we know $1 < R^*(z)$. Hence, it suffices to show that $K(R; z) < X(R; z)$ for any $R > 1$. Because both $K(R; z)$ and $X(R; z)$ are decreasing in R , it suffices to show $\frac{\partial K(R; z)}{\partial R}\bigg|_{R=1} < \frac{\partial X(R; z)}{\partial R}\bigg|_{R=1}$. This holds because

$$\frac{\partial K(R; z)}{\partial R}\bigg|_{R=1} = -\frac{1}{(1 - \alpha)\delta} \left(\frac{z\alpha}{\delta}\right)^{\frac{1}{1-\alpha}} < -\left(\frac{z\alpha}{\delta}\right)^{\frac{1}{1-\alpha}} = \frac{\partial X(R; z)}{\partial R}\bigg|_{R=1}. \quad \blacksquare$$