Disclosure Services and Endogenous Segmentation in Takeover Markets

Kei Kawakami*

April 28th, 2020

Abstract

We present a competitive model of takeovers among heterogeneous firms. Each firm owns a tradeable "project" and non-tradeable "skill". The complementarity between them generates takeovers. We construct an equilibrium with two segmented markets. In one market, firms pay a fee to an intermediary to fully disclose their project quality. In the other market, firms reveal at no cost that their project quality is above a minimum standard. The latter market matches projects to skill randomly. Yet, it significantly improves welfare by raising the elasticity of the demand for the full disclosure service. Regulations necessary to support this equilibrium are discussed. (*JEL* L1, G3, D8)

Key words: Asymmetric information, Disclosure, Market segmentation, Takeovers.

*College of Economics, Aoyama Gakuin University, 4-4-25, Shibuya, Tokyo 150-0002, Japan (e-mail: kei.kawakami@gmail.com; https://sites.google.com/site/econkeikawakami/). This paper builds on a paper titled "Disclosure and Efficiency in Takeover Markets". I thank seminar participants at Aoyama Gakuin University, Hitotsubashi University, Nagoya University, NASMES 2018 at UC Davis, and University of Tokyo for comments and suggestions. All errors are mine. This work was supported by JSPS KAKENHI Grant Number JP18K12745. Financial support from the College of Economics at Aoyama Gakuin University is gratefully acknowledged.

1 Introduction

Recent studies show that a significant amount of resources are reallocated through takeover activities.[1] While the literature on takeovers is extensive, we know little about the *reallocational efficiency* of takeover markets. Standard macroeconomic models study reallocation through factor markets, and assume away a market for corporate control.[2] On the other hand, models of takeovers designed to study the managerial efficiency of public firms are not well-suited to study the effect on the aggregate production.[3] In this paper, we aim to fill this gap by presenting a model of takeovers among a continuum of heterogenous firms.

While a large and increasing number of firms engage in takeovers, it remains true that only a small fraction of firms participate in this market. Why is the participation so limited? We argue that because a unit of trading in takeover markets is an entire firm (or at least its large indivisible component not tradeable via factor markets), the importance and difficulty of *credibly disclosing its quality* is significant. For many firms, such disclosure is too costly to do on their own and they need to rely on a costly service provider. In practice, the disclosure is often facilitated by large for-profit intermediaries (e.g. investment banks, law firms, accounting firms, consulting firms etc.). Aside from occasional "mega" deals, most takeovers involve firms with little bargaining power against these intermediaries. Two questions arise. To improve the reallocational efficiency of takeover markets, should such information intermediaries be regulated? If so, how should such regulations be designed?

To shed light on these issues, we present a competitive matching model of takeovers in which firms are subject to information frictions and need to rely on a third party to disclose private information.[4] Each firm owns two indivisible factors: a tradeable "project" and non-tradeable "skill" necessary to manage a project. Firms are heterogeneous in both dimensions, and complementarity between the two factors creates potential gains from trade. As a first friction in the model, we assume that firms are privately informed about their project quality and skill. Hence, all firms have private information not only about their prospect for takeovers (as targets or bidders), but also about their stand-alone values. Nevertheless, we show that competitive takeover markets can achieve the efficient reallocation *if firms can credibly disclose their project quality at no cost*. The amount of increase in production

[1] For example, see David (2020) and Eckbo (2014).

[2] For example, see Restuccia and Rogerson (2008). Hopenhayn (2013) surveys this literature. A recent work in this line of research, David (2020), is an exception.

[3] Studies in this area of research do not model firms' production side and/or analyze only firms directly involved in takeovers. For example, see Grossman and Hart (1980a) and the literature that followed.

[4] A competitive matching framework allows for transparent treatment of inefficiency arising from information frictions. While strategic aspects of firms may be more important in some industries than others, we believe that our modelling choice is a reasonable first step given our focus on the effect of takeovers on the aggregate production.

relative to no takeover is used as our benchmark welfare gain from takeovers.

As a second friction in the model, we assume that target firms must rely on a third party to credibly disclose their project quality. We study two types of disclosure services: 1) *a full disclosure service* provided by a monopoly intermediary, and 2) *a minimum disclosure service* provided by a regulatory body. The former allows a target firm to fully disclose its project quality by paying a fixed fee. The latter is free, but a target firm can only reveal that its project quality is above a common minimum standard.[5] After studying the welfare consequence of each service independently, we investigate the possibility of endogenous market segmentation, and whether it can improve the welfare.

We show that if the minimum disclosure service is appropriately designed, markets are segmented into two parts. Firms endogenously select into either a full disclosure market or a minimum disclosure market (or staying out of takeovers), and also choose a side (to be a target or a bidder) in each market. Targets in the full disclosure market have better projects than those in the minimum disclosure market. Similarly, bidders in the full disclosure market have better skills than those in the minimum disclosure market. A competitive price schedule in the full disclosure market leads to assortative matching: a project of better quality is transferred to a firm with a better skill. On the other hand, matching in the minimum disclosure market is random: all the projects of quality above the minimum standard but below the endogenous quality level that separates the two markets trade at a single price.

We calculate the welfare loss associated with each type of discloser services as a fraction to the benchmark welfare gain. Independently, the welfare loss caused by the monopoly intermediary is 28.5%, while the welfare loss caused by the minimum disclosure service is 70.6%, if no standard is imposed. The welfare loss with the minimum disclosure service can be reduced to 17.4% by optimally setting a minimum quality standard. When the minimum standard is structured such that the two services coexist in equilibrium, the welfare loss is reduced to 6.7%. This is a significant improvement over the situation with only either one of the two disclosure services.

To achieve the coexistence of the two services, the minimum standard must respond to the monopoly fee in the following way. If the fee is above a certain threshold level, then the minimum standard is set at a sufficiently high level that drives the profit of the full disclosure service down to zero. If the fee is below this threshold, then the minimum standard is set to ensure that each service is used by some firms. We show that when the regulatory body commits to this plan, the monopoly intermediary chooses a fee level that is significantly lower

[5] Focusing on these polar cases allows us to study a key trade-off in a transparent way. While multiple intermediaries compete one another to some degree, they charge a high fee for their service, indicating their large bargaining power relative to client firms. A monopoly assumption is a convenient way to capture this relative bargaining power.

than the fee it would choose in the absence of the minimum disclosure service. Intuitively, the presence of a minimum disclosure service *designed to be used by some firms* makes demand for the full disclosure service more elastic to its fee. This forces the intermediary not only to lower its fee but also to cater to a smaller group of firms with higher gains from trade. The reduction in the welfare loss is significant because both a price margin (the fee) and a quantity margin (the participation of firms) work against the intermediary. The analysis suggests that a minimum disclosure service, although it appears very inefficient on its own, might be a powerful regulatory tool.

The balance of the paper is organized as follows. After reviewing the related literature, Section 2 describes a model. Section 3 studies competitive takeovers when firms can fully disclose their project quality at no cost. This allocation serves as our welfare benchmark. In Section 4 we introduce two disclosure services. After studying each service in isolation in Section 4.1 and Section 4.2, in Section 4.3 we construct an equilibrium where both services are used by firms. Section 5 concludes. All proofs are gathered in the Appendix.

1.1 Related literature

The literature on takeovers is extensive, spanning many fields from financial economics, organizational economics, to macroeconomics. Because we focus on normative issues, we discuss related works with normative implications.[6] Lambrecht and Myers (2007) study a model of consolidating takeovers in declining industries in which inefficiency can arise due to an agency friction. Legros and Newman (2013) study a model of firms' integration decision in which the shared ownership may cause inefficiency. We focus on takeovers that exploit synergies and growth opportunities. Also, a source of inefficiency is different because we ignore any agency or ownership friction. David (2020) embeds a takeover process in a dynamic macro model to study its aggregate implication. The key friction in his model is search frictions. None of these works studies information frictions and the role of disclosure services. In this respect, our model is closest to Jovanovic and Braguinsky (2004). Relative to this work, our model features a non-degenerate distribution of project quality and multiple modes of disclosures.[7] These features allow us to study an interaction of multiple disclosure services. Beyond the literature on takeovers, we share with Kim (2012) that the endogenous market segmentation can improve welfare in the presence of information frictions. However, the mechanism that generates segmentation is different. In our model it arises from a cost-

[6] Theoretical and empirical studies on positive issues are vast and not reviewed here. Positive issues include determinants of takeovers as well as implications for asset prices and subsequent firm performances.

[7] In their model a project quality takes either zero or one. It simplifies their analysis but limits its scope.

benefit trade-off of different disclosure services, while Kim (2012) relies on search frictions.[8] Finally, we contribute to the literature on voluntary disclosure.[9] To our knowledge, we are the first to study heterogeneous firms sorting into different disclosure services.

2 Model

In this section, we describe a model of competitive takeovers among a continuum of firms, and define a competitive equilibrium. We wait until Section 4 to introduce disclosure services.

Endowment. Firms utilize two factors of production. The first factor is tradeable but indivisible, while the second factor is non-tradeable. Firms are heterogeneous in the quality of both factors. We call the tradeable factor *a project*, and the non-tradeable factor *skill*. We interpret a project as a collection of *tangible* assets, that remain productive when their ownership changes. Examples include a large plant (manufacturing), a customer base (retail), and an access to specific locations (services). On the other hand, we interpret skill as a collection of *intangible* assets, that are productive only within the current firm boundary.[10] Examples include organization-specific knowledge and team capital embodied in a large network of people. These are useful assets but it is difficult, if not impossible, to sustain their productivity after takeovers due to various coordination issues.[11]

We assume that each firm can effectively manage at most one project. This implies that when a firm buys another firm to obtain a new project, it must sell, or simply abandon, its original project. While large firms that manage many independent projects do exist, most small firms face resource constrains that prevent them from managing multiple projects. For these small firms, the relevant decision is not *how many projects* to manage but *which project* to manage. In this sense, the constraint that each firm manages at most one project captures an aspect of small firms, consistent with a notion of competitive takeovers.

Technology. A firm with a project of quality A and the skill level X produces output using the technology

$$F(A, X) = AX. \tag{1}$$

We assume that A and X are independently uniformly distributed in a closed interval $[0, 1]$,

[8] Also, buyers and sellers are exogenous in Kim (2012), while firms choose their roles in our model.

[9] See Grossman and Hart (1980b), Grossman (1981), Jovanovic (1982), and Lizzeri (1999).

[10] Some intangible assets, such as intellectual properties and human capital of workers, may be tradeable via takeovers. However, to be productive, these assets may require a firm-specific factor such as local reputation and history. If the latter is not tradeable, neither is the former.

[11] A simple example is the skill of the single owner-manager selling his firm to do other business. Generally, a biding firm does not pay for anything it cannot utilize, no matter how valuable it is for a target firm.

and that there is a continuum measure one of firms. The form of technology and the distributional assumption are made to make our analysis transparent.[12]

Welfare measure. The aggregate production without takeovers is

$$\int_0^1 \int_0^1 (AX)\, dAdX = \left(\int_0^1 AdA\right)\left(\int_0^1 XdX\right) = \frac{1}{4}.$$

Given the form of technology (1) and the constraint that each firm can manage at most one project, the first best allocation features positive assortative matching between skills and projects: for any $a \in (0,1]$, a project of quality $A = a$ should be managed by a firm with skill level $X = a$. This leads to the maximum aggregate production of

$$\int_0^1 a^2 da = \frac{1}{3}.$$

Thus, the maximum welfare gain is $\frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{4}} \times 100 = 33.3\%$.

Takeover markets. Firms can operate on both sides of markets, i.e., they can sell its project and/or buy a new project. However, we assume that firms need to prioritize these two activities. For example, if a firm tries to sell its project as soon as it can, then less internal resources will be allocated to buying and managing a new project. Similarly, if a firm tries to buy a new project as soon as it can, then less internal resources will be allocated to selling its initial project. To capture this internal resource constraint on acting on both sides of the market, we assume that the payoff from the activity chosen as secondary is reduced by a constant rate $\beta \in [0,1]$. This makes firms optimally rank the two activities such that a "primary" activity should not be sacrificed for a "secondary" activity. When the secondary activity is unprofitable (i.e., $\beta = 0$), each firm acts on only one side of markets.

In short, options available for each firm are: (i) sell its project as a primary activity, and buy a new project as a secondary activity (*S-strategy*), or (ii) buy a new project as a primary activity, and sell its project as a secondary activity (*B-strategy*), or (iii) not trade and produce with its project (*N-strategy*). We call firms that choose S-strategy *S-firms*. Similarly, we call firms that choose B-strategy *B-firms*, and firms that choose N-strategy *N-firms*. We formally describe firms' problem below by assuming that project quality is public information. We show later that target firms indeed have an incentive to make this information public, if they can do it at no cost. Firms' problem with a costly disclosure service will be described in Section 4.1.

[12] Kawakami (2019) studies a model with more general technology and distribution.

First, a payoff from S-strategy is

$$\Pi_S(A,X) \equiv P(A) + \beta \max_{a\in[0,1]} \{aX - P(a)\},$$

where $\{P(a)\}_{a\in(0,1]}$, is a competitive price schedule, and $\beta \in [0,1]$ is a rate at which the payoff from the secondary activity is reduced.[13] Second, a payoff from B-strategy[14] is

$$\Pi_B(A,X) \equiv \max_{a\in[0,1]} \{aX - P(a)\} + \beta P(A).$$

A payoff from the N-strategy is AX. Note that both $\Pi_S(A,X)$ and $\Pi_B(A,X)$ depend on the entire price schedule $P(a)$, $a \in (0,1]$. Firms' problem is

$$\Pi(A,X) \equiv \max\{\Pi_S(A,X), \Pi_B(A,X), AX\}. \tag{2}$$

Market-clearing. Given a price $P(a)$, a supply and a demand of projects of quality a are determined as follows. A supply of projects of quality a coming from S-firms is $S_1(a) \equiv \int\int \mathbf{1}_{\mathcal{S}_1(a)} dAdX$, where $\mathbf{1}_{\mathcal{S}_1(a)}$ is an indicator function for the set

$$\mathcal{S}_1(a) \equiv \left\{(A,X) \in [0,1]^2 \,|A = a, \Pi(A,X) = \Pi_S(A,X)\right\}.$$

A supply of projects of quality a coming from B-firms is $S_2(a) \equiv \int \mathbf{1}_{\mathcal{S}_2(a)} dAdX$, where

$$\mathcal{S}_2(a) \equiv \left\{(A,X) \in [0,1]^2 \,|A = a, \Pi(A,X) = \Pi_B(A,X)\right\}.$$

A demand of projects of quality a coming from B-firms is $D_1(a) \equiv \int\int \mathbf{1}_{\mathcal{D}_1(a)} dAdX$, where

$$\mathcal{D}_1(a) \equiv \left\{(A,X) \in [0,1]^2 \,|a = \arg\max_{a\in[0,1]} \{aX - P(a)\}, \Pi(A,X) = \Pi_B(A,X)\right\}.$$

A demand of projects of quality a coming from S-firms is $D_2(a) \equiv \int\int \mathbf{1}_{\mathcal{D}_2(a)} dAdX$, where

$$\mathcal{D}_2(a) \equiv \left\{(A,X) \in [0,1]^2 \,|a = \arg\max_{a\in[0,1]} \{aX - P(a)\}, \Pi(A,X) = \Pi_S(A,X)\right\}.$$

[13]An alternative specification $\max_{a\in[0,1]} \{\beta aX - P(a)\}$ yields a qualitatively similar result. What is important for our analysis is that a fraction $1-\beta$ of the social value of the secondary activity is not realized due to an internal resource constraint.

[14]This is where the assumption that each firm can manage only one project matters. Without this assumption, $P(A)$ in the second term should be $\max\{P(A), AX\}$, and B-firms would choose AX rathern than $P(A)$, i.e., manage two projects.

For $\beta = 0$, we set $S_2(a) = D_2(a) = 0$ for any $a \in [0,1]$. We say that $P(a)$ clears a market for projects of quality a if and only if

$$S_1(a) + S_2(a) = D_1(a) + D_2(a). \tag{3}$$

Competitive equilibrium. A competitive equilibrium is a pair of price function $\{P(a)\}_{a\in(0,1]}$ and firms' strategies such that (i) firms' strategies solve (2) taking $\{P(a)\}_{a\in(0,1]}$ as given, and (ii) (3) holds for each $a \in (0,1]$. In the next section, we characterize a competitive equilibrium and show that equilibrium allocation is efficient.

Interpretation of the model. The model is static and markets open only once. However, two different economic activities occur in markets. The first activity is *takeovers*. S-firms are target firms whose first priority is to sell their project. B-firms are bidder firms whose first priority is to buy a new project. For this activity, projects are transferred from S-firms to B-firms. The second activity is *restructuring*. As a secondary activity, B-firms would like to sell its initial project, because they lack resources to manage two projects. S-firms would like to buy these projects to utilize their skill. For this activity, projects are transferred from B-firms to S-firms. In this interpretation, $S_1(a)$ is a supply by *takeover targets* while $D_1(a)$ is a demand by *takeover bidders*. On the other hand, $S_2(a)$ is a supply coming from the restructuring process of B-firms while $D_2(a)$ is a demand coming from the restructuring process of S-firms.

Interpretation of the behavior of S-firms needs a care — because they are target firms, *they disappear as a firm*. Accordingly, we interpret S-firms' secondary activity as *starting a new firm after selling the original firm*. For example, an owner-manager of a start-up firm, after selling his share to another firm, may start a new firm by buying a small project from other firms.[15] We follow this interpretation of S-firms, but our main results do not hinge on this particular interpretation because our focus is the effect on the aggregate production.[16]

3 Competitive equilibrium and disclosure

We first characterize competitive equilibrium assuming that the information about the project quality is public. After establishing its efficiency property in Section 3.1, in Sec-

[15] B-firms have no interest in paying for two projects due to their capacity constraint. Hence, S-firms must sell themselves *before* buying a new project (otherwise they will have to hand over a control right of the two projects without sufficient compensation). While we do not explicitly model dynamics, the way we formalize firms' problem is consistent with this timing.

[16] It does affect positive implications. For example, to observe announcement returns for takeover deals, firms must be publicly traded. We study positive issues in Kawakami (2019).

tion 3.2 we show that the competitive equilibrium is robust to information frictions as long as firms can disclose their project quality at no cost.

3.1 Competitive equilibrium

We use a guess-and-verify method to jointly solve for firms' optimal strategy and an equilibrium price schedule. We first conjecture that the price function of the form

$$P(a) = \lambda a^2 \tag{4}$$

with an unknown parameter $\lambda \in (0,1)$. After deriving firms' optimal response to this price function, we show that markets clear for any $a \in (0,1]$ if and only if $\lambda = \frac{1}{2}$.[17]

Given the conjecture (4), the optimal choice of a project and the associated payoff are

$$\arg\max_{a\in[0,1]} \left\{aX - \lambda a^2\right\} = \frac{X}{2\lambda} \qquad \text{and} \qquad \max_{a\in[0,1]} \left\{aX - \lambda a^2\right\} = \frac{X^2}{4\lambda}.$$

First, we consider the choice between S-strategy and B-strategy.

$$\Pi_S(A,X) \gtrless \Pi_B(A,X) \quad \Leftrightarrow \quad \lambda A^2 + \frac{\beta}{4\lambda}X^2 \gtrless \frac{X^2}{4\lambda} + \beta\lambda A^2.$$

If $\beta = 1$, all firms are indifferent between S-strategy and B-strategy. Otherwise,

$$\Pi_S(A,X) \gtrless \Pi_B(A,X) \quad \Leftrightarrow \quad \frac{A}{X} \gtrless \frac{1}{2\lambda}. \tag{5}$$

Next, we fix $a \in (0,1]$ and study the choice of firms with $A = a$ and $X < 2\lambda a$. By (5), S-strategy dominates B-strategy. S-strategy also dominates N-strategy if and only if

$$\Pi_S(a,X) = \lambda a^2 + \frac{\beta}{4\lambda}X^2 \geq aX \quad \Leftrightarrow \quad \frac{\beta}{4\lambda}X^2 - aX + \lambda a^2 \geq 0,$$

which is equivalent to

$$\begin{aligned} &X \leq \lambda a && \text{for } \beta = 0,\\ &X \leq X^-(a) \text{ or } X^+(a) \leq X && \text{for } \beta > 0, \end{aligned}$$

where $X^{\pm}(a) \equiv \frac{2\lambda a}{1\mp\sqrt{1-\beta}}$. Therefore, for $\beta = 0$, $S_1(a) = \lambda a$ and $S_2(a) = 0$. For $\beta > 0$, notice

[17] There is an alternative approach. For a given $a \in (0,1]$, we could start by conjecturing that firms with $A = a$ and $X = a$ trade each other, derive a corresponding market-clearing price $\widetilde{P}(a)$, and then verify that $\arg\max_{a\in[0,1]} \left\{aX - \widetilde{P}(a)\right\} = X$ for any $X \in (0,1]$. We use this alternative approach in Section 4.

that $X < 2\lambda a \in \left(X^{-}(a), X^{+}(a)\right)$.[18] Hence these firms choose S-strategy if and only if

$$X \leq X^{-}(a) = \frac{2\lambda a}{1+\sqrt{1-\beta}}.$$

This implies $S_1(a) = X^{-}(a)$ for $\beta > 0$.

For the same $a \in (0,1]$, we similarly study the choice of firms with $X = 2\lambda a$ and $A < a$. By (5), these firms choose B-strategy over S-strategy and demand a project of quality a. They choose B-strategy over N-strategy if and only if

$$\Pi_B(A, 2\lambda a) = \frac{1}{4\lambda}(2\lambda a)^2 + \beta\lambda A^2 \geq A2\lambda a \quad \Leftrightarrow \quad \beta A^2 - 2aA + a^2 \geq 0,$$

which is equivalent to

$$\begin{array}{ll} A \leq \frac{1}{2}a & \text{for } \beta = 0, \\ A \leq A^{-}(a) \text{ or } A^{+}(a) \leq A & \text{for } \beta > 0, \end{array}$$

where $A^{\pm}(a) \equiv \frac{a}{1\mp\sqrt{1-\beta}}$. Therefore, for $\beta = 0$, $D_1(a) = \frac{1}{2}a$ and $D_2(a) = 0$. For $\beta > 0$, notice that $A < a \in \left(A^{-}(a), A^{+}(a)\right)$.[19] Hence these firms choose B-strategy if and only if

$$A \leq A^{-}(a) = \frac{a}{1+\sqrt{1-\beta}}.$$

This establishes $D_1(a) = A^{-}(a)$ for $\beta > 0$.

The above analysis shows that, for any $\beta \in [0,1]$, $S_1(a) = D_1(a)$ holds if and only if $\lambda = \frac{1}{2}$. Finally, for $\beta > 0$ we show that $P(a) = \frac{a^2}{2}$ also implies $S_2(a) = D_2(a)$. Given $P(a) = \frac{a^2}{2}$, firms with $A = a$ and $X \in \left[0, \frac{a}{1+\sqrt{1-\beta}}\right]$ become S-firms, while firms with $X = a$ and $A \in \left[0, \frac{a}{1+\sqrt{1-\beta}}\right]$ become B-firms. Because this holds for any $a \in (0,1]$, firms with $\frac{A}{X} \geq 1+\sqrt{1-\beta}$ become S-firms while firms with $\frac{A}{X} \leq \frac{1}{1+\sqrt{1-\beta}}$ become B-firms. Recall that S-firms and B-firms reverse their roles for their secondary activity. Then, for any $a \in \left(0, \frac{1}{1+\sqrt{1-\beta}}\right]$, the restructuring supply comes from B-firms with $A = a$, so $S_2(a) = 1 - \left(1+\sqrt{1-\beta}\right)a$. On the other hand, the demand comes form S-firms with $X = a$, so $D_2(a) = 1 - \left(1+\sqrt{1-\beta}\right)a$.

In sum, for any $a \in (0,1]$, $P(a) = \frac{a^2}{2}$ clears a market. Using this price function, solving for $\Pi_S(A,X)$ and $\Pi_B(A,X)$ is straightforward. We denote by $Q_1(a)$ an equilibrium measure of S-firms with $A = a$. This is a number of takeover deals. Similarly we denote by $Q_2(a)$ an equilibrium measure of B-firms with $A = a$. This is a number of restructuring deals. Note

[18] This is implied by $\frac{\beta}{4\lambda}(2\lambda a)^2 - a(2\lambda a) + \lambda a^2 < 0$.

[19] This is implied by $\beta a^2 - 2aa + a^2 < 0$.

that $Q_2(a) = 0$ for $\beta = 0$. Finally, our welfare measure is the aggregate production

$$Y_B + Y_S + Y_N \equiv Y(\beta),$$

where Y_i, $i \in \{B, S, N\}$, is the aggregate production by i-firms. For $i \in \{B, S\}$, firms with skill X produce X^2. Therefore, by denoting a measure of i-firms with skill X by $m_i(X)$, the sum of their production is $\int_0^1 X^2 m_i(X)\, dX$. More precisely,

$$\begin{aligned} Y_B &= \int_0^1 X^2 \left(\frac{X}{1+\sqrt{1-\beta}} \right) dX, \\ Y_S &= \beta \int_0^{\frac{1}{1+\sqrt{1-\beta}}} X^2 \left(1 - \left(1+\sqrt{1+\beta}\right) X \right) dX, \\ Y_N &= 2 \int_0^1 \left(\int_{\frac{A}{1+\sqrt{1-\beta}}}^{A} (AX)\, dX \right) dA. \end{aligned}$$

Proposition 1 summarizes the above analysis.

Proposition 1 (competitive equilibrium)

(a) *Firms' equilibrium payoff is*

$$\Pi(A,X) = \begin{cases} \frac{1}{2}\left(A^2 + \beta X^2\right) & \text{for } \frac{A}{X} \geq 1+\sqrt{1-\beta}, & \text{(S-firms)} \\ \frac{1}{2}\left(X^2 + \beta A^2\right) & \text{for } \frac{A}{X} \leq \frac{1}{1+\sqrt{1-\beta}}, & \text{(B-firms)} \\ AX & \text{for } \frac{A}{X} \in \left(\frac{1}{1+\sqrt{1-\beta}}, 1+\sqrt{1-\beta} \right). & \text{(N-firms)} \end{cases}$$

(b) *For each $a \in (0,1]$, equilibrium volume is*

$$\begin{aligned} Q_1(a) &= \tfrac{a}{1+\sqrt{1-\beta}} & \text{for } \beta \in [0,1], \\ Q_2(a) &= \max\left\{0, 1 - \left(1+\sqrt{1-\beta}\right) a\right\} & \text{for } \beta \in (0,1]. \end{aligned}$$

(c) *$Y(\beta)$ increases in β. A welfare gain relative to no reallocation is*

$$Y(\beta) - \frac{1}{4} = \frac{1}{12} \frac{\beta + 3\sqrt{1-\beta} + 3(1-\beta)}{\left(1+\sqrt{1-\beta}\right)^3} \qquad \in \left[\frac{1}{16}, \frac{1}{3} - \frac{1}{4} \right].$$

Proposition 1(a) shows that firms' sorting pattern only depends on $\frac{A}{X}$. Firms with $\frac{A}{X} \geq 1 + \sqrt{1-\beta}$ become S-firms, while firms with $\frac{A}{X} \leq \frac{1}{1+\sqrt{1-\beta}}$ become B-firms. Naturally, N-firms are concentrated along the line $\frac{A}{X} = 1$. These are the firms whose initial allocation benefits the most from the complementarity in the technology. In **Figure 1** below, N-firms reside in the area between the two indifference lines, $A = \left(1 + \sqrt{1-\beta}\right) X$ (a solid blue line) and $A = \frac{X}{1+\sqrt{1-\beta}}$ (a dashed red line). As β increases, a measure of N-firms decreases.

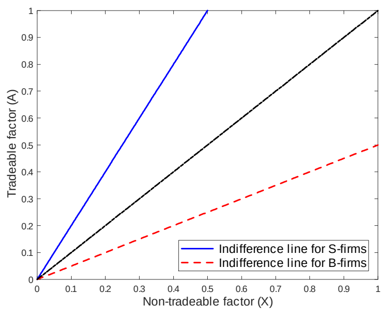

Figure 1(a) $\beta = 0$.

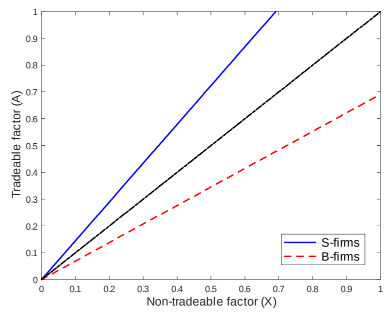

Figure 1(b) $\beta = 0.8$.

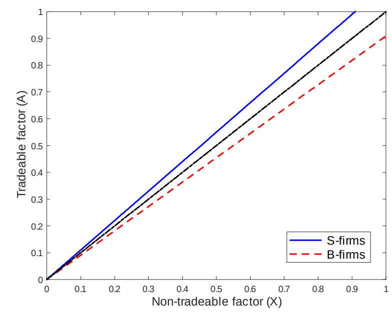

Figure 1(c) $\beta = 0.99$.

Proposition 1(b) shows that a number of takeover deals, $Q_1(a)$, increases in a while that of restructuring deals, $Q_2(a)$, decreases in a. Intuitively, projects of higher quality receive a higher priority to trade, and those of lower quality are traded in the restructuring process. Both types of trading activities increase in β, because higher β induces more N-firms to become either S-firms or B-firms, increasing both takeovers and restructuring.

Proposition 1(c) shows that the aggregate production increases in β. In particular, $Y(0) = \frac{1}{4} + \frac{1}{16}$ and $Y(1) = \frac{1}{3}$. Recall that the production without any reallocation yields $\frac{1}{4}$ while $Y(1) = \frac{1}{3}$ is achieved by the first best allocation. We show in the Appendix[20] that $Y(0) = \frac{1}{4} + \frac{1}{16}$ can be interpreted as the second best allocation where the planner cannot force firms to trade twice (corresponding to full discounting of the secondary activity). In Section 4, we focus on the case with $\beta = 0$. Accordingly, we use $Y(0) - \frac{1}{4} = \frac{1}{16}$ as our benchmark welfare gain from takeovers.

3.2 Information friction and disclosure

To derive the competitive market allocation in **Proposition 1**, we assumed that project quality is public information.[21] What happens if (A, X) is firms' private information? Following the literature on disclosure, suppose that firms can credibly disclose quality of their projects at no cost. Because the equilibrium price function $P(a)$ is increasing in a, voluntary

[20] See the first section titled "Planner's solution".

[21] Skill does not have to be public information, because given that the project quality is publicly observable, bidders' optimal choice of a project leads to an efficient matching between projects and skill.

disclosure leads to a standard *unravelling result* for all firms.[22] Therefore, **Proposition 1** holds true even if (A, X) is firms' private information, as long as *the full disclosure of project quality is possible for free.* Since this result is standard, we state it without a proof.

Lemma 1 *Assume that (A, X) is firms' private information.* ***Proposition 1*** *holds if firms can disclose their project quality at no cost.*

As we discussed in the introduction, the assumption of *full and free disclosure* is unlikely to hold in actual takeover markets. In the next section, we relax this assumption step-by-step. First, we relax the "free" part by studying a monopoly intermediary who charges a fee for their full disclosure service. Second, we relax the "full" part by studying a minimum disclosure service offered to firms for free by a regulatory body. Finally, we construct an equilibrium where the two disclosure services coexist, and investigate how the intermediary and the regulatory body interact in equilibrium.

4 Disclosure services in takeover markets

To produce and communicate the relevant information, firms need to rely on disclosure services provided by a third party with credibility and expertise. In this section, we introduce two types of such disclosure services: (i) a full disclosure service offered by a monopoly intermediary, and (ii) a minimum disclosure service offered for free by a regulatory body. We focus on these two services because they capture a key trade-off – quality and cost – in a stark way. In the next two subsections, we characterize a competitive equilibrium with each type of disclosure service independently. In the last subsection, we construct an equilibrium in which the two disclosure services coexist, and discuss how the minimum standard must be designed to support this equilibrium. To make our analysis transparent, we set $\beta = 0$ in this section. Accordingly, a benchmark welfare gain is $Y(0) - \frac{1}{4} = \frac{1}{16}$. Because there is no restructuring activity, we simply call S-firms targets, and B-firms bidders.

4.1 Full disclosure service by a monopoly intermediary

As an opposite extreme to a free disclosure service, we study a monopoly intermediary who charges a fee to target firms for its service. As our main scenario, we assume that the

[22] Among potential sellers, firms trying to sell a projects of the best quality would like to separate from others, and hence disclose its quality. The same reasoning applies for the remaining group of potential sellers. See Grossman and Hart (1980b).

intermediary uses a fixed fee for all target firms. More generally, we may think of (i) a fee for bidder firms, and (ii) a flexible fee structure that depends on firms' private information. First, we ignore a fee for bidder firms because the intermediary in our model provides a disclosure service, not a matching service. When matching is done in a competitive market, bidder firms do not need any disclose service.[23] Second, we restrict our attention to a set of fee structures that do not distort bidders' choice of projects. This allows us to preserve the structure of competitive matching, separating a role of information production from that of matching. We show below that a fixed fee is the only fee structure to achieve this separation if the fee cannot depend on skill X. We also study a fee contingent on (A, X) that can extract full surplus from targets. However, we will show in Section 4.3 that this fee is not robust to the presence of the minimum disclosure service.[24]

Suppose that the intermediary charges each target a fee $\phi \geq 0$. Firms take prices in the takeover markets as well as ϕ as given. Because ϕ affects prices of target firms, we denote it by $P(a;\phi)$. We first solve firms' problem and derive equilibrium $P(a;\phi)$ for a given $\phi \geq 0$. Then we study the monopoly intermediary's choice of ϕ.

Firms' problem. With $\beta = 0$, firms' problem simplifies to

$$\max\left\{P(A;\phi) - \phi, \max_a \{aX - P(a;\phi)\}, AX\right\}.$$

Instead of conjecturing a form of $P(a;\phi)$, we conjecture that firms with $X = a$ demand a project of quality a for an arbitrary price P. Then we derive a unique market-clearing price and verify that under this price (i) firms with $A = a$ and $X \leq \frac{a-\frac{\phi}{a}}{2}$ choose to be targets and (ii) firms with $X = a$ and $A \leq \frac{a-\frac{\phi}{a}}{2}$ choose to be bidders demanding a project of quality a.

Given a price P, targets' participation constraint (to sell a project of quality a) is $P-\phi \geq aX \Leftrightarrow X \leq \frac{P-\phi}{a}$. Therefore, a measure of targets is

$$S_a(P;\phi) = \frac{P-\phi}{a}.$$

Bidders' participation constraint (to buy a project of quality a) is $a^2 - P \geq Aa \Leftrightarrow A \leq \frac{a^2-P}{a}$.

[23] If the intermediary actually matches targets to bidders, then it can make bidders pay for bringing a right target to them. We can show that the intermediary has an incentive do so, *given that it fully extracts surplus from targets.* This would be a model of investment banks who not only produce information but also match targets to bidders. In this paper, to separate a role of information production from matching, we assume that the intermediary interacts only with targets.

[24] Another reason why we do not treat this full surplus extraction as our main scenario is that contracting upon non-tradeable skill seems extremely difficult in practice.

Therefore, a measure of bidders is

$$B_a(P) = \frac{a^2 - P}{a}.$$

A market-clearing condition $S_a(P;\phi) = B_a(P)$ defines a unique market-clearing price $P(a;\phi)$ and a number of takeovers:

$$P(a;\phi) = \frac{a^2+\phi}{2} \qquad \text{and} \qquad Q(a;\phi) = \frac{1}{2}\left(a - \frac{\phi}{a}\right). \tag{6}$$

$P(a;\phi)$ given in (6) is increasing in a and also satisfies $X = \arg\max_a \{aX - P(a;\phi)\}$. Therefore, the unravelling force exists and firms with $X = a$ indeed demand projects of quality a under this price function. This analysis also shows that any fee that depends on project quality a (but not on skill X) distorts bidders' demand for projects.[25]

From the expression of $Q(a;\phi)$ in (6), projects of low quality are not traded.

$$Q(a;\phi) > 0 \Leftrightarrow a > \sqrt{\phi}.$$

For each $a > \sqrt{\phi}$, bidders and targets have the same payoff

$$P(a;\phi) - \phi = a^2 - P(a;\phi) = \frac{a^2-\phi}{2}.$$

Using this, equilibrium participation constraint is $\frac{a-\frac{\phi}{a}}{2} \geq X$ for targets, and $\frac{a-\frac{\phi}{a}}{2} \geq A$ for bidders, as we anticipated.

Monopoly intermediary. The intermediary solves $\max_{\phi\in[0,1]} \{\phi Q(\phi)\}$, where $Q(\phi)$ is the aggregate demand for the full disclosure service given by

$$Q(\phi) \equiv \int_{\sqrt{\phi}}^{1} Q(a;\phi)\,da = \frac{1}{4}(1 - \phi + \phi\ln\phi).$$

Establishing the existence of a unique solution $\phi_M \equiv \arg\max_{\phi} \{\phi Q(\phi)\}$ is straightforward.[26]

[25] For exmple, a fee $\frac{a^2}{2} = \arg\max_{\phi} \{\phi Q(a;\phi)\}$ cannot constitute a competitive equilibrium. This fee implies $P\left(a;\frac{a^2}{2}\right) = \frac{3}{4}a^2$ and $Q\left(a;\frac{a^2}{2}\right) = \frac{a}{4}$ but the price distorts bidders' choice of projects, invalidating the conjectured sorting pattern of firms.

[26] An implicit assumption behind this formulation is that there is only a fixed cost for information production. This can be relaxed. For example, an alternative formulation $\max_{\phi\in[0,1]} \{\phi Q(\phi) - C(Q(\phi))\}$ with a cost function $C(q) = cq$ or $C(q) = \frac{c}{2}q^2$ remains tractable. This makes the welfare loss larger, but does not offer much additional insight.

Because $\phi_M \approx 0.285$, we obtain the lower bound for the disclosed project quality $\sqrt{\phi_M} \approx 0.534$. The sorting pattern is shown in **Figure 2** below.

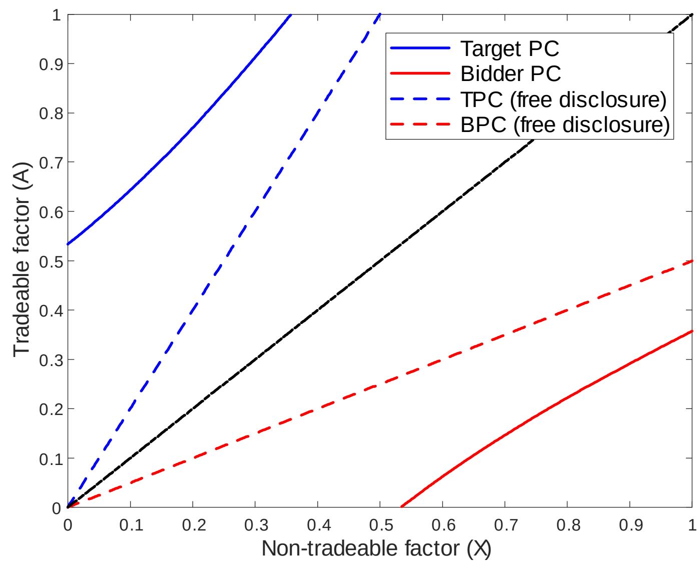

Figure 2. Takeovers with a monopoly intermediary

Solid lines represent firms' participation constraints with ϕ_M, while dashed lines represent participation constraints in the benchmark case (i.e., free disclosure $\phi = 0$). The sum of fees $\phi_M Q(\phi_M)$ paid by target firms to the intermediary is a pure transfer, and hence not a social cost. Monopoly pricing distorts reallocation through takeovers by affecting firms' participation decisions. First, projects with $A \leq \sqrt{\phi_M}$ are not reallocated. Second, for each $A > \sqrt{\phi_M}$, smaller number of projects are reallocated. We compute the welfare gain from this reallocation by substituting ϕ_M into (6) for each $A > \sqrt{\phi_M}$.

Full surplus extraction. If we allow the intermediary to use a more elaborate fee structure, the full surplus extraction from target firms is possible, and it is in fact efficient. To see this, recall that for a given price P, a potential target with (A, X) faces a participation constraint (to sell a project of quality A) $P - \phi \geq AX$. If the intermediary charges $\phi = P - AX$, this firm is indifferent between receiving $P - \phi$ (target) and aX (staying out). Suppose that the intermediary does this to firms with $A = a$ and $X \leq \frac{P}{a}$ (such that fees are non-negative). Then a measure of targets is $\frac{P}{a}$. On the other side of the market, the participation constraint is same as before, so a measure of bidders is $\frac{a^2-P}{a}$. The market clears at $P = \frac{a^2}{2}$. Therefore, the fee structure that extracts full surplus from targets is

$$\phi(A, X) = \frac{A^2}{2} - AX \qquad \text{for firms with } X \leq \frac{A}{2}.$$

Because firms' sorting pattern is same with that in **Proposition 1** (with $\beta = 0$), this is efficient. However, the payoff to targets is reduced to AX.[27] The analysis of this subsection is summarized as below.

Proposition 2 (Full disclosure)

(a) *A monopoly intermediary can implement the efficient reallocation by charging* $\phi(A, X) = \frac{A^2}{2} - AX$ *for targets. If a fee cannot depend on skill* X*, then the only fee structure that does not distort bidders' choice of projects is a fixed fee for all targets.*

(b) *With a fixed fee, the welfare gain is* $\frac{1}{16}(1 - \phi_M)$*, where* ϕ_M *is a smaller solution to* $\phi(1 - 2\ln\phi) = 1$*.*

Proposition 2(a) shows that by making a fee dependent on the private information of each target, the intermediary can achieve the efficient reallocation. However, adjusting a disclosure fee for the project to non-tradeable skill seems difficult for both a practical and an ethical reason.[28]

Proposition 2(b) shows that ϕ_M measures a welfare loss as a fraction of the benchmark welfare gain $\frac{1}{16}$. Because $\phi_M \approx 0.285$, the welfare loss due to a monopoly intermediary is 28.5%. An obvious drawback of this reallocation is that it leaves many gains from trade unrealized. A standard approach would be to encourage more entries. However, as we discussed in the introduction, disclosure for takeovers requires expertise and reputation both of which take a long time to build. In fact, the industry seems to be dominated by highly profitable large firms. Accordingly, we take an alternative approach. How much can we gain by setting up a public (non-profit) institution that offers a disclosure service *of lower quality for free* ? As a bridge to answering this question, we study a competitive equilibrium with a minimum disclosure service in the next section.

[27] All targets are indifferent between disclosure and non-disclosure. A strict preference for disclosure can be created if the intermediary leaves a small surplus to targets that is increasing in A.

[28] We can show that the intermediary charging $\phi(A, X) = \frac{A^2}{2} - AX$ for targets can increase its profit by charging a fixed fee ϕ_B for *bidders*. This results in price $P(a; \phi_B) = \frac{a^2 - \phi_B}{2}$ and quantity $Q(a; \phi_B) = \frac{1}{2}\left(a - \frac{\phi_B}{a}\right)$. Thus, the full surplus extraction from targets combined with an ability to charge bidders results in a sorting pattern qualitatively similar to our main scenario.

4.2 Minimum disclosure by a regulatory body

In this subsection we study a minimum disclosure service. We assume that a regulatory body can only ensure that the quality of projects for sale is no lower than a publicly specified minimum standard $A_{\min} \in [0,1]$. As a result, projects of different qualities will be traded at the same price. We first study a case with $A_{\min} = 0$ (no disclosure), and then explain how the equilibrium changes as $A_{\min}$ increases. It turns out that even with $A_{\min} = 0$ some information is revealed through the endogenous participation of firms. A detailed analysis with a general $A_{\min} \in [0,1]$ is relegated to the Appendix.

A case with $A_{\min} = 0$. Suppose that all projects for sale are traded at a single price P. Because the expected quality of projects for sale is endogenous, we denote it by

$$\widehat{a} \equiv E\left[A|A \text{ is for sale}\right].$$

Firms' problem is simply $\max\{P, \widehat{a}X - P, AX\}$. For a firm with (A, X), a participation constraint as a target is

$$AX \leq P, \tag{7}$$

while that as a bidder is

$$AX \leq \widehat{a}X - P. \tag{8}$$

In **Figure 3** below, (7) is illustrated by a blue line decreasing in X, while (8) is illustrated by a red dotted line increasing in X. The intersection of the two lines exhibits a threshold skill level $X^* \equiv \frac{2P}{\widehat{a}}$, with which firms with any $A < A^* \equiv \frac{\widehat{a}}{2}$ are indifferent between being targets and bidders but strictly prefer trading to not trading.

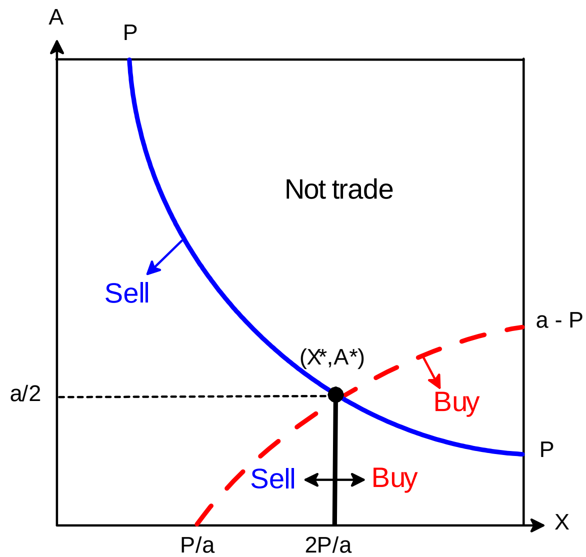

Figure 3. Sorting with $A_{\min} = 0$.

A measure of targets and a measure of bidders for a given P are

$$\begin{aligned} S(P) &= A^*X^* + \int_{A^*}^{1} \frac{P}{A} dA = P(1 - \ln A^*), \\ B(P) &= \int_{X^*}^{1} \left(\widehat{a} - \frac{P}{X}\right) dX = \widehat{a} - P\left(2 - \ln \frac{2P}{\widehat{a}}\right). \end{aligned}$$

A market-clearing condition $S(P) = B(P)$ is equivalent to

$$\widehat{a} = P(3 - \ln P), \tag{9}$$

which defines a unique market-clearing price $P(\widehat{a}) \in \left(0, \frac{\widehat{a}}{3}\right)$ for any conjectured $\widehat{a} \in (0, 1)$. Finally, given the conjectured $\widehat{a}$, the expected quality of projects for sale is

$$\Gamma(\widehat{a}) \equiv \frac{\int_0^{A^*} AX^* dA + \int_{A^*}^1 A\frac{P(\widehat{a})}{A} dA}{S(P(\widehat{a}))} = \frac{1 - \frac{\widehat{a}}{4}}{1 - \ln \frac{\widehat{a}}{2}}.$$

In the Appendix we show that $\Gamma(\widehat{a}) = \widehat{a}$ has a unique solution $a^* \in (0, 1)$, which is the expected quality of projects for sale in equilibrium. Using a^* and $P^* \equiv P(a^*)$, we compute equilibrium objects such as volume (i.e., $S(P^*) = B(P^*) = Q^*$), dollar volume (i.e., P^*Q^*) and the expected welfare gain.

A case with $A_{\min} \in (0,1]$. The above analysis can be generalized for any $A_{\min} \in [0,1]$. **Figure 4** illustrates the two possible cases: $A_{\min} < A^*$ and $A_{\min} > A^*$.

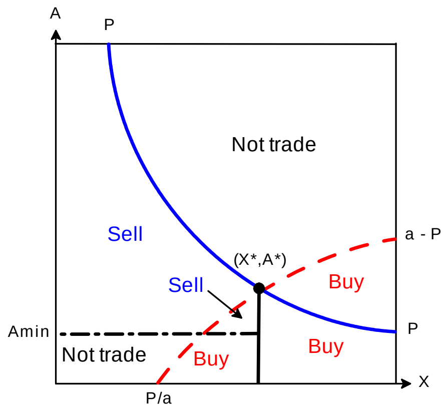

Figure 4 (a) $0 < A_{\min} < A^*$

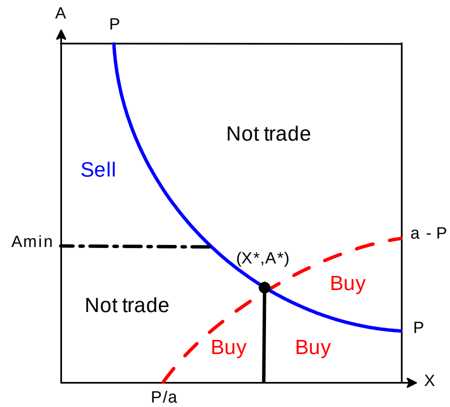

Figure 4 (b) $A^* < A_{\min} < 1$

A qualitative difference between the two cases is that with $A_{\min} < A^*$ (the panel (a)), targets and bidders are separated by the indifference condition $X = X^*$, while with $A_{\min} > A^*$ (the panel (b)) the two groups are separated by N-firms. Because both a^* and P^* are endogenous and depend on $A_{\min}$, which case in **Figure 4** occurs in equilibrium depends on the value of $A_{\min}$. The next result summarizes this analysis.

Proposition 3 (minimum disclosure)

(a) $A_{\min} \lesseqgtr A^* \Leftrightarrow A_{\min} \lesseqgtr A_0$, *where* $A_0 \in (0,1)$ *is a smaller solution to* $1 = A\,(1 - 2\ln A)$.

(b) *A unique market-clearing equilibrium exists for all* $A_{\min} \in [0,1]$. *The expected welfare gain is* $\left(\frac{a^* - P^*}{2}\right)^2$.

By **Proposition 3**, we can compute the expected welfare gain as a function of $A_{\min} \in [0,1]$. Importantly, **Proposition 3(b)** shows that maximizing the expected welfare gain amounts to maximize $a^* - P^*$, which is the equilibrium payoff of the best bidders with $X = 1$. Numerically, we obtained the following result.

Claim *There is an optimal* $A^*_{\min} \in (A_0, 1)$ *that maximizes the expected welfare gain.*

Three panels below show how the sorting pattern changes as $A_{\min}$ increases.

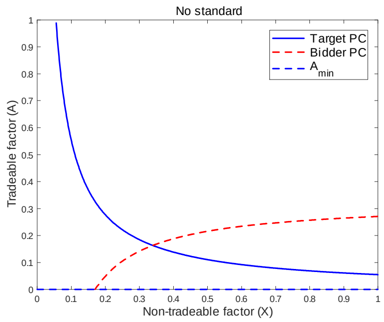

Figure 5 (a) $A_{\min} = 0$.

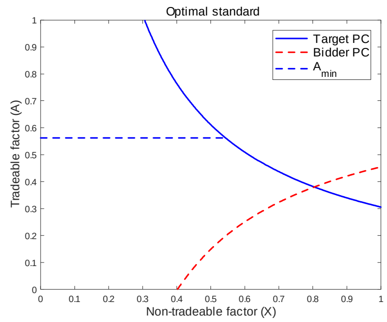

Figure 5 (b) $A_{\min} = A^*_{\min}$.

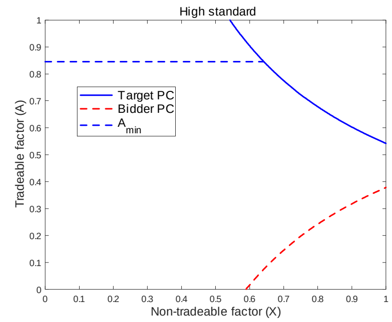

Figure 5 (c) $A_{\min} > A^*_{\min}$.

In each panel, the minimum standard $A_{\min}$ is represented by a horizontal dashed line, and it increases from the panel (a) to (c). The panel (a) is the case with $A_{\min} = 0$ such that projects of all qualities are traded in equilibrium. The panel (b) is the case with the optimal $A^*_{\min} \approx 0.56$. The panel (c) is the case with $A_{\min} > A^*_{\min}$. For all panels, targets are located below a decreasing solid blue line, which represents their participation constraint $AX \leq P^*$, but above the minimum standard $A_{\min}$. Bidders are located below an increasing dashed red line, which represents their participation constraint $A \leq a^* - \frac{P^*}{X}$.

As shown in **Figure 5**, as $A_{\min}$ increases, a mass of targets move up and also spread to the right, while a mass of bidders becomes compressed to the right. We show in the Appendix that both the average quality a^* and price P^* increase in $A_{\min}$, while $\frac{a^*}{A_{\min}}$ and $\frac{a^*}{P^*}$ decrease in $A_{\min}$ (see **Lemma A** in the Appendix). Intuitively, a higher standard reduces the quality uncertainty, hence it leads to smaller price discount. As $A_{\min}$ increases, more firms with high X find it profitable to sell their projects, and bidders are more concentrated among those with high X.

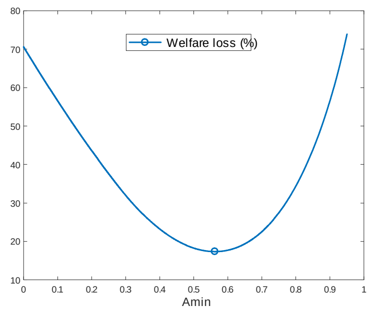

Figure 6 (a) Welfare loss.

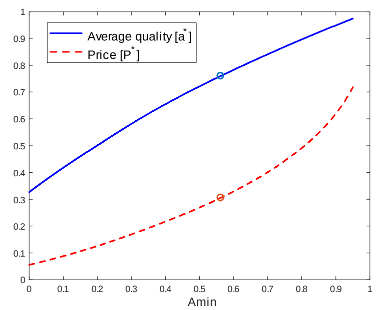

Figure 6 (b) a^* and P^*.

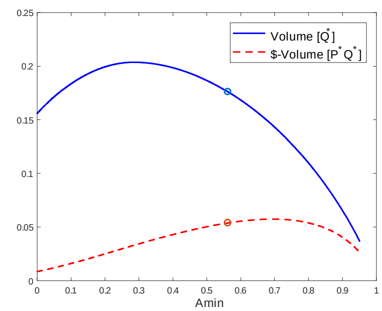

Figure 6 (c) Q^* and P^*Q^*.

Figure 6 shows equilibrium objects as a function of $A_{\min}$. The panel (a) shows that the welfare loss (as a percentage of the benchmark gain of $\frac{1}{16}$) is U-shaped in $A_{\min}$. In particular, with $A_{\min} = 0$, the welfare loss is 70.6%, but it can be reduced to 17.4% by using $A^*_{\min} \approx 0.56$. The panel (b) plots average quality a^* as well as the price of projects P^*. Circle markers indicate the position of $A^*_{\min}$. **Proposition 3(b)** shows that the welfare gain is monotonic in $a^* - P^*$. The panel (b) shows that both a^* and P^* increase in $A_{\min}$, but as $A_{\min}$ becomes sufficiently large, P^* increases at a faster rate, reflecting the reduced quality uncertainty and the demand from more skilled bidders. Accordingly, $a^* - P^*$ has a unique maximum.

The panel (c) plots volume Q^* and dollar volume P^*Q^*. It shows that both exhibit a hump-shaped pattern. Interestingly, it also shows that choosing $A_{\min}$ to maximize volume or dollar-volume does *not* maximize the gain from trade. In fact, $A^*_{\min}$ lies in between: $A_{\min}$ should be set higher than the value that maximizes volume, but it should be set lower than the value that maximizes dollar volume. In other words, if $A_{\min}$ were chosen to maximize the *number* of takeover deals, the average quality and the price of targets would be *too low*, while if it were chosen to maximize the *monetary value* of takeover deals, the average quality and the price would be *too high*.

Inefficiency of the minimum disclosure market is qualitatively different from that of the full disclosure market. In the full disclosure market, the matching that occurs was done efficiently, but the amount of matching was too small because of the monopoly power of intermediary. Here, there is no surplus extraction, but random matching creates inefficiency. Also, the ex post inefficiency arises because projects of the highest quality $A = 1$ could be matched to firms with low skill, while firms with $X = 1$ might draw a project of low quality.

Implications for the full disclosure service. The minimum disclosure service has an important implication for the viability of the full disclosure service. When the minimum disclosure service exists, it guarantees a minimum payoff P^* for any firms with $A \geq A_{\min}$ or $X \geq X^*$. This immediately implies that the full surplus extraction is not feasible in the presence of the minimum disclosure service. Next, consider a full disclosure service with a fixed fee ϕ. Using the minimum disclosure service with a fee $\phi > 0$, target firms with $A = 1$ or bidder firms with $X = 1$ obtain the payoff $\frac{1-\phi}{2}$. If this payoff is no greater than P^*, then no firm would choose the full disclosure service. Therefore, to attract *some* firms away from the minimum disclosure service, the intermediary must choose ϕ such that

$$P^* < \frac{1-\phi}{2} \Leftrightarrow \phi < 1 - 2P^*.$$

Recall that P^* is increasing in $A_{\min}$. As shown in **Figure 6(b)** (and formally proven in **Lemma 2** below), there exists a unique value $\underline{A}_{\min} \in (0, 1)$ defined by $P^* = \frac{1}{2}$. As a result, we obtain the following result.

Lemma 2 *There exists a unique* $\underline{A}_{\min} \in (0, 1)$ *defined by* $P^* = \frac{1}{2}$. *A minimum disclosure service with* $A_{\min} \in [\underline{A}_{\min}, 1)$ *makes the full disclosure service unprofitable.*

Lemma 2 shows that a minimum disclosure service can be a powerful tool that makes a full disclosure service completely unprofitable. Numerical evaluation yields $\underline{A}_{\min} \approx 0.81$.[29] We use this result to construct an equilibrium with the two services in the next subsection.

4.3 Market segmentation

In the previous two subsections, we investigated a welfare consequence of each of the two disclosure services in isolation. Two natural questions are whether they can coexist in equilibrium and whether they should. In this subsection, we answer these questions. A short answer is "yes and yes". We first construct an equilibrium where each service is used by some target firms. Then we discuss what a regulatory body needs to do to support this equilibrium.

Minimum disclosure market. We conjecture that projects of quality $A \geq \overline{A} \in (0, 1)$ are traded in the full disclosure markets, while projects of quality $A \in \left[A_{\min}, \overline{A}\right)$ are traded in the minimum disclosure market. We denote a price in the minimum disclosure

[29] The associated welfare loss is 35.8%. See **Figure 6(a)**.

market by P_0, and prices in the full disclosure markets by $\{P(A)\}_{A\geq\overline{A}}$. The quality threshold $\overline{A}$ will be endogenously determined. Importantly, we will show that $A_{\min}$ must take a particular value to be consistent with firms' choice over two disclosure services.

The analysis of the minimum disclosure market follows the same steps described in the Appendix (see the proof of **Proposition 3**) with two modifications. First, the best type on both sides of the market is $\overline{A} < 1$ in stead of 1. Second, the best bidder with $X = \overline{A}$ must be indifferent about buying a project of known quality $\overline{A}$ in a full disclosure market, while the best target with $A = \overline{A}$ must be indifferent about selling its project at $P\left(\overline{A}\right)$ (and paying a fee) in a full disclosure market. Hence, these two groups of firms must have the same payoff. In the minimum disclosure market, the payoff for all targets is P_0, while bidders have different payoffs depending on their skill (i.e., $a^*X - P_0$). This implies that all bidders except the best ones must be worse off than targets, i.e., the case $A_{\min} > A^*$ must hold (as illustrated in **Figure 4(b)**). Accordingly, a supply curve and a demand curve are given by

$$\begin{aligned} S(P_0) &= \int_{A_{\min}}^{\overline{A}} \frac{P_0}{A} dA = P_0 \ln \frac{\overline{A}}{A_{\min}}, \\ B(P_0) &= \int_{\frac{P_0}{a^*}}^{\overline{A}} \left(a^* - \frac{P_0}{X}\right) dX = a^*\overline{A} - P_0 - P_0 \ln \frac{\overline{A}a^*}{P_0}. \end{aligned}$$

A market-clearing condition $S(P_0) = B(P_0)$ yields

$$a^*\overline{A} = P_0 \left(1 - \ln P_0 + \ln \frac{a^*}{A_{\min}} + 2 \ln \overline{A}\right). \tag{10}$$

The right-hand side of (10) is increasing in P_0, and it is greater than $a^*\overline{A}$ at $P_0 = a^*\overline{A}$. This implies the existence of a unique market-clearing price $P_0 \in \left(0, a^*\overline{A}\right)$. In the Appendix we show that the expected quality of projects for sale is

$$a^* = \frac{\overline{A} - A_{\min}}{\ln \overline{A} - \ln A_{\min}}. \tag{11}$$

Market segmentation

The characterization of the full disclosure markets for a given ϕ are the same as in Section 4.1. In the full disclosure market, the "worst" target with project quality $A = \overline{A}$ and the "worst" bidder with skill $X = \overline{A}$ obtain the same payoff $\frac{\overline{A}^2 - \phi}{2}$. In addition, they must be indifferent about using the minimum disclosure service. This implies the following two indifference conditions:

$$\frac{\overline{A}^2 - \phi}{2} = P_0 \qquad \text{and} \qquad \frac{\overline{A}^2 - \phi}{2} = a^* \overline{A} - P_0.$$

These conditions imply

$$P_0 = \frac{a^* \overline{A}}{2}, \tag{12}$$

$$\overline{A}^2 - a^* \overline{A} - \phi = 0. \tag{13}$$

Substituting (12) into the market-clearing condition (10) yields

$$A_{\min} = \frac{2}{e} \overline{A} < \overline{A}, \qquad \text{where } \frac{2}{e} \approx 0.736. \tag{14}$$

By substituting (14) into (11) to eliminate $A_{\min}$,

$$a^* = \kappa \overline{A}, \qquad \text{where } \kappa \equiv \frac{1 - \frac{2}{e}}{\ln \frac{e}{2}} \approx 0.861. \tag{15}$$

Note that $a^* \in \left(A_{\min}, \overline{A}\right)$. Finally, we use (13) and (15) to derive $\overline{A}$ as a function of ϕ.

$$\overline{A}^2 - \left(\kappa \overline{A}\right) \overline{A} - \phi = 0 \quad \Leftrightarrow \quad \overline{A} = \sqrt{\frac{\phi}{1 - \kappa}}. \tag{16}$$

This implies

$$A_{\min} = \frac{2}{e} \sqrt{\frac{\phi}{1 - \kappa}} > \sqrt{\phi} \qquad \text{and} \qquad a^* = \frac{\kappa}{\sqrt{1 - \kappa}} \sqrt{\phi}. \tag{17}$$

In the following analysis, we use the notation $A_{\min}(\phi)$ and $\overline{A}(\phi)$ to make their dependence on ϕ explicit. From the expression of $A_{\min}(\phi)$ in (17), we have

$$A_{\min}(\phi) < 1 \quad \Leftrightarrow \quad \phi < \overline{\phi} \equiv (1 - \kappa) \left(\frac{e}{2}\right)^2 \approx 0.2565.$$

This means that for the two services to coexist, the monopoly fee ϕ must be smaller than $\overline{\phi} = (1 - \kappa) \left(\frac{e}{2}\right)^2$. How can we ensure that the monopoly intermediary sets $\phi < \overline{\phi}$? Suppose that the regulatory body commits to the following action:

$$A_{\min} = \begin{cases} \underline{A}_{\min} & \text{if } \phi \geq \overline{\phi}, \\ A_{\min}(\phi) & \text{otherwise.} \end{cases} \tag{18}$$

By **Lemma 2**, setting any $A_{\min} \in [\underline{A}_{\min}, 1)$ makes the full disclosure service unprofitable. Therefore, if the intermediary anticipates (18), then it will choose $\phi \in \left(0, \overline{\phi}\right)$ to make some profit by providing its service to targets with $A \in \left[\overline{A}(\phi), 1\right]$.[30]

Monopoly intermediary.

The intermediary solves

$$\max_{\phi \in \left[0, \overline{\phi}\right)} \left\{ \phi Q^H(\phi) \right\}, \tag{19}$$

where $Q^H(\phi)$ is the aggregate demand for the intermediary given by

$$Q^H(\phi) \equiv \int_{\overline{A}(\phi)}^{1} Q(a, \phi)\, da = \frac{1}{4}\left(1 - \frac{\phi}{1-\kappa} + \phi \ln \frac{\phi}{1-\kappa}\right).$$

The super script H stands for "hybrid". A monopoly intermediary chooses ϕ subject to a demand constraint imposed by $\overline{A}(\phi)$.[31] A unique solution $\phi_H \equiv \arg\max_{\phi} \left\{\phi Q^H(\phi)\right\}$ is characterized by

$$\phi\left(\frac{1+\kappa}{1-\kappa} - 2\ln\frac{\phi}{1-\kappa}\right) = 1. \tag{20}$$

We calculate the welfare gain using $A_{\min}(\phi_H)$ and $\overline{A}(\phi_H)$, where ϕ_H is characterized by (20).[32] The reallocation pattern is shown in **Figure 7** below.

[30] In fact, $\overline{A}(\phi) < 1 \Leftrightarrow \phi < 1 - \kappa$ implies that the intermediary chooses $\phi \in (0, 1-\kappa)$.

[31] Note that $Q^H(\phi) = 0$ for $\phi \in \left[1-\kappa, \overline{\phi}\right)$.

[32] Why not modify (18) by replacing $\overline{\phi}$ with $\phi' \in (0, \phi_H)$? This would improve the welfare if the regulatory knows ϕ_H and the intermediary complies to this tougher "threat" by solving $\max_{\phi \in [0, \phi']} \left\{\phi Q^H(\phi)\right\}$. The use of (18) may be more desirable because it is less hostile to the intermediary and induces it to find an unconstrained solution to (19).

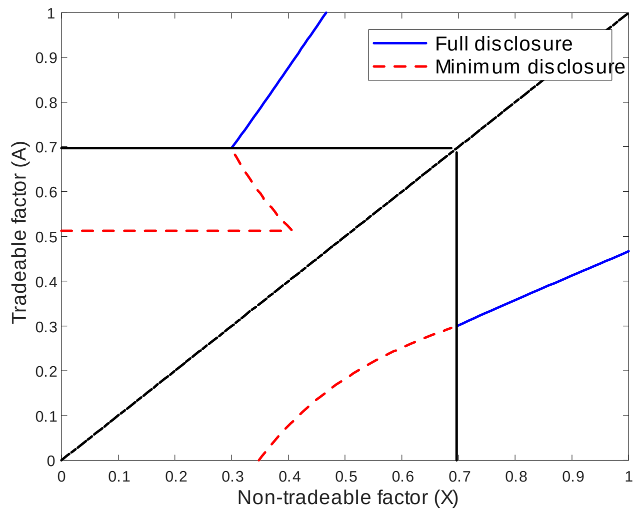

Figure 7. Segmented markets.

A solid black square represents $\overline{A}(\phi_H)$, i.e., the boundary between the minimum disclosure market and the full disclosure markets. Firms on this line are indifferent between the two disclosure services. Dashed red lines inside the black square show the sorting in the minimum disclosure market: bidders reside in the bottom-right region, while targets reside in the top-left region. The horizontal part of the dashed red line corresponds to $A_{\min}(\phi_H)$. Inside the black square, bidders and targets are matched randomly and trade projects at a single price $P_0 = \frac{\phi_H}{2}\frac{\kappa}{1-\kappa}$. Finally, solid blue lines outside the black square show the selection in the full disclosure markets. Outside of the black square, the positive assortative matching between bidders and targets is achieved. **Proposition 4** summarizes this analysis.

Proposition 4 (Endogenous market segmentation)

(a) *Let* $\kappa \equiv \frac{1-\frac{2}{e}}{\ln\frac{e}{2}}$. *For any* $\phi \in (0, 1-\kappa)$, *the two disclosure services coexist. Targets using the minimum disclosure service have projects of quality* $A \in \left[A_{\min}(\phi), \overline{A}(\phi)\right)$, *while targets using the full disclosure service have projects of quality* $A \in \left[\overline{A}(\phi), 1\right]$.

(b) *Given the regulatory body's strategy (18), the monopoly intermediary chooses* $\phi_H \in (0, 1-\kappa)$, *where* ϕ_H *is a solution to* $\phi\left(\frac{1+\kappa}{1-\kappa} - 2\ln\frac{\phi}{1-\kappa}\right) = 1$. *The welfare gain is* $\frac{1}{16}(1-\phi_H)$.

Because $\phi_H = 0.067$, the welfare loss is 6.7% of the benchmark welfare gain. This is a significant improvement over $\phi_M = 0.285$, i.e., 28.5% when the minimum disclosure market does not exist. The improvement is significant even relative to 17.4% when the minimum standard is set optimally (i.e., $A^*_{\min}$) in the absence of the monopoly intermediary. The key for this welfare improvement is that the aggregate demand facing the intermediary, $Q^H(\phi) \equiv \int_{\overline{A}(\phi)}^{1} Q(a,\phi)\, da$, becomes much more elastic to its fee through $\overline{A}(\phi)$.

The analysis in this section suggest that the minimum disclosure service, although inefficient on its own, might be a useful regulatory tool to indirectly control non-competitive behaviors of disclosure service providers. While setting up and implementing the minimum disclosure service takes real resources, the suggested magnitude of the welfare gain indicates that the benefit may exceed the cost.

5 Conclusion

We presented a competitive matching model of takeover markets. We showed that the full disclosure service by a monopoly intermediary and the minimum disclosure service by a regulatory body can coexist, if the minimum standard is appropriately designed. A presence of the minimum disclosure service significantly improves the welfare, because it raises the demand elasticity facing the intermediary. As a result, the intermediary reduces the fee and more gains from trade are realized.

In this paper we remained in a static model, studied only two types of disclosure services, and ignored reallocation through factor markets. How much do we gain by allowing for a second round of trading? Do intermediaries have an incentive to provide a more general disclosure service?[33] Under what conditions do firms and/or intermediaries have an incentive to disclose noisy or biased information? Does a market for corporate control substitute or complement factor markets? We believe that our model of takeovers is a useful first step toward answering these questions.

[33] Lizzeri (1999) studies this issue in a single seller model.

References

[1] David, J. M. (2020): "The Aggregate Implications of Mergers and Acquisitions." *Working paper.*

[2] Eckbo, B. E. (2014): "Corporate Takeovers and Economic Efficiency." *Annual Review of Economics*, 6, 51-74.

[3] Grossman, S. J. (1981) : "The Informational Role of Warranties and Private Disclosure about Product Quality." *The Journal of Law and Economics*, 24, 3, 461-483.

[4] Grossman, S. J. and O. D. Hart (1980a): "Takeover Bids, The Free-Rider Problem, and the Theory of the Corporation." *The Bell Journal of Economics*, 11, 1, 42-64.

[5] Grossman, S. J. and O. D. Hart (1980b): "Disclosure Laws and Takeover Bids." *The Journal of Finance*, 35, 2, Papers and Proceedings, 323-334.

[6] Hopenhayn, H. A. (2013): "Firm Microstructure and Aggregate Productivity." *Journal of Money, Credit and Banking*, 43, 5, 111-145.

[7] Jovanovic (1982): "Truthful Disclosure of Information." *The Bell Journal of Economics*, 13, 1, 36-44.

[8] Jovanovic, B. and S. Braguinsky (2004): "Bidder Discounts and Target Premia in Takeovers." *American Economic Review*, 94, 46-56.

[9] Kawakami, K. (2019): "Sorting, Selection, and Announcement Returns in Takeover Markets." *Working Paper.*

[10] Kim, K. (2012): "Endogenous market segmentation for lemons." *Rand Journal of Economics*, 43, 3, 562–576.

[11] Lambrecht and Myers (2007): "A Theory of Takeovers and Disinvestment." *The Journal of Finance*, 62, 2, 809–845.

[12] Legros and Newman (2013): "A Price Theory of Vertical and Lateral Integration." *The Quarterly Journal of Economics*, 128, 2, 725-770.

[13] Lizzeri (1999): "Information Revelation and Certification Intermediaries." *Rand Journal of Economics*, 30, 2, 214-231.

[14] Restuccia, D., and R. Rogerson. (2008): "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments." *Review of Economic Dynamics*, 11, 707–20.

6 Appendix: For Online Publication

6.1 Planner's solution

We characterize a constrained efficient allocation, where the planner can take or give a project for each firm, but cannot do both. This corresponds to competitive market allocation with $\beta = 0$ such that each firm trades only once. Under this constraint, transferring a project from a firm with (A_1, X_1) to another firm with (A_2, X_2) yields net gains $A_1X_2 - (A_1X_1 + A_2X_2)$. Consider transferring projects of quality $A = a$ to firms with skill level $X = a$. Because "target" firms' original production would be lost, it is better to collect projects of quality a from firms with lower skills. Also, because "bidder" firms' original production would be lost as well, it is better to transfer the collected projects to firms with lower project quality. Suppose that the planner collects projects of quality a from firms with skill level in $[0, y]$ ("targets"), and transfer them to firms with skill level $X = a$ and initial project quality in $[0, y]$ ("bidders"). The new production achieved by this reallocation is ya^2. The lost production by the targets is $\int_0^y (Aa)\, dA$, while the lost production by the bidders is $\int_0^y (aX)\, dX$. The planner chooses $y \in [0, 1]$ to maximize the net gain from the reallocation:

$$\max_{y\in[0,1]} \left\{ ya^2 - \int_0^y (Aa)\, dA - \int_0^y (aX)\, dX \right\}.$$

The solution to this problem is $y = \frac{a}{2}$.[34] The planner conducts this reallocation for each $a \in [0, 1]$. According to this reallocation, firms whose (A, X) satisfy $\frac{A}{X} \geq 2$ will give away their projects, while firms whose (A, X) satisfy $\frac{A}{X} \leq \frac{1}{2}$ will obtain a new project of quality X and produce X^2. Other firms will keep their initial (A, X). This is same with the competitive market allocation with $\beta = 0$. The welfare gain achieved by this reallocation is

$$\underbrace{\int_0^1 \frac{a}{2} a^2 da}_{\text{New production}} - \underbrace{\int_0^1 \left(A \int_0^{\frac{A}{2}} X dX \right) dA}_{\text{Lost production for "targets"}} - \underbrace{\int_0^1 \left(X \int_0^{\frac{X}{2}} A dA \right) dX}_{\text{Lost production for "bidders"}} = \frac{1}{16}.$$

6.2 Proofs

6.2.1 Competitive equilibrium

Proof of Proposition 1

(a)(b) These follow from the analysis in the main text.

[34] This result generalizes to the case where A and X have any identical (but imperfectly correlated) distribution.

(c) B-firms' production is

$$Y_B = \int_0^1 X^2 \frac{X}{1+\sqrt{1-\beta}} dX = \frac{1}{1+\sqrt{1-\beta}} \frac{1}{4}.$$

S-firms produce as a secondary activity, so their production is

$$\begin{aligned} Y_S &= \beta \int_0^{\frac{1}{1+\sqrt{1-\beta}}} X^2 \left(1 - \left(1+\sqrt{1+\beta}\right) X\right) dX \\ &= \beta \left\{ \int_0^{\frac{1}{1+\sqrt{1-\beta}}} X^2 dX - \left(1+\sqrt{1+\beta}\right) \int_0^{\frac{1}{1+\sqrt{1-\beta}}} X^3 dX \right\} \\ &= \frac{\beta}{\left(1+\sqrt{1-\beta}\right)^3} \frac{1}{12}. \end{aligned}$$

N-firms' production is

$$\begin{aligned} Y_N &= 2 \int_0^1 \left(\int_{\frac{A}{1+\sqrt{1-\beta}}}^A AX dX \right) dA = 2 \int_0^1 A \left(\int_{\frac{A}{1+\sqrt{1-\beta}}}^A X dX \right) dA \\ &= \left\{ 1 - \left(\frac{1}{1+\sqrt{1-\beta}} \right)^2 \right\} \int_0^1 A^3 dA \\ &= \left\{ 1 - \left(\frac{1}{1+\sqrt{1-\beta}} \right)^2 \right\} \frac{1}{4}. \end{aligned}$$

Therefore,

$$\begin{aligned} Y_B + Y_S + Y_N &= \frac{1}{4} \left\{ \frac{1}{1+\sqrt{1-\beta}} + \frac{\beta}{\left(1+\sqrt{1-\beta}\right)^3} \frac{1}{3} + 1 - \left(\frac{1}{1+\sqrt{1-\beta}} \right)^2 \right\} \\ &= \frac{1}{4} + \frac{1}{4} \left\{ \frac{1}{1+\sqrt{1-\beta}} + \frac{\beta}{\left(1+\sqrt{1-\beta}\right)^3} \frac{1}{3} - \left(\frac{1}{1+\sqrt{1-\beta}} \right)^2 \right\} \\ &= \frac{1}{4} + \frac{1}{4} \frac{\left(1+\sqrt{1-\beta}\right)^2 + \frac{\beta}{3} - \left(1+\sqrt{1-\beta}\right)}{\left(1+\sqrt{1-\beta}\right)^3} \\ &= \frac{1}{4} + \frac{1}{12} \frac{\beta + 3\left(2 - \beta + 2\sqrt{1-\beta} - 1 - \sqrt{1-\beta}\right)}{\left(1+\sqrt{1-\beta}\right)^3} \\ &= \frac{1}{4} + \frac{1}{12} \frac{\beta + 3\left(\sqrt{1-\beta} + 1 - \beta\right)}{\left(1+\sqrt{1-\beta}\right)^3} \equiv Y(\beta). \end{aligned}$$

Let $x \equiv \sqrt{1-\beta} \in [0,1] \Leftrightarrow \beta = 1 - x^2$. Then

$$\begin{aligned}
\frac{\beta + 3\left(\sqrt{1-\beta} + 1 - \beta\right)}{\left(1+\sqrt{1-\beta}\right)^3} &= \frac{1 - x^2 + 3\left(x + x^2\right)}{\left(1+x\right)^3} \\
&= \frac{1 + 3x + 2x^2}{1 + 3x + 2x^2 + x^2 + x^3} \\
&= \frac{1}{1 + \frac{2x^2 + x^2 + x^3}{1+3x+2x^2}}.
\end{aligned}$$

Because $\frac{2x^2+x^2+x^3}{1+3x+2x^2}$ is increasing in $x \in [0,1]$, $Y(\beta)$ is increasing in β. ■

6.2.2 Full disclosure

Proof of Proposition 2

(a) This follows from the analysis in the main text.

(b) The first order condition for the monopoly intermediary's problem is $\phi\left(1 - 2\ln\phi\right) = 1$. While this has two solutions, only the smaller solution satisfies the second order condition $2\ln\phi + 1 \leq 0 \Leftrightarrow \phi \leq \exp\left(-\frac{1}{2}\right)$. For a given ϕ, a measure of traded projects in market a is $Q\left(a;\phi\right) = \frac{1}{2}\left(a - \frac{\phi}{z}\right)$. The total gain from trade is

$$\underbrace{\int_{\sqrt{\phi}}^{1} a^2 Q\left(a;\phi\right) da}_{\text{New production}} - \underbrace{\int_{\sqrt{\phi}}^{1}\left(A\int_0^{Q(A;\phi)} X dX\right) dA}_{\text{Lost production for targets}} - \underbrace{\int_{\sqrt{\phi}}^{1}\left(X\int_0^{Q(X;\phi)} A dA\right) dX}_{\text{Lost production for bidders}}.$$

Due to symmetry, the second term equals the third term. The first term is

$$\begin{aligned}
\frac{1}{2}\int_{\sqrt{\phi}}^{1} a^2\left(a - \frac{\phi}{a}\right) da &= \frac{1}{2}\left[\frac{1}{4}a^4 - \frac{\phi}{2}a^2\right]_{\sqrt{\phi}}^{1} \\
&= \frac{1}{8}\left\{1 - \phi^2 - 2\phi\left(1-\phi\right)\right\} \\
&= \frac{1}{8}\left(1-\phi\right)^2.
\end{aligned}$$

The second term, the lost production for targets, is

$$\begin{aligned}
\frac{1}{2}\int_{\sqrt{\phi}}^{1}\left(A\left\{Q\left(A;\phi\right)\right\}^{2}\right)dA &= \frac{1}{8}\int_{\sqrt{\phi}}^{1}A\left(A-\frac{\phi}{A}\right)^{2}dA \\
&= \frac{1}{8}\left[\frac{1}{4}\left(1-\phi^{2}\right)-\phi\left(1-\phi\right)-\phi^{2}\ln\sqrt{\phi}\right] \\
&= \frac{1}{8}\left[\frac{1}{4}-\phi+\frac{3}{4}\phi^{2}-\phi^{2}\ln\sqrt{\phi}\right].
\end{aligned}$$

Combining these together, the gain from trade is

$$\begin{aligned}
G\left(\phi\right) &\equiv \frac{1}{8}\left[\left(1-\phi\right)^{2}-2\left\{\frac{1}{4}-\phi+\frac{3}{4}\phi^{2}-\phi^{2}\ln\sqrt{\phi}\right\}\right] \\
&= \frac{1}{8}\left(\frac{1}{2}-\frac{1}{2}\phi^{2}+\phi^{2}\ln\phi\right) \\
&= \frac{1}{16}\left\{1-\phi^{2}\left(1-2\ln\phi\right)\right\}.
\end{aligned}$$

Because ϕ_M satisfies $\phi_M\left(1-2\ln\phi_M\right)=1$, $G\left(\phi_M\right)=\frac{1}{16}\left(1-\phi_M\right)$. ■

6.2.3 Minimum disclosure

Recall that in the main text we used a notation $\widehat{a}\equiv E\left[A|A \text{ for sale}\right]$ for a conjectured average value of projects for sale. Its equilibrium counterpart is a^*. Here, for notational simplicity we use a for $\widehat{a}$. The following **Lemma A** is used to prove **Lemma 2** and **Proposition 3**.

Lemma A *Define $A_0\in(0,1)$ by a smaller solution to $A\left(1-2\ln A\right)=1$.*

(a) *If $A_{\min}\in[0,A_0)$, then $a^*\in(0,2A_0)$ is given by a unique solution to $a=\Gamma\left(a;A_{\min}\right)$, where*

$$\Gamma\left(a;A_{\min}\right)\equiv\frac{1-\frac{a}{4}-\frac{1}{a}A_{\min}^{2}}{1-\ln\frac{a}{2}-\frac{2}{a}A_{\min}}, \tag{21}$$

and $A_{\min}<A^*=\frac{a^*}{2}$ holds. A market-clearing price is a unique solution to

$$a^*=P\left\{3-\ln P-\left(4\frac{A_{\min}}{a^*}+\ln\left(1-\frac{A_{\min}}{a^*}\right)\right)\right\}. \tag{22}$$

(b) *If $A_{\min}\in[A_0,1)$, then $a^*=-\frac{1-A_{\min}}{\ln A_{\min}}\in[2A_0,1)$ and $A_{\min}\geq A^*=\frac{a^*}{2}$ holds. A market-clearing price is a unique solution to*

$$a^*=P\left(1-\ln P+\ln\frac{a^*}{A_{\min}}\right). \tag{23}$$

(c) *a^* is increasing in $A_{\min}$. It satisfies $a^* = 2A_0$ when $A_{\min} = A_0$ and $\lim_{A_{\min} \nearrow 1} a^* = 1$.*

(d) *P^*, $\frac{A_{\min}}{a^*}$ and $\frac{P^*}{a^*}$ are increasing in $A_{\min}$. Also, $\lim_{A_{\min} \nearrow 1} P^* = 1$.*

Proof of Lemma A

(a) Conjecture $A_{\min} < A^* = \frac{a}{2}$. We verify later that this occurs if and only if $A_{\min} < A_0$. Given $A_{\min} < A^*$ and (a, P) such that $0 < P < a < 1$, the sorting pattern implies that targets satisfy $X \leq \frac{P}{A}$, $X \leq X^*$, and $A \geq A_{\min}$. Hence, a supply curve is

$$S(P) = \int_{A_{\min}}^{A^*} X^* dA + \int_{A^*}^{1} \frac{P}{A} dA = X^* (A^* - A_{\min}) - P \ln A^*.$$

Substituting $A^* = \frac{a}{2}$ and $X^* = \frac{2P}{a}$,

$$S(P) = P\left(1 + \ln\frac{2}{a} - A_{\min}\frac{2}{a}\right).$$

Bidders satisfy $A \leq a - \frac{P}{X}$, and additionally, if $A \in [A_{\min}, A^*]$, $X > X^*$. Note that $a - \frac{P}{X} = 0$ defines a skill threshold $X = \frac{P}{a}$ (below which no bidder exists), while $a - \frac{P}{X} = A_{\min}$ defines another threshold $X = \frac{P}{a - A_{\min}} \in \left(\frac{P}{a}, X^*\right)$ (above which $A \geq A_{\min}$ becomes a binding constraint for some firms). Using these, a demand curve is

$$\begin{aligned} B(P) &= \int_{\frac{P}{a}}^{1} \left(a - \frac{P}{X}\right) dX - \int_{\frac{P}{a - A_{\min}}}^{X^*} \left(a - \frac{P}{X} - A_{\min}\right) dX \\ &= P\left\{\frac{a}{P} - \ln\frac{a}{P} + \ln\left(2\frac{a - A_{\min}}{a}\right) - 2\frac{a - A_{\min}}{a}\right\}. \end{aligned}$$

For $P > 0$, a market-clearing condition $S(P) = B(P)$ is equivalent to

$$a = P\left\{3 - \ln P - \left(4\frac{A_{\min}}{a} + \ln\left(1 - \frac{A_{\min}}{a}\right)\right)\right\},$$

which is (22). Denote the right hand side of (22) by $\Phi_1(P; a, A_{\min})$. A brief inspection of Φ_1 yields

$$\frac{\partial \Phi_1}{\partial P} = 2\left(1 - 2\frac{A_{\min}}{a}\right) + \ln\frac{a}{a - A_{\min}} - \ln P.$$

This is positive for any $0 < P < a < 1$ such that $A_{\min} < A^* = \frac{a}{2}$, because

$$2\left(1 - 2\frac{A_{\min}}{a}\right) > 0 > \ln P - \ln\frac{a}{a - A_{\min}}.$$

Also, $\Phi_1(0; a, A_{\min}) = 0$ and $\Phi_1(a; a, A_{\min}) = a\left\{3 - 4\frac{A_{\min}}{a} - \ln(a - A_{\min})\right\}$. Note that $\Phi_1(a; a, A_{\min}) > a \Leftrightarrow 2 - 4\frac{A_{\min}}{a} > \ln(a - A_{\min})$ holds because $A_{\min} < A^* = \frac{a}{2}$ implies $2\left(1 - 2\frac{A_{\min}}{a}\right) > 0 > \ln(a - A_{\min})$. This establishes a unique solution $P(a) \in (0, a)$ to (22).

Given the sorting pattern, the expected quality of projects for sale is

$$\Gamma(a; A_{\min}) \equiv \frac{\int_{A_{\min}}^{A^*} (AX^*)\, dA + \int_{A^*}^{1} \left(A\frac{P}{A}\right) dA}{S(P)}.$$

The numerator can be evaluated as

$$P\left\{\frac{1}{2}\frac{2}{a}\left(A^{*2} - A_{\min}^2\right) + 1 - A^*\right\} = P\left(1 - \frac{a}{4} - \frac{1}{a}A_{\min}^2\right).$$

Combining this with $S(P) = P\left(1 + \ln\frac{2}{a} - A_{\min}\frac{2}{a}\right)$ yields (21).

We prove that the following three properties hold for any $A_{\min} < A_0$:

$$\text{(i) } \Gamma(2A_{\min}; A_{\min}) > 2A_{\min}, \qquad \text{(ii) } \Gamma(1; A_{\min}) < 1, \qquad \text{(iii) } \frac{\partial \Gamma(a; A_{\min})}{\partial a}\Big|_{a=a*} < 1.$$

(i) - (iii) imply that $\Gamma(a; A_{\min}) = a$ has a unique solution $a^* \in (2A_{\min}, 1)$.

For the property (i),

$$\Gamma(2A_{\min}; A_{\min}) = \frac{1 - \frac{A_{\min}}{2} - \frac{A_{\min}}{2}}{-\ln A_{\min}} > 2A_{\min}$$

$$\Leftrightarrow 1 > A_{\min}(1 - 2\ln A_{\min}) \Leftrightarrow A_{\min} < A_0.$$

For the property (ii),

$$\Gamma(1; A_{\min}) = \frac{\frac{3}{4} - A_{\min}^2}{1 + \ln 2 - 2A_{\min}} < 1 \Leftrightarrow \frac{3}{4} - \ln 2 < (1 - A_{\min})^2.$$

This holds because $\frac{3}{4} - \ln 2 = 0.057 < (1 - A_0)^2 = (1 - 0.285)^2 = 0.511 < (1 - A_{\min})^2$.

For the property (iii), let $N \equiv 1 - \frac{a}{4} - \frac{1}{a}A_{\min}^2$ and $D \equiv 1 - \ln\frac{a}{2} - \frac{2}{a}A_{\min}$ so that $\Gamma(a; A_{\min}) = \frac{N}{D}$. Then

$$\frac{\partial \Gamma(a; A_{\min})}{\partial a} < 1 \Leftrightarrow \frac{\partial N}{\partial a}D - N\frac{\partial D}{\partial a} < D^2 \Leftrightarrow \frac{\partial N}{\partial a} - D < \frac{N}{D}\frac{\partial D}{\partial a}$$

$$\Leftrightarrow \left(\frac{A_{\min}}{a}\right)^2 - \frac{1}{4} - D < \frac{N}{D}\left(\frac{2A_{\min}}{a} - 1\right)\frac{1}{a}.$$

Because both sides are negative for $A_{\min} < \frac{a}{2}$, this is equivalent to

$$\frac{N}{D} = \Gamma\left(a; A_{\min}\right) < a \frac{1 - \ln\frac{a}{2} - \frac{2}{a}A_{\min} + \frac{1}{4}\left\{1 - \left(\frac{2A_{\min}}{a}\right)^2\right\}}{1 - \frac{2A_{\min}}{a}}.$$

The right hand side can be written as $a + a\frac{\ln\frac{2}{a} + \frac{1}{4}\left(1 - \frac{2A_{\min}}{a}\right)\left(1 + \frac{2A_{\min}}{a}\right)}{1 - \frac{2A_{\min}}{a}}$, so

$$\frac{\partial \Gamma\left(a; A_{\min}\right)}{\partial a} < 1 \Leftrightarrow \frac{\Gamma\left(a; A_{\min}\right) - a}{a} < \frac{\ln\frac{2}{a}}{1 - \frac{2A_{\min}}{a}} + \frac{1}{4}\left(1 + \frac{2A_{\min}}{a}\right).$$

Because the right hand side is positive for $A_{\min} < \frac{a}{2}$ while the left hand side is zero at $a = a^*$, this implies that $\frac{\partial \Gamma(a; A_{\min})}{\partial a} < 1$ holds at $a = a^*$.

To show that a^* is increasing in $A_{\min}$, it suffices to show that $\frac{\partial \Gamma(a; A_{\min})}{\partial A_{\min}}|_{a=a^*} > 0$.

$$\frac{\partial \Gamma\left(a; A_{\min}\right)}{\partial A_{\min}} > 0 \Leftrightarrow \frac{\partial N}{\partial A_{\min}} D > N \frac{\partial D}{\partial A_{\min}}.$$

Because $\frac{\partial N}{\partial A_{\min}} = -\frac{2A_{\min}}{a}$ and $\frac{\partial D}{\partial A_{\min}} = -\frac{2}{a}$ are both negative,

$$\frac{\partial \Gamma\left(a; A_{\min}\right)}{\partial A_{\min}} > 0 \Leftrightarrow \frac{N}{D} = \Gamma\left(a; A_{\min}\right) > A_{\min}.$$

This holds at $a = a^*$, because $A_{\min} < \frac{a^*}{2} < a^* = \Gamma\left(a^*; A_{\min}\right)$.

Finally, we already know that, for $A_{\min} < 1$, $\Gamma\left(2A_{\min}; A_{\min}\right) = 2A_{\min} \Leftrightarrow 1 = A_{\min}\left(1 - 2\ln A_{\min}\right) \Leftrightarrow A_{\min} = A_0$. Therefore, $\lim_{A_{\min} \nearrow A_0} a^* = 2A_0$.

(b) Conjecture $A_{\min} \geq A^* = \frac{a}{2}$. We verify later that this occurs if and only if $A_{\min} \geq A_0$. A supply curve is

$$S\left(P\right) = \int_{A_{\min}}^{1} \frac{P}{A} dA = -P \ln A_{\min}.$$

A demand curve is

$$B\left(P\right) = \int_{\frac{P}{a}}^{1} \left(a - \frac{P}{X}\right) dX = a - P + P \ln\frac{P}{a}.$$

A market-clearing condition $S\left(P\right) = B\left(P\right)$ is equivalent to $a = P\left(1 - \ln P + \ln\frac{a}{A_{\min}}\right)$ as shown in (23). Denote the right hand side of (23) by $\Phi_2\left(P; a, A_{\min}\right)$. A brief inspection of Φ_2 yields

$$\frac{\partial \Phi_2}{\partial P} = \ln\frac{a}{P A_{\min}}.$$

This is positive if and only if $\frac{a}{P} > A_{\min}$, which is true for any $A_{\min} < 1$ and $P < a$. Also, $\Phi_2(0; a, A_{\min}) = 0$ and $\Phi_2(a; a, A_{\min}) = a(1 - \ln A_{\min}) > a$ for any $A_{\min} < 1$. This establishes a unique solution $P \in (0, a)$ to (23).

We now pin down a^*. Given the sorting pattern, the expected quality of projects for sale is

$$\frac{\int_{A_{\min}}^{1} \left(A \frac{P}{A}\right) dA}{S(P)} = \frac{P(1 - A_{\min})}{-P \ln A_{\min}} = \frac{A_{\min} - 1}{\ln A_{\min}}.$$

Therefore, given $A_{\min} \geq A^* = \frac{a}{2}$, $a^* = \frac{A_{\min}-1}{\ln A_{\min}}$. To verify the conjecture $A_{\min} \geq A^* = \frac{a}{2}$,

$$A_{\min} \geq \frac{1}{2} \frac{A_{\min} - 1}{\ln A_{\min}} \Leftrightarrow 1 \leq A_{\min}(1 - 2\ln A_{\min}) \Leftrightarrow A_{\min} \geq A_0.$$

That $a^* = \frac{A_{\min}-1}{\ln A_{\min}}$ is increasing in $A_{\min}$ is immediate from

$$\ln A_{\min} - \frac{A_{\min} - 1}{A_{\min}} > 0 \Leftrightarrow 1 > A_{\min}(1 - \ln A_{\min}),$$

where the right hand side is increasing in $A_{\min}$ and approaches one as $A_{\min} \nearrow 1$. Note also that $\lim_{A_{\min} \nearrow 1} \frac{A_{\min}-1}{\ln A_{\min}} = 1$. At $A_{\min} = A_0$, $a^* = \frac{A_0 - 1}{\ln A_0} = 2A_0$ holds because this is equivalent to $1 = A_0(1 - 2\ln A_0)$. This means that a^* is continuous in $A_{\min} \in [0, 1)$.

(c) This was proved in the proof of (a)(b).

(d)

[For $A_{\min} < A_0$] For $\frac{A_{\min}}{a^*}$, consider

$$\frac{\Gamma(a; A_{\min})}{A_{\min}} = \frac{a}{A_{\min}} \Leftrightarrow \frac{1}{A_{\min}}\left(1 - \frac{a}{4} - \frac{1}{a}A_{\min}^2\right) = \frac{a}{A_{\min}}\left(1 - \ln\frac{a}{2} - \frac{2}{a}A_{\min}\right)$$

$$\Leftrightarrow \frac{1}{a}\left(1 - \frac{a}{4} - \frac{1}{a}A_{\min}^2\right) = 1 - \ln\frac{a}{2} - 2\frac{A_{\min}}{a}$$

$$\Leftrightarrow \frac{1}{a} - \frac{1}{4} + \ln\frac{a}{2} = \left(1 - \frac{A_{\min}}{a}\right)^2.$$

Because the left hand side is decreasing in a, $\frac{A_{\min}}{a}$ must be increasing in a. Since a^* is increasing in $A_{\min}$, $\frac{A_{\min}}{a^*}$ is increasing in $A_{\min}$ as well.

For P^*, consider a market-clearing condition (22) with a^*, i.e., $a^* = \Phi_1(P; a^*, A_{\min})$. The left hand side a^* increases in $A_{\min}$. We want to show that for a fixed P the right hand side $\Phi_1(P; a^*, A_{\min})$ decreases in $A_{\min}$. It suffices to show that $4\frac{A_{\min}}{a^*} + \ln\left(1 - \frac{A_{\min}}{a^*}\right)$ increases in

$\frac{A_{\min}}{a^*}$. This is true because

$$\frac{\partial\left\{4X+\ln\left(1-X\right)\right\}}{\partial X}=4-\frac{1}{1-X}=\frac{3-4X}{1-X}$$

and $\frac{3-4\frac{A_{\min}}{a^*}}{1-\frac{A_{\min}}{a^*}}>0$ holds for $A_{\min}<\frac{a^*}{2}<\frac{3}{4}a^*$.

For $\frac{P^*}{a^*}$, rewrite (22) as

$$\frac{a^*}{P^*}=3-\ln P^*-\left(4\frac{A_{\min}}{a^*}+\ln\left(1-\frac{A_{\min}}{a^*}\right)\right).$$

Clearly the right hand side decreases in $A_{\min}$.

[For $A_{\min}\geq A_0$] For $\frac{P^*}{a^*}$, rewrite a market-clearing condition (23) as

$$\frac{a}{P}=\ln\frac{a}{P}+\ln\frac{e}{A_{\min}}.$$

For the domain $\frac{a}{P}>1$, the above equation in $\frac{a}{P}$ has a unique solution and it decreases in $A_{\min}$. Hence $\frac{P^*}{a^*}$ increases in $A_{\min}$ with $\lim_{A_{\min}\nearrow 1}\frac{P^*}{a^*}=1$.

Because a^* increases in $A_{\min}$, $P^*=a^*\frac{P^*}{a^*}$ increases in $A_{\min}$. To show that $\frac{A_{\min}}{a^*}=\frac{A_{\min}}{-\frac{1-A_{\min}}{\ln A_{\min}}}=\frac{A_{\min}\ln A_{\min}}{A_{\min}-1}$ increases in $A_{\min}$, take its derivative:

$$\frac{\left(\ln A_{\min}+1\right)\left(A_{\min}-1\right)-A_{\min}\ln A_{\min}}{\left(A_{\min}-1\right)^2}=\frac{A_{\min}-\ln A_{\min}-1}{\left(A_{\min}-1\right)^2}.$$

The numerator is decreasing in $A_{\min}<1$ and takes zero when $A_{\min}=1$. So $A_{\min}-\ln A_{\min}-1>0$ for any $A_{\min}<1$. Finally, $\lim_{A_{\min}\nearrow 1}a^*=1$ and $\lim_{A_{\min}\nearrow 1}\frac{P^*}{a^*}=1$ imply $\lim_{A_{\min}\nearrow 1}P^*=1$. ■

Proof of Lemma 2

We use a notation $P^*\left(A_{\min}\right)$ for the market-clearing price for a given $A_{\min}\in[0,1]$. First, a market-clearing condition (9) implies $P^*(0)<\frac{1}{3}$. Second, from **Lemma A(d)**, $P^*\left(A_{\min}\right)$ is increasing in $A_{\min}$ with $\lim_{A_{\min}\nearrow 1}P^*\left(A_{\min}\right)=1$. A continuity implies that $P^*\left(A_{\min}\right)=\frac{1}{2}$ uniquely defines $\underline{A}_{\min}\in(0,1)$. The rest follows from the analysis in the main text. ■

Proof of Proposition 3

(a) This follows from **Lemma A(a,b)**.

(b)

[**Case with** $A_{\min}=0$]

For notational simplicity, we use (a,P) to mean (a^*,P^*). The expected welfare gains are given by $NP-(TL+BL)$, where NP is the expected value of new production by bidders,

TL is targets' lost production, and BL is bidders' lost production. These are given by

$$NP = a \int_{X^*}^{1} X \left(a - \frac{P}{X} \right) dX = a \left(\frac{a}{2} - P \right),$$

$$\begin{aligned} TL &= \int_0^{A^*} \int_0^{X^*} (AX)\, dA dX + \int_{A^*}^{1} \left(A \int_0^{\frac{P}{A}} X dX \right) dA \\ &= \frac{A^{*2}}{2} \frac{X^{*2}}{2} + \frac{P^2}{2} \int_{A^*}^{1} \frac{1}{A} dA \\ &= \frac{P^2}{2} \left(\frac{1}{2} - \ln \frac{a}{2} \right). \end{aligned}$$

$$\begin{aligned} BL &= \int_{X^*}^{1} \left(X \int_0^{a - \frac{P}{X}} A dA \right) dX \\ &= \frac{1}{2} \int_{X^*}^{1} \left(a^2 X - 2aP + \frac{P^2}{X} \right) dX \\ &= \left(\frac{a}{2} - P \right)^2 + \frac{P^2}{2} \left(\ln \frac{a}{2} - \ln P \right). \end{aligned}$$

Hence, the lost production is

$$TL + BL = \left(\frac{a}{2} - P \right)^2 + \frac{P^2}{2} \left(\frac{1}{2} - \ln P \right).$$

Finally, the expected welfare gain is

$$G(a, P) \equiv NP - (TL + BL) = \left(\frac{a}{2} - P \right) \left(\frac{a}{2} + P \right) + \frac{P^2}{2} \left(\ln P - \frac{1}{2} \right). \tag{24}$$

From the market-clearing condition (9),

$$a = P(3 - \ln P) \Leftrightarrow \ln P = 3 - \frac{a}{P}.$$

Substituting this into (24),

$$NP - (TL + BL) = \left(\frac{a}{2} - P \right) \left(\frac{a}{2} + P \right) + \frac{P^2}{2} \left(\frac{5}{2} - \frac{a}{P} \right) = \left(\frac{a - P}{2} \right)^2.$$

[**General case with** $A_{\min} \in [0, 1]$]

We proceed in two steps. First, we compute the expected welfare gains as a function of

$(a, P, A_{\min})$. Second, we show that using a market-clearing condition the expected welfare gain can always be expressed as $\left(\frac{a-P}{2}\right)^2$.

[Step 1. The expected welfare gains as a function of $(a, P, A_{\min})$]

For $A_{\min} < A^*$, the expected value of new production is given by

$$NP = a\int_{X^*}^{1} X\left(a - \frac{P}{X}\right)dX + a\int_{\frac{P}{a}}^{\frac{P}{a-A_{\min}}} X\left(a-\frac{P}{X}\right)dX + a\int_{\frac{P}{a-A_{\min}}}^{X^*} (XA_{\min})\,dX.$$

The first term is evaluated to be $a\left(\frac{a}{2} - P\right)$ as in the case with $A_{\min} = 0$. The second term is

$$\begin{aligned} a\int_{\frac{P}{a}}^{\frac{P}{a-A_{\min}}} X\left(a-\frac{P}{X}\right)dX &= a\left[\frac{a}{2}X^2 - PX\right]_{\frac{P}{a}}^{\frac{P}{a-A_{\min}}} \\ &= aP^2\left(\frac{1}{a-A_{\min}} - \frac{1}{a}\right)\left(\frac{1}{2}\frac{a}{a-A_{\min}} - \frac{1}{2}\right) \\ &= \frac{P^2}{2}\left(\frac{A_{\min}}{a-A_{\min}}\right)^2. \end{aligned}$$

The third term is

$$\begin{aligned} a\int_{\frac{P}{a-A_{\min}}}^{X^*} (XA_{\min})\,dX &= a\frac{A_{\min}}{2}\left\{X^{*2} - \left(\frac{P}{a-A_{\min}}\right)^2\right\} \\ &= a\frac{A_{\min}}{2}P^2\left\{\left(\frac{2}{a}\right)^2 - \left(\frac{1}{a-A_{\min}}\right)^2\right\} \\ &= \frac{P^2}{2}\frac{A_{\min}}{a}\left(4 - \left(\frac{a}{a-A_{\min}}\right)^2\right). \end{aligned}$$

Therefore,

$$\begin{aligned} NP &= a\left(\frac{a}{2} - P\right) + \frac{P^2}{2}\frac{A_{\min}}{a}\left\{\frac{a}{A_{\min}}\left(\frac{A_{\min}}{a-A_{\min}}\right)^2 + 4 - \left(\frac{a}{a-A_{\min}}\right)^2\right\} \\ &= a\left(\frac{a}{2} - P\right) + \frac{P^2}{2}\frac{A_{\min}}{a}\left(4 - \frac{a}{a-A_{\min}}\right) \\ &= a\left(\frac{a}{2} - P\right) + \frac{P^2}{2}\frac{A_{\min}}{a}\left(\frac{3 - 4\frac{A_{\min}}{a}}{1 - \frac{A_{\min}}{a}}\right). \end{aligned}$$

Targets' lost production is

$$
\begin{aligned}
TL &= \int_{A_{\min}}^{A^*}\int_0^{X^*}(AX)\,dAdX + \int_{A^*}^{1}\left(A\int_0^{\frac{P}{A}} XdX\right)dA \\
&= \frac{1}{2}\left(A^{*2}-A_{\min}^2\right)\frac{X^{*2}}{2} - \frac{P^2}{2}\ln\frac{a}{2} \\
&= \frac{P^2}{2}\left(\frac{1}{2}-\ln\frac{a}{2}\right) - P^2\left(\frac{A_{\min}}{a}\right)^2.
\end{aligned}
$$

Bidders' lost production is

$$
BL = \int_{X^*}^{1}\left(X\int_0^{a-\frac{P}{X}} AdA\right)dX + \int_{\frac{P}{a}}^{X^*}\left(X\int_0^{a-\frac{P}{X}} AdA\right)dX - \int_{\frac{P}{a-A_{\min}}}^{X^*}\left(X\int_{A_{\min}}^{a-\frac{P}{X}} AdA\right)dX.
$$

The first term is $\left(\frac{a}{2}-P\right)^2 + \frac{P^2}{2}\left(\ln\frac{a}{2}-\ln P\right)$. To compute the remaining two terms,

$$
\begin{aligned}
\int_{\frac{P}{a}}^{X^*}\left(X\int_0^{a-\frac{P}{X}} AdA\right)dX &= \frac{1}{2}\int_{\frac{P}{a}}^{X^*} X\left(a-\frac{P}{X}\right)^2 dX \\
&= \frac{1}{2}\int_{\frac{P}{a}}^{X^*}\left(a^2X - 2aP + \frac{P^2}{X}\right)dX,
\end{aligned}
$$

$$
\begin{aligned}
\int_{\frac{P}{a-A_{\min}}}^{X^*}\left(X\int_{A_{\min}}^{a-\frac{P}{X}} AdA\right)dX &= \frac{1}{2}\int_{\frac{P}{a-A_{\min}}}^{X^*} X\left\{\left(a-\frac{P}{X}\right)^2 - A_{\min}^2\right\}dX \\
&= \frac{1}{2}\int_{\frac{P}{a-A_{\min}}}^{X^*}\left(a^2X - 2aP + \frac{P^2}{X}\right)dX - \frac{A_{\min}^2}{2}\int_{\frac{P}{a-A_{\min}}}^{X^*} XdX.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
& \int_{\frac{P}{a}}^{X^*}\left(X\int_0^{a-\frac{P}{X}} AdA\right)dX - \int_{\frac{P}{a-A_{\min}}}^{X^*}\left(X\int_{A_{\min}}^{a-\frac{P}{X}} AdA\right)dX \\
= & \frac{1}{2}\int_{\frac{P}{a}}^{\frac{P}{a-A_{\min}}}\left(a^2X-2aP+\frac{P^2}{X}\right)dX + \frac{A_{\min}^2}{4}\left\{X^{*2}-\left(\frac{P}{a-A_{\min}}\right)^2\right\} \\
= & \frac{P^2}{2}\left[\frac{A_{\min}}{(a-A_{\min})\,a}\left\{\frac{a}{2}\frac{a}{a-A_{\min}}-\frac{3}{2}a\right\}+\ln\frac{a}{a-A_{\min}}+\frac{A_{\min}^2}{2}\left(\frac{2}{a}+\frac{1}{a-A_{\min}}\right)\left(\frac{2}{a}-\frac{1}{a-A_{\min}}\right)\right] \\
= & \frac{P^2}{4}\left[\left(\frac{A_{\min}}{a-A_{\min}}\right)^2-2\frac{A_{\min}}{a-A_{\min}}+2\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)+\left(\frac{A_{\min}}{a}\right)^2\left(4-\left(\frac{a}{a-A_{\min}}\right)^2\right)\right] \\
= & \frac{P^2}{2}\left[2\left(\frac{A_{\min}}{a}\right)^2-\frac{A_{\min}}{a-A_{\min}}+\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)\right].
\end{aligned}
$$

Bidders' lost production is

$$
\left(\frac{a}{2}-P\right)^2+\frac{P^2}{2}\left(\ln\frac{a}{2}-\ln P\right)+\frac{P^2}{2}\left[2\left(\frac{A_{\min}}{a}\right)^2-\frac{A_{\min}}{a-A_{\min}}+\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)\right].
$$

Total lost production is

$$
\begin{aligned}
TL+BL = & \frac{P^2}{2}\left(\frac{1}{2}-\ln\frac{a}{2}\right)-P^2\left(\frac{A_{\min}}{a}\right)^2+\left(\frac{a}{2}-P\right)^2+\frac{P^2}{2}\left(\ln\frac{a}{2}-\ln P\right) \\
& +\frac{P^2}{2}\left[2\left(\frac{A_{\min}}{a}\right)^2-\frac{A_{\min}}{a-A_{\min}}+\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)\right] \\
= & \left(\frac{a}{2}-P\right)^2+\frac{P^2}{2}\left(\frac{1}{2}-\ln P\right)+\frac{P^2}{2}\left[\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)-\frac{A_{\min}}{a-A_{\min}}\right].
\end{aligned}
$$

The expected welfare gain is

$$
\begin{aligned}
NP-(TL+BL) = & a\left(\frac{a}{2}-P\right)-\left\{\left(\frac{a}{2}-P\right)^2+\frac{P^2}{2}\left(\frac{1}{2}-\ln P\right)\right\} \\
& +\frac{P^2}{2}\frac{A_{\min}}{a}\left(\frac{3-4\frac{A_{\min}}{a}}{1-\frac{A_{\min}}{a}}\right)-\frac{P^2}{2}\left[\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)-\frac{A_{\min}}{a-A_{\min}}\right] \\
= & G(a,P)+\frac{P^2}{2}\left\{\frac{A_{\min}}{a}\left(\frac{3-4\frac{A_{\min}}{a}}{1-\frac{A_{\min}}{a}}\right)+\frac{A_{\min}}{a-A_{\min}}-\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)\right\} \\
= & G(a,P)+\frac{P^2}{2}\left\{4\frac{A_{\min}}{a}+\ln\left(1-\frac{A_{\min}}{a}\right)\right\}.
\end{aligned}
$$

Note that $G(a,P)$ was defined in (24).

For $A_{\min} \geq A^{*}$, the expected value of new production is given by

$$\begin{aligned} NP &= a\int_{\frac{P}{a}}^{1} X\left(a-\frac{P}{X}\right)dX = a\left[\frac{a}{2}\left\{1-\left(\frac{P}{a}\right)^{2}\right\}-P\left(1-\frac{P}{a}\right)\right] \\ &= \frac{(a-P)^{2}}{2}. \end{aligned}$$

Targets' lost production is

$$\begin{aligned} TL &= \int_{A_{\min}}^{1}\left(A\int_{0}^{\frac{P}{A}} XdX\right)dA = \frac{1}{2}\int_{A_{\min}}^{1} A\left(\frac{P}{A}\right)^{2}dA \\ &= -\frac{P^{2}}{2}\ln A_{\min}. \end{aligned}$$

Bidders' lost production is

$$\begin{aligned} BL &= \int_{\frac{P}{a}}^{1}\left(X\int_{0}^{a-\frac{P}{X}} AdA\right)dX = \frac{1}{2}\int_{\frac{P}{a}}^{1} X\left(a-\frac{P}{X}\right)^{2}dX \\ &= \frac{1}{2}\int_{\frac{P}{a}}^{1}\left(a^{2}X-2aP+\frac{P^{2}}{X}\right)dX \\ &= \frac{1}{4}(a-P)(a-3P)-\frac{P^{2}}{2}\ln\frac{P}{a}. \end{aligned}$$

Total lost production is

$$\begin{aligned} TL+BL &= \frac{1}{4}(a-P)(a-3P)-\frac{P^{2}}{2}\ln\frac{P}{a}-\frac{P^{2}}{2}\ln A_{\min} \\ &= \frac{1}{4}(a-P)(a-3P)-\frac{P^{2}}{2}\ln\left(A_{\min}\frac{P}{a}\right). \end{aligned}$$

The expected welfare gains are

$$\begin{aligned} NP-(TL+BL) &= \frac{(a-P)^{2}}{2}-\frac{1}{4}(a-P)(a-3P)+\frac{P^{2}}{2}\ln\left(A_{\min}\frac{P}{a}\right) \\ &= \frac{1}{4}(a-P)\{2(a-P)-(a-3P)\}+\frac{P^{2}}{2}\ln\left(A_{\min}\frac{P}{a}\right) \\ &= \frac{1}{4}(a-P)(a+P)+\frac{P^{2}}{2}\ln\left(A_{\min}\frac{P}{a}\right), \end{aligned}$$

where $A_{\min} \geq A^{*} = \frac{a}{2}$.

[Step 2. Use a market-clearing condition to get rid of $A_{\min}$]

We use **Lemma A** to show that the expected welfare gain is always $\left(\frac{a-P}{2}\right)^2$.

For $A_{\min} < A_0$, from the market-clearing condition (22),

$$4\frac{A_{\min}}{a} + \ln\left(1 - \frac{A_{\min}}{a}\right) = 3 - \ln P - \frac{a}{P}. \tag{25}$$

Recall that $NP - (TL + BL) = G(a, P) + \frac{P^2}{2}\left\{4\frac{A_{\min}}{a} + \ln\left(1 - \frac{A_{\min}}{a}\right)\right\}$, where the expression of $G(a, P)$ is given in (24). Eliminating $A_{\min}$ using (25) yields

$$NP - (TL + BL) = \left(\frac{a}{2}\right)^2 - P^2 + \frac{P^2}{2}\left(\ln P - \frac{1}{2} + 3 - \ln P - \frac{a}{P}\right) = \left(\frac{a-P}{2}\right)^2.$$

For $A_{\min} \geq A_0$, from the market-clearing condition (23),

$$\ln\left(A_{\min}\frac{P}{a}\right) = -\frac{a-P}{P}. \tag{26}$$

Recall that $NP - (TL + BL) = \frac{1}{4}(a - P)(a + P) + \frac{P^2}{2}\ln\left(A_{\min}\frac{P}{a}\right)$. Eliminating $A_{\min}$ using (26) yields

$$NP - (TL + BL) = \frac{1}{4}\left(a^2 - P^2\right) - \frac{P}{2}(a - P) = \left(\frac{a-P}{2}\right)^2. \quad \blacksquare$$

6.2.4 Market segmentation

Proof of Proposition 4

(a) This was proved in the main text.

(b) The first part was proved in the main text. Here we prove the result for the gain from trade. Denote the right hand side of the market-clearing condition (10) by

$$\Phi_0(P_0) \equiv P_0\left(1 - \ln P_0 + \ln\frac{a^*}{A_{\min}} + 2\ln\overline{A}\right).$$

This is increasing in $P \leq a^*\overline{A}$ because

$$\Phi_0'(P_0) = \ln\left(\frac{a^*}{A_{\min}P_0}\overline{A}^2\right) \geq \ln\frac{\overline{A}}{A_{\min}} > 0.$$

$\Phi_0(0) = 0$ and $\Phi_0\left(a\overline{A}\right) = a^*\overline{A}\left(1 + \ln\frac{\overline{A}}{A_{\min}}\right) > a^*\overline{A}$ imply the existence of a unique solution $P_0(a^*) \in \left(0, a^*\overline{A}\right)$ to (10). Next, the expected quality of projects for sale is

$$\frac{\int_{A_{\min}}^{\overline{A}} A\frac{P_0}{A}dA}{S(P_0)} = \frac{P_0\left(\overline{A} - A_{\min}\right)}{P_0 \ln \frac{\overline{A}}{A_{\min}}} = \frac{\overline{A} - A_{\min}}{\ln \overline{A} - \ln A_{\min}} = a^*.$$

The derivation of $\overline{A}$, $A_{\min}$, and a^* as functions of ϕ were explained in the main text.

The gain from trade in the full disclosure markets is

$$G^{FD}(\phi) \equiv \underbrace{\int_{\overline{A}}^{1} a^2 Q(a;\phi)\, dz}_{\text{New production}} - \underbrace{\int_{\overline{A}}^{1}\left(A\int_0^{Q(A;\phi)} XdX\right) dA}_{\text{Lost production for targets}} - \underbrace{\int_{\overline{A}}^{1}\left(X\int_0^{Q(X;\phi)} AdA\right) dX}_{\text{Lost production for bidders}}.$$

Due to symmetry, the second term equals the third term. The first term is

$$\begin{aligned}
\frac{1}{2}\int_{\overline{A}}^{1} a^2\left(a - \frac{\phi}{a}\right) da &= \frac{1}{8}\left\{1 - \overline{A}^4 - 2\phi\left(1 - \overline{A}^2\right)\right\} \\
&= \frac{1}{8}\left(1 - \frac{\phi}{1-\kappa}\right)\left(1 + \phi\frac{2\kappa - 1}{1-\kappa}\right).
\end{aligned}$$

The second term is

$$\begin{aligned}
\frac{1}{2}\int_{\overline{A}}^{1}\left(A\left\{Q_A(\phi)\right\}^2\right) dA &= \frac{1}{8}\int_{\overline{A}}^{1}\left(A^3 - 2\phi A + \frac{\phi^2}{A}\right) dA \\
&= \frac{1}{8}\left[\frac{1}{4}\left(1 - \overline{A}^4\right) - \phi\left(1 - \overline{A}^2\right) - \phi^2 \ln \overline{A}\right] \\
&= \frac{1}{8}\left[\frac{1}{4} - \phi + \left(\frac{\phi}{1-\kappa}\right)^2 \frac{3-\kappa}{4} - \phi^2 \ln\sqrt{\frac{\phi}{1-\kappa}}\right].
\end{aligned}$$

Combining these together,

$$\begin{aligned}
G^{FD}(\phi) &= \frac{1}{8}\left[\left(1 - \frac{\phi}{1-\kappa}\right)\left(1 + \phi\frac{2\kappa-1}{1-\kappa}\right) - 2\left\{\frac{1}{4} - \phi + \left(\frac{\phi}{1-\kappa}\right)^2\frac{3-\kappa}{4} - \phi^2\ln\sqrt{\frac{\phi}{1-\kappa}}\right\}\right] \\
&= \frac{1}{16}\left\{1 - \left(\frac{\phi}{1-\kappa}\right)^2 + 2\phi^2 \ln\frac{\phi}{1-\kappa}\right\}.
\end{aligned}$$

Next, we compute the gain from trade in the minimum disclosure market. The expected value of new production is

$$NP_0 = a^*\int_{\frac{P_0}{a^*}}^{\overline{A}} X\left(a^* - \frac{P_0}{X}\right) dX = \frac{\left(\overline{A}a^* - P_0\right)^2}{2}.$$

Targets' lost production is

$$TL_0 = \int_{A_{\min}}^{\overline{A}} \left(A \int_0^{\frac{P_0}{A}} X dX \right) dA = \frac{P_0^2}{2} \ln \frac{\overline{A}}{A_{\min}}.$$

Bidders' lost production is

$$BL_0 = \int_{\frac{P_0}{a^*}}^{\overline{A}} \left(X \int_0^{a^* - \frac{P_0}{X}} A dA \right) dX = \frac{1}{4} \left(\overline{A} a^* - P_0 \right) \left(\overline{A} a^* - 3P_0 \right) + \frac{P_0^2}{2} \ln \frac{\overline{A} a^*}{P_0}.$$

Total lost production is

$$TL_0 + BL_0 = \frac{1}{4} \left(\overline{A} a^* - P_0 \right) \left(\overline{A} a^* - 3P_0 \right) + \frac{P_0^2}{2} \ln \left(\frac{a^*}{P_0 A_{\min}} \overline{A}^2 \right).$$

The expected welfare gain in the minimum disclosure market is

$$\begin{aligned} G^{MD}(\phi) &\equiv NP_0 - (TL_0 + BL_0) \\ &= \frac{\left(\overline{A} a - P_0 \right)^2}{2} - \frac{1}{4} \left(\overline{A} a - P_0 \right) \left(\overline{A} a - 3P_0 \right) - \frac{P_0^2}{2} \ln \left(\frac{a}{P_0 A_{\min}} \overline{A}^2 \right) \\ &= \frac{1}{4} \left(\overline{A} a - P_0 \right) \left(\overline{A} a + P_0 \right) - \frac{P_0^2}{2} \ln \left(\frac{a}{P_0 A_{\min}} \overline{A}^2 \right). \end{aligned}$$

Using the market-clearing condition (10), $\ln \left(\frac{a}{P_0 A_{\min}} \overline{A}^2 \right) = \frac{\overline{A} a}{P_0} - 1$. Substituting this into the above expression of $G^{MD}(\phi)$,

$$G^{MD}(\phi) = \frac{1}{4} \left(\overline{A} a^* - P_0 \right) \left(\overline{A} a^* + P_0 \right) - \frac{P_0^2}{2} \left(\frac{\overline{A} a^*}{P_0} - 1 \right) = \left(\frac{\overline{A} a^* - P_0}{2} \right)^2.$$

Using $P_0 = \frac{\overline{A} a^*}{2}$ and $a^* = \kappa \overline{A}$,

$$G^{MD}(\phi) = \frac{\kappa^2 \overline{A}^4}{16} = \frac{1}{16} \left(\frac{\kappa \phi}{1 - \kappa} \right)^2.$$

Finally, the total gain from trade is

$$\begin{aligned}
G^{H}(\phi) &\equiv G^{FD}(\phi)+G^{MD}(\phi) \\
&= \frac{1}{16}\left\{1-\left(\frac{\phi}{1-\kappa}\right)^{2}+2\phi^{2}\ln\frac{\phi}{1-\kappa}+\left(\frac{\kappa\phi}{1-\kappa}\right)^{2}\right\} \\
&= \frac{1}{16}\left\{1-\phi^{2}\left(\frac{1-\kappa^{2}}{(1-\kappa)^{2}}-2\ln\frac{\phi}{1-\kappa}\right)\right\} \\
&= \frac{1}{16}\left\{1-\phi^{2}\left(\frac{1+\kappa}{1-\kappa}-2\ln\frac{\phi}{1-\kappa}\right)\right\}.
\end{aligned}$$

Because ϕ_H satisfies $\phi_H\left(\frac{1+\kappa}{1-\kappa}-2\ln\frac{\phi_H}{1-\kappa}\right)=1$, $G^{H}(\phi_H)=\frac{1}{16}(1-\phi_H)$. ■