Fair Criteria for Social Decisions under Uncertainty*

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Abstract

In this paper, we study the welfare economics of uncertainty. In a simple model where agents have ordinal and interpersonally noncomparable subjective expected utility preferences over uncertain future incomes, we analyze the implications of equity, efficiency, separability, and social rationality. Our efficiency conditions are fairly weak, because the standard ex ante Pareto principle conflicts with other desirable properties and not compelling under uncertainty. We derive social welfare criteria based on certainty equivalents by using the weaker efficiency conditions, equity requirements and separability axioms. Our results are essentially relevant to tensions between equity, efficiency, and separability. We also discuss incompatibility between our principles

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and *Statewise Dominance*, often referred to as "the minimal criterion" of rationality. The social criteria derived from our axioms respect ex ante equity, which is typically incompatible with *Statewise Dominance*.

1 Introduction

It is important to construct a reasonable welfare criterion for social decisions involving uncertainty such as policies on social security and redistribution. Although welfare economics has provided various social criteria, there is still disagreement concerning which should be adopted. A major reason for this disagreement is tension between social rationality, equity, and efficiency. The pathbreaking work is Harsanyi's (1955) aggregation theorem stating that a social welfare function satisfying the following conditions should be an affine combination of the agents' expected utility functions:
(i) The social expected utility (*SEU*) condition as social rationality: the social planner is an expected utility maximizer; and
(ii) the ex ante Pareto principle (*XAP*): if all agents prefer one prospect to another, the former is socially preferred to the latter.[1]

Diamond (1967) criticizes Harsanyi's (1955) social welfare function on the ground that *SEU* is incompatible with ex ante equity. It is also well known that Harsanyi's social welfare function is inconsistent with ex post equity (Broome, 1991; Adler and Sanchirico, 2006). These arguments show that if *SEU* and *XAP* are required, reasonable equity principles are violated. Based on these arguments, some researchers have explored alternative social welfare functions that do not satisfy *SEU* or *XAP*. For example, Epstein and Segal (1992) derived a quadratic social welfare function using *XAP* and an ex ante equity under a weaker social rationality. Fleurbaey (2010) characterized a class of social welfare functions called

[1]Since the derived social welfare criterion is the same as a weighted utilitarian social welfare function, this theorem was viewed as a justification of utilitarianism. There are, however, criticisms that the result is not relevant to utilitarianism; see Sen (1986) and Weymark (1991). More recently, Fleurbaey and Mongin (2015) showed the relevance of Harsanyi's theorem to utilitarianism.

the equally distributed equivalent criterion, using the following conditions.

(i) the Pareto principle when agents are under no risk;

(ii) *Pareto for Equal Risk*: Pareto condition confined to the case where all agents have equal risks; and

(iii) *Statewise Dominance*: A weak social rationality.[2]

Another principle, *separability*, is also considered as an important requirement in the literature (Fleming, 1952; Broome, 1991; Adler and Sanchirico, 2006). This principle states that social decisions should not be influenced by individuals irrelevant to the decisions.[3] Several papers have shown there are conflicts between efficiency, equity, separability, and social rationality (Fleurbaey, 2010; Fleurbaey and Zuber, 2013; Fleurbaey et al., 2014). However, for constructing useful social welfare criteria, it is crucial to explore the implications of these four principles.

We adopt a simple model where agents' future monetary prospects (called *acts*) are uncertain, and preferences are represented by ordinal and interpersonally noncomparable subjective expected utility functions. It is broadly recognized that interpersonal comparisons of utility have no sound empirical basis. We derive resource-based criteria for interpersonal comparison from certain axioms, following the fair social welfare function approach (Fleurbaey and Maniquet, 2011). By adopting this approach, we can clarify ways of comparing agents' levels of well-being and the value judgments behind the interpersonal comparisons. The domain of expected utility functions is standard in the literature, but any experimental results show that individuals' preferences typically violate the independence axiom of

[2] Mongin and Pivato (2016) provide a comprehensive survey of the literature.

[3] In the context of risk and uncertainty, it is often required that if an individual is under the same riskless situation in two prospects, the individual should not influence social judgments. This requirement is justified based on the argument that in a dynamic setting, social decisions should be made independently of the utility of the dead (Blackorby et al., 2005). By contrast, our separability conditions preclude influences to social decisions by irrelevant individuals under uncertainty (as well as certainty), because it would also be unreasonable if those individuals were to affect the decisions that do not influence them. For instance, Fleurbaey and Maniquet (2011, section 6.2) and Sprumont (2013) required a separability of this kind.

von Neumann and Morgenstern (vNM) or Savage's sure-thing principle. However, our results do not depend on the domain restriction: The results hold on broader domains where preference orderings satisfy, for instance, only monotonicity and continuity.[4]

In the environment described above, we explore implications of social rationality, equity, efficiency, and separability. As social rationality conditions, we first consider only completeness and transitivity to derive social welfare criteria. Although one may think that these conditions are too weak, as shown by Fleurbaey (2010) and Fleurbaey et al. (2015), it is difficult to find social criteria satisfying *Statewise Dominance*, *Pareto for Equal Risk*, and reasonable equity and separability conditions. It is also well known that *Statewise Dominance* is incompatible with ex ante equity. Moreover, as we argue in Section 5, there are inconsistencies between *Statewise Dominance* and ex post equity under a weak efficiency condition. Hence, it is worth studying social criteria that merely completeness and transitivity as social rationality.

We also consider fairly weak efficiency axioms, rather than *XAP*. This is because *XAP* is incompatible with reasonable equity principles (Fleurbaey and Trannoy, 2003; Fleurbaey and Voorhoeve, 2013). Moreover, as argued by Hammond (1981), decision making under risk and uncertainty is quite difficult for individuals because of misperceptions of probability, overconfidence, and other heuristics. Then, it is not compelling to fully respect agents' ex ante preferences, and a degree of paternalism may be justified. For example, mandatory social insurance programs are partly based on this idea.

Another reason to relax *XAP* is that if agents have different beliefs, ex ante unanimity may be spurious (Mongin, 1997).[5] For instance, consider two individuals, Ann and Bill. Ann believes that the price of a company's stock will increase while Bill expects that the price will decrease.[6] They both seem to gain by trade according to their ex ante preferences, and thus *XAP* requires that the trade should be implemented. It is not clear, however, that the unanimity is socially desirable because it is impossible that both Ann and Bill are

[4]The proofs are upon request.

[5]See also Gayer et al. (2014).

[6]Almost the same example was given by Gilboa et al. (2014).

right.[7] Together, these arguments mean that *XAP* is not compelling and it is plausible to relax the condition.

Our main contributions are as follows. We first study social criteria satisfying fairly weak efficiency and equity conditions, and a standard separability axiom. One of the efficiency conditions is inspired by *Pareto for Equal Risk* (Fleurbaey, 2010; Fleurbaey and Zuber, 2015). Another efficiency axiom is *Social Monotonicity*, which states that increases in all agents' incomes in every state should imply a social improvement. The equity condition is *Transfer Principle*, which requires that for a pair of agents with the same preference, if one agent has more income in each state than the other, then the inequality should be reduced by a transfer in every state. We also introduce an independence of risk preferences whenever riskless allocations are compared (Chambers and Echenique, 2012). Then, we derive a maximin social criterion based on certainty equivalents using these axioms. This criterion was characterized by Fleurbaey and Maniquet (2011, Section 6.2) and Ertemel (2016) in related models. A difference is that whereas these researches used standard ex ante Pareto principles, we use much weaker conditions that are compelling in the environment under study.

Another finding is that the separability, the Pareto principles above and *Transfer Principle* together imply *XAP*, and thus are incompatible with *Dominance Averse Transfer* by the result of Fleurbaey and Trannoy (2003). This result uncovers a tension between equity, efficiency, and separability in our environment. Notice that those conditions of social rationality, equity and efficiency are very weak. Thus, if *Dominance Averse Transfer* is considered more compelling than *XAP*, the separability axiom should be weakened.

Our next step is to consider a combination of weaker separability termed *Well-off Separability* (Fleurbaey and Maniquet, 2011, Axiom 5.3), *Dominance Averse Transfer*, and two efficiency principles. *Well-off Separability* requires that an irrelevant agent should not af-

[7] Recent contributions to the relevant issue include Chambers and Hayashi (2014), Danan et al. (2015), Mongin and Pivato (2015), and Zuber (2016). Hayashi and Lombardi (2016) consider aggregation of beliefs and tastes taking into account inequality aversion and responsibility for beliefs.

fect social judgements if the agent is unambiguously better off than some other agent. This form of separability would be more reasonable than the standard version introduced above. This is because information on the worse-off agents may be important for egalitarian social evaluations. Two other Pareto conditions are introduced. The first is *Pareto for Riskless Acts*, which requires that, other things equal, if some agents prefer riskless prospects to other (possibly uncertain) prospects, the former should be socially weakly preferred to the latter. The second is *Pareto for Consensual Risk-taking*, which states that if each agent can move from a riskless situation to a risky situation and this risk-taking behavior is supported by all agents, the risk-taking is also socially supported.

Using these axioms and the independence of risk preferences in riskless situations, we derive another form of maximin criterion. This social criterion satisfies *Dominance Averse Transfer* but violates *XAP*, and hence places more importance on equity. Furthermore, we introduce a stronger equity axiom named *Dominance Aversion*, which requires that for a pair of agents, if one agent has more money in every state than the other, the reduction of inequality should be acceptable. Then, we show that the social criterion is derived from *Pareto for Consensual Risk-taking*, *Pareto for Riskless Acts*, and *Dominance Aversion*.

We also consider *Statewise Dominance*, often referred to as "the minimal criterion" of social rationality. We show incompatibility between equity, efficiency, separability, and *Statewise Dominance* in our environment. We also argue that the criteria developed in this paper respect ex ante equity rather than *Statewise Dominance*.[8]

The remainder of the paper is organized as follows. In Section 2, we present the model. Section 3 analyzes the implications of the first set of axioms including the separability, *Transfer Principle*, and *Pareto for Equal Risk*. In Section 4, we consider social criteria satisfying *Well-off Separability* and *Dominance Averse Transfer*. In Section 5, we discuss inconsistencies between our principles and *Statewise Dominance*. Section 6 offers concluding

[8]Miyagishima (2017) drives a social welfare criterion using axioms of ex post equity, efficiency, and *Statewise Dominance*. The social criterion compares allocations by the statewise minimum incomes evaluated by the certainty equivalences.

remarks. A discussion of the independence of axioms is included in the appendix.

2 The Model

Let $\overline{N}$ be an infinite set of possible agents. $\mathcal{N}$ is the family of finite subsets of $\overline{N}$ such that for each $N \in \mathcal{N}$, $|N| \geq 2$. $S = \{s_1, ..., s_m\}$ is the finite set of states with $m \geq 2$. We denote by $x_{is} \in \mathbb{R}_+$ the amount of money agent i receives under state $s \in S$. An act of agent i is denoted by $\boldsymbol{x}_i = (x_{is})_{s\in S} \in \mathbb{R}_+^S$, which is a vector of state-contingent monetary payoffs. Let $X = \mathbb{R}_+^S$ be the set of acts. $\boldsymbol{x} = (x_s)_{s\in S} \in X$ is called a *constant act* if $x_s = x_{s'}$ for all $s, s' \in S$. Let $\bar{X}$ be the set of constant acts. For each $\boldsymbol{x} \in \bar{X}$, the value of money in every state is denoted by x. An allocation is denoted by $\boldsymbol{x}_N = (\boldsymbol{x}_i)_{i\in N} \in X^N$ for each $N \in \mathcal{N}$.

R_i is agent i's preference relation over X, with the strict part P_i and the indifference part I_i. A binary relation is an ordering if it is complete and transitive. Let $\mathcal{R}$ denote a set of preferences represented by subjective expected utility functions. More specifically, for each $R_i \in \mathcal{R}$, there exists a subjective expected utility function $E_{p_i}u_i$ such that, for all $\boldsymbol{x}_i, \boldsymbol{y}_i \in X$,

$$\boldsymbol{x}_i R_i \boldsymbol{y}_i \Longleftrightarrow E_{p_i}(u_i \circ \boldsymbol{x}_i) = \sum_{s\in S} p_{is}u_i(x_{is}) \geq \sum_{s\in S} p_{is}u_i(y_{is}) = E_{p_i}(u_i \circ \boldsymbol{y}_i),$$

where p_i is a probability distribution over S and $u_i : \mathbb{R}_+ \to \mathbb{R}$ is a Bernoulli utility function.

Given $\boldsymbol{x}_j \in X$ and $R_j \in \mathcal{R}$, define $I(\boldsymbol{x}_j, R_j) = \{\boldsymbol{z} \in X | \boldsymbol{z} I_j \boldsymbol{x}_j\}$, $L(\boldsymbol{x}_j, R_j) = \{\boldsymbol{z} \in X | \boldsymbol{x}_j R_j \boldsymbol{z}\}$, $\mathring{L}(\boldsymbol{x}_j, R_j) = \{\boldsymbol{z} \in X | \boldsymbol{x}_j P_j \boldsymbol{z}\}$, $U(\boldsymbol{x}_j, R_j) = \{\boldsymbol{z} \in X | \boldsymbol{z} R_j \boldsymbol{x}_j\}$, and $\mathring{U}(\boldsymbol{x}_j, R_j) = \{\boldsymbol{z} \in X | \boldsymbol{z} P_j \boldsymbol{x}_j\}$. Let $\mathbf{1} = (1, \ldots, 1) \in \bar{X}$. Given $N' \subset N$, let us denote by $(\boldsymbol{x}_{N'}, \boldsymbol{y}_{N\setminus N'})$ an allocation such that each agent $i \in N'$ has $\boldsymbol{x}_i$ and each agent $j \in N\setminus N'$ has $\boldsymbol{y}_j$.

A social ordering function (SOF), $\boldsymbol{R}$, is a mapping that for every preference profile determines a complete and transitive binary relation over the set of allocations. The domain is denoted by $\mathcal{D} = \bigcup_{N\in\mathcal{N}} (\mathcal{R}^E)^N$. Given a preference profile $R_N \in \mathcal{D}$, $\boldsymbol{R}(R_N)$ is a *social ordering* over X^N. In addition, let $\boldsymbol{P}(R_N)$ and $\boldsymbol{I}(R_N)$ be the strict and indifference parts of $\boldsymbol{R}(R_N)$, respectively.

3 Ex ante Pareto, Transfer Principle, and Separability

In this section, we introduce several axioms and provide the first characterization. The first axiom is the standard ex ante Pareto condition.

Ex Ante Pareto (XAP). For all $R_N \in \mathcal{D}$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, if $\boldsymbol{x}_i P_i \boldsymbol{x}'_i$ for all $i \in N$, then $\boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N$.

This axiom implies that a unanimous improvement in terms of ex ante preferences should be socially preferred.

We also introduce an equity axiom.

Dominance Averse Transfer (DAT). For all $R_N \in \mathcal{D}$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, if there exist j, k such that $\boldsymbol{x}_i = \boldsymbol{x}'_i$ for all $i \neq j, k$, and for all $\Delta \in \mathbb{R}^S_{++}$ such that $\Delta = \lambda(\boldsymbol{x}_j - \boldsymbol{x}_k)$ for some $\lambda \in \mathbb{R}_{++}$,

$$\left[\boldsymbol{x}_j = \boldsymbol{x}'_j - \Delta \geq \boldsymbol{x}_k = \boldsymbol{x}'_k + \Delta\right] \Rightarrow \boldsymbol{x}_N \boldsymbol{R}(R_N) \boldsymbol{x}'_N.$$

This axiom states that for two agents, if one has more income in every state than the other, a transfer in each state to reduce the inequality is acceptable. This axiom could be interpreted as an ex post equity condition because ex post inequality is reduced in each state.

As shown by Fleurbaey and Trannoy (2003), there exists no social ordering satisfying *XAP* and *DAT*. Thus, in order to have some possibility results, we must weaken at least one of these conditions. In what follows, we introduce weak and compelling versions of efficiency and equity, and elaborate on what orderings would be obtained by combining other axioms.

We introduce an efficiency requirement relevant to *Pareto for Equal Risk* first developed by Fleurbaey (2010) in a model of interpersonally comparable expected utility, and considered by Fleurbaey and Zuber (2015) in a model of economic environments with non-comparable expected utility.

Weak Pareto for Equal Risk (WPER). For all $R_N \in \mathcal{D}$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$ such that $R_i = R_j$, $\boldsymbol{x}_i = \boldsymbol{x}_j$, $\boldsymbol{x}'_i = \boldsymbol{x}'_j$ for all $i, j \in N$, if $\boldsymbol{x}_i P_i \boldsymbol{x}'_i$ for all $i, j \in N$, then $\boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N$.

This axiom requires that an ex ante unanimous agreement should be judged as social improvement only when all agents have equal preference and acts in the allocations. In this situation, because all agents are subject to the same conditions, there would be no concern that the agreement is spurious or inconsistent with any equity conditions. Moreover, this requirement avoids the problem of spurious unanimity because the agents have the same outcome ex post. Since this axiom considers the situations where all agents have the same preference, *WPER* is weaker than the *Pareto for Equal Risk* condition of Fleurbaey and Zuber (2017).

We introduce another efficiency principle.

Social Monotonicity (SM). For all $R_N \in \mathcal{D}$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, if $\boldsymbol{x}_i \gg \boldsymbol{x}'_i$ for all $i \in N$, then $\boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N$.

According to this axiom, increases in all agents' future incomes should be socially preferred. This requirement is clearly compelling.

The next axiom is an equity principle that is much weaker than *DAT*.

Transfer Principle (TP). For all $R_N \in \mathcal{D}$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, if there exist j, k such that $R_j = R_k$, and $\boldsymbol{x}_i = \boldsymbol{x}'_i$ for all $i \neq j, k$, then for all $\Delta \in \mathbb{R}^S_{++}$ such that $\Delta = \lambda(\boldsymbol{x}_j - \boldsymbol{x}_k)$ for some $\lambda \in \mathbb{R}_{++}$,

$$\left[\boldsymbol{x}_j = \boldsymbol{x}'_j - \Delta \geq \boldsymbol{x}_k = \boldsymbol{x}'_k + \Delta\right] \Rightarrow \boldsymbol{x}_N \boldsymbol{R}(R_N) \boldsymbol{x}'_N.$$

This requirement insists that among two agents with the same preference, if one has more income in every state than the other, a redistribution to decrease the inequality should be socially acceptable.

The next invariance axiom was essentially introduced by Chambers and Echenique (2012).

Invariance to Risk Attitudes and Beliefs for Constant Acts (IRBC). For all R_N, $R'_N \in \mathcal{D}$ represented by $(E_{p_i}u_i)_{i\in N}$ and $(E_{p'_i}u'_i)_{i\in N}$ respectively, and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in \bar{X}^N$,

$$\boldsymbol{x}_N \boldsymbol{R}(R_N)\boldsymbol{x}'_N \Longleftrightarrow \boldsymbol{x}_N \boldsymbol{R}(R'_N)\boldsymbol{x}'_N.$$

This axiom claims that social judgments over allocations of constant acts should be invariant of risk preferences and beliefs. The idea is that as long as riskless outcomes are compared, agents' risk preferences and beliefs are irrelevant for the comparisons, because only riskless outcomes are compared. This axiom could also be justified from the strategic point of view that social decisions over certain outcomes should be robust to agents' misreporting of their risk preferences.

Next, we introduce two forms of separability.

Separability (SEP). For all $R_N \in \mathcal{D}$ such that $|N| \geq 3$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, if $\boldsymbol{x}_i = \boldsymbol{x}'_i$ for some $i \in N$, then

$$\boldsymbol{x}_N \boldsymbol{R}(R_N)\boldsymbol{x}'_N \Longleftrightarrow \boldsymbol{x}_{N\setminus\{i\}} \boldsymbol{R}(R_{N\setminus\{i\}})\boldsymbol{x}'_{N\setminus\{i\}}.$$

Weak Separability (WS). For all $R_N \in \mathcal{D}$ such that $|N| \geq 3$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, if $\boldsymbol{x}_i = \boldsymbol{x}'_i$ for some $i \in N$, then

$$\boldsymbol{x}_N \boldsymbol{P}(R_N)\boldsymbol{x}'_N \Longrightarrow \boldsymbol{x}_{N\setminus\{i\}} \boldsymbol{P}(R_{N\setminus\{i\}})\boldsymbol{x}'_{N\setminus\{i\}}.$$

These axioms require that if one agent has the same act under two allocations, the social ranking over the two allocations should be invariant of excluding this indifferent agent. Clearly, *SEP* is stronger than *WS*. As discussed below, these requirements have different implications.

Based on the axioms described above, we derive a social welfare criterion based on certainty equivalence. Given $\boldsymbol{x}_i \in X$ and $R_i \in \mathcal{R}$, let $C(\boldsymbol{x}_i, R_i) = \inf\{c \in \mathbb{R}_+ | (c, \cdots, c) R_i \boldsymbol{x}_i\}$, which is the certainty equivalence of $\boldsymbol{x}_i$ with respect to R_i. Then, we obtain the following result.

Theorem 1. Suppose that an SOF $\boldsymbol{R}$ satisfies *WPER, SM, TP, IRBC*, and *WS*. Then, for all $R_N \in \mathcal{D}$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$,

$$\min_{i \in N} C(\boldsymbol{x}_i, R_i) > \min_{i \in N} C(\boldsymbol{x}'_i, R_i) \Longrightarrow \boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N.$$

We offer remarks before proving the theorem. As is standard in the literature on fair social ordering, the characterization is partial. For instance, if a standard continuity condition is also required, we immediately have a full characterization of the certainty equivalence maximin ordering, $\boldsymbol{R}_C$, which is defined below: For all $R_N \in \mathcal{D}$, all $\boldsymbol{x}_N$, $\boldsymbol{x}'_N \in X^N$,

$$\boldsymbol{x}_N \boldsymbol{R}_C(R_N) \boldsymbol{x}'_N \Longleftrightarrow \min_{i \in N} C(\boldsymbol{x}_i, R_i) \geq \min_{i \in N} C(\boldsymbol{x}'_i, R_i).$$

The proof of Theorem 1 makes use of the following two lemmas, which are interesting in their own right. The first lemma reveals tension between equity, efficiency, and separability under risk and uncertainty. Because requiring *WS* in addition to the fairly weak axioms of equity and efficiency yields *XAP*, and we must sacrifice *DAT* (Fleurbaey and Trannoy, 2003).[9]

Lemma 1. *WPER, SM, TP*, and *WS* together imply *XAP*.

Proof. Let $\boldsymbol{x}_N, \boldsymbol{x}'_N$ be such that $\boldsymbol{x}_i P_i \boldsymbol{x}'_i$ for all $i \in N$. For each $i \in N$, let $\boldsymbol{x}^*_i \in I(\boldsymbol{x}_i, R_i)$ be such that $\boldsymbol{x}^*_i \gg \boldsymbol{x}'_i$.

We first consider $1_a, 1_b \in \bar{N} \setminus N$ such that $R_{1_a} = R_{1_b} = R_1$. Let us denote $N^1 = \{1, 1_a, 1_b\}$ and $\boldsymbol{x}_{1_a} = \boldsymbol{x}_{1_b} = \boldsymbol{x}^*_1$. Define $\boldsymbol{y} = \boldsymbol{x}_1 + 2(\boldsymbol{x}^*_1 - \boldsymbol{x}'_1)/3$ and $\boldsymbol{y}_{N^1} = (\boldsymbol{y}_1, \boldsymbol{y}_{1_a}, \boldsymbol{y}_{1_b}) = (\boldsymbol{y}, \boldsymbol{y}, \boldsymbol{y})$. By *TP*, we have $\boldsymbol{y}_{N^1} \boldsymbol{R}(R_{N^1})(\boldsymbol{x}'_1, \boldsymbol{x}_{1_a}, \boldsymbol{x}_{1_b})$. Let $\boldsymbol{y}'_{N^1} = (\boldsymbol{x}_1 - (n+1)\epsilon^1 \mathbf{1}, \boldsymbol{x}_1 - (n+1)\epsilon^1 \mathbf{1}, \boldsymbol{x}_1 - (n+1)\epsilon^1 \mathbf{1})$, where $\epsilon^1 > 0$ is small enough that $[\boldsymbol{x}_1 - (n+1)\epsilon^1 \mathbf{1}] P_1 \boldsymbol{y}$. Then, by *WPER*, we obtain $\boldsymbol{y}'_{N^1} \boldsymbol{P}(R_{N^1}) \boldsymbol{y}_{N^1}$.

Next, define $\boldsymbol{y}'' \in X$ such that $\boldsymbol{x}^*_1 \gg \boldsymbol{y}''$ and $\boldsymbol{y}'' P_1 \boldsymbol{y}'$. Denote $\boldsymbol{y}''_{N^1} = (\boldsymbol{x}_1 - (n + 1)\epsilon^1 \mathbf{1}, \boldsymbol{y}'', \boldsymbol{y}'')$. Since $(\boldsymbol{y}'', \boldsymbol{y}'') \boldsymbol{P}(R_{\{1_a, 1_b\}})(\boldsymbol{x}_1 - (n+1)\epsilon^1 \mathbf{1}, \boldsymbol{x}_1 - (n+1)\epsilon^1 \mathbf{1})$ by *WPER*, we obtain $\boldsymbol{y}''_{N^1} \boldsymbol{R}(R_{N^1}) \boldsymbol{y}'_{N^1}$ from *WS*. Transitivity implies $\boldsymbol{y}''_{N^1} \boldsymbol{P}(R_{N^1})(\boldsymbol{x}'_1, \boldsymbol{x}_{1_a}, \boldsymbol{x}_{1_b})$.

[9]Note that $\boldsymbol{R}_C$ satisfies *XAP* and violates *DAT*.

Here we consider $N^{1+} = \{1, 2, \cdots, n, 1_a, 1_b\}$. From *WS* and $\boldsymbol{y}''_{N^1}\boldsymbol{P}(R_{N^1})(\boldsymbol{x}'_1, \boldsymbol{x}_{1_a}, \boldsymbol{x}_{1_b})$, we have

$$(\boldsymbol{x}_1 - (n+1)\epsilon^1\mathbf{1}, \boldsymbol{x}'_2, \cdots, \boldsymbol{x}'_n, \boldsymbol{y}'', \boldsymbol{y}'')\boldsymbol{R}(R_{N^{1+}})(\boldsymbol{x}'_1, \boldsymbol{x}'_2, \cdots, \boldsymbol{x}'_n, \boldsymbol{x}_{1_a}, \boldsymbol{x}_{1_b}),$$

and *SM* implies

$$(\boldsymbol{x}_1 - n\epsilon^1\mathbf{1}, \boldsymbol{x}'_2 + \epsilon^*\mathbf{1}, \cdots, \boldsymbol{x}'_n + \epsilon^*\mathbf{1}, \boldsymbol{x}_{1_a}, \boldsymbol{x}_{1_b})\boldsymbol{P}(R_{N^{1+}})(\boldsymbol{x}_1 - (n+1)\epsilon^1\mathbf{1}, \boldsymbol{x}'_2, \cdots, \boldsymbol{x}'_n, \boldsymbol{y}'', \boldsymbol{y}''),$$

where ϵ^* is small enough that $\boldsymbol{x}^*_i \gg \boldsymbol{x}'_i + n\epsilon^*\mathbf{1}$ for all $i \neq 1$. By transitivity and *WS*, we have

$$(\boldsymbol{x}_1 - n\epsilon^1\mathbf{1}, \boldsymbol{x}'_2 + \epsilon^*\mathbf{1}, \cdots, \boldsymbol{x}'_n + \epsilon^*\mathbf{1})\boldsymbol{P}(R_N)\boldsymbol{x}'_N.$$

Similarly, considering $N^2 = \{2, 2_a, 2_b\}$ and $\boldsymbol{x}_{2_a} = \boldsymbol{x}_{2_b} = \boldsymbol{x}^*_2$, we can show

$$(\boldsymbol{x}_2 - n\epsilon^2\mathbf{1}, \boldsymbol{x}_{2_a} - \epsilon'\mathbf{1}, \boldsymbol{x}_{2_b} - \epsilon'\mathbf{1})\boldsymbol{P}(R_{N^2})(\boldsymbol{x}'_2 + \epsilon^*\mathbf{1}, \boldsymbol{x}_{2_a}, \boldsymbol{x}_{2_b}),$$

where $\epsilon^2 > 0$ is defined similarly to ϵ^1 and $\epsilon' > 0$ is sufficiently small. Let $N^{2+} = \{1, \cdots, n, 2_a, 2_b\}$. Consider the following allocations.

$$\begin{aligned}
\boldsymbol{z}_{N^{2+}} &= (\boldsymbol{x}_1 - n\epsilon^1\mathbf{1}, \boldsymbol{x}'_2 + \epsilon^*\mathbf{1}, \boldsymbol{x}'_3 + \epsilon^*\mathbf{1}, \cdots, \boldsymbol{x}'_n + \epsilon^*\mathbf{1}, \boldsymbol{x}_{2_a}, \boldsymbol{x}_{2_b}),\\
\boldsymbol{z}'_{N^{2+}} &= (\boldsymbol{x}_1 - n\epsilon^1\mathbf{1}, \boldsymbol{x}_2 - n\epsilon^2\mathbf{1}, \boldsymbol{x}'_3 + \epsilon^*\mathbf{1}, \cdots, \boldsymbol{x}'_n + \epsilon^*\mathbf{1}, \boldsymbol{x}_{2_a} - \epsilon'\mathbf{1}, \boldsymbol{x}_{2_b} - \epsilon'\mathbf{1}),\\
\boldsymbol{z}''_{N^{2+}} &= (\boldsymbol{x}_1 - (n-1)\epsilon^1\mathbf{1}, \boldsymbol{x}_2 - (n-1)\epsilon^2\mathbf{1}, \boldsymbol{x}'_3 + 2\epsilon^*\mathbf{1}, \cdots, \boldsymbol{x}'_n + 2\epsilon^*\mathbf{1}, \boldsymbol{x}_{2_a}, \boldsymbol{x}_{2_b}).
\end{aligned}$$

Applying *WS* to the last social ranking, we obtain $\boldsymbol{z}'_{N^{2+}}\boldsymbol{R}(R_{N^{2+}})\boldsymbol{z}_{N^{2+}}$. Moreover, by *SM*, we have $\boldsymbol{z}''_{N^{2+}}\boldsymbol{P}(R_{N^{2+}})\boldsymbol{z}'_{N^{2+}}$. Transitivity implies $\boldsymbol{z}''_{N^{2+}}\boldsymbol{P}(R_{N^{2+}})\boldsymbol{z}_{N^{2+}}$. Then, *WS* yields $\boldsymbol{z}''_N\boldsymbol{P}(R_N)\boldsymbol{z}_N$, where

$$\begin{aligned}
\boldsymbol{z}_N &= (\boldsymbol{x}_1 - n\epsilon^1\mathbf{1}, \boldsymbol{x}'_2 + \epsilon^*\mathbf{1}, \boldsymbol{x}'_3 + \epsilon^*\mathbf{1}, \cdots, \boldsymbol{x}'_n + \epsilon^*\mathbf{1}),\\
\boldsymbol{z}''_N &= (\boldsymbol{x}_1 - (n-1)\epsilon^1\mathbf{1}, \boldsymbol{x}_2 - (n-1)\epsilon^2\mathbf{1}, \boldsymbol{x}'_3 + 2\epsilon^*\mathbf{1}, \cdots, \boldsymbol{x}'_n + 2\epsilon^*\mathbf{1}).
\end{aligned}$$

Remember

$$\boldsymbol{z}_N = (\boldsymbol{x}_1 - n\epsilon^1\mathbf{1}, \boldsymbol{x}'_2 + \epsilon^*\mathbf{1}, \cdots, \boldsymbol{x}'_n + \epsilon^*\mathbf{1})\boldsymbol{P}(R_N)\boldsymbol{x}'_N.$$

Then, by transitivity again, we obtain

$$\boldsymbol{z}''_N = (\boldsymbol{x}_1 - (n-1)\epsilon^1 \mathbf{1}, \boldsymbol{x}_2 - (n-1)\epsilon^2 \mathbf{1}, \boldsymbol{x}'_3 + 2\epsilon^* \mathbf{1}, \cdots, \boldsymbol{x}'_n + 2\epsilon^* \mathbf{1}) \boldsymbol{P}(R_N) \boldsymbol{x}'_N.$$

By repeating the same procedure, we have

$$(\boldsymbol{x}_1 - \epsilon^1 \mathbf{1}, \boldsymbol{x}_2 - \epsilon^2 \mathbf{1}, \cdots, \boldsymbol{x}_n - \epsilon^n \mathbf{1}) \boldsymbol{P}(R_N) \boldsymbol{x}'_N.$$

The desired result can be obtained from *SM* and transitivity. $\square$

The next lemma establishes infinite ex post inequality aversion, which is captured by the next axiom.

Certainty Inequality Aversion (CIA). For all $R_N \in \mathcal{D}$ and $\boldsymbol{x}_N, \boldsymbol{x}'_N \in \bar{X}^N$, if there exist j, k such that $\boldsymbol{x}_i = \boldsymbol{x}'_i$ for all $i \neq j, k \in N$, then

$$[x'_j > x_j > x_k > x'_k] \Rightarrow \boldsymbol{x}_N \boldsymbol{R}(R_N) \boldsymbol{x}'_N.$$

Lemma 2. *WPER, SM, TP, WS*, and *IRBC* together imply *CIA*.

Proof. Let $\boldsymbol{x}_N, \boldsymbol{x}'_N \in \bar{X}^N$ be allocations such that $x'_j > x_j > x_k > x'_k$ and $x_i = x'_i$ for all $i \neq j, k$. Since these allocations are composed of constant acts, we can invoke *IRBC* to arbitrarily modify the preferences. Let R_j and R_k be such that $R_j = R_k = R_0$ represented by $E_{p_0} u_0$ defined below.

Let us denote $a, b, c, \alpha, \beta, \gamma, \delta, \epsilon_1, \epsilon_2, \epsilon_3 \in \mathbb{R}_{++}$ as parameters such that

$$\alpha x_k + \beta(a - x_k) = m\alpha x'_k + \epsilon_1, \tag{1}$$

$$\alpha x_k + \beta(a - x_k) + \gamma(b - a) = m\alpha x_k - \epsilon_2, \tag{2}$$

$$\alpha x_k + \beta(a - x_k) + \gamma(b - a) + \delta(c - b) = m[\alpha x_k + \beta(x'_j - x_k)] + \epsilon_3, \tag{3}$$

where $a > x_k$, $c - a = x'_j - x'_k$, $b = \frac{c+a}{2}$, and $\epsilon_1, \epsilon_2, \epsilon_3$ are arbitrarily close to 0. By simple calculations, we have

$$\beta = \frac{m\alpha x'_k - \alpha x_k + \epsilon_1}{a - x_k},$$
$$\gamma = \frac{m\alpha(x_k - x'_k) - \epsilon_1 - \epsilon_2}{b - a},$$
$$\delta = \frac{\beta(x'_j - x_k) - \epsilon_2 + \epsilon_3}{c - b}.$$

Note that

$$b - a = \frac{1}{2}(x'_j - x'_k),\ c - b = \frac{1}{2}(x'_j - x'_k).$$

For $\alpha, \beta, \gamma, \delta$ to be positive, we can set, for instance,

$$a = 2x_k,\ \alpha = \frac{\epsilon_1}{2|mx''_k - x_k|}.$$

Define $u_0 : \mathbb{R}_+ \to \mathbb{R}_+$ as follows.

$$u_0(x) = \begin{cases} \alpha x \text{ for } x \in [0, x_k], \\ \alpha x_k + \beta(x - x_k) \text{ for } x \in (x_k, a], \\ \alpha x_k + \beta(a - x_k) + \gamma(x - a) \text{ for } x \in (a, b], \\ \alpha x_k + \beta(a - x_k) + \gamma(b - a) + \delta(x - b) \text{ for } x > b, \end{cases}$$

Let us also define $p_{0s} = 1/m$ for all $s \in S$.[10] Then, $E_{p_0}u_0$ is defined by p_0 and u_0. It is straightforward to check that this is consistent with conditions (1) to (3) above. Note that $E_{p_0}u_0$ satisfies

$$\frac{u_0(a)}{m} > u_0(x'_k),\ \frac{u_0(b)}{m} < u_0(x_k),\ \frac{u_0(c)}{m} > u_0(x'_j).$$

Let $R_N \in \mathcal{D}$ denote the preference profile where all agents have $E_{p_0}u_0$.

Consider $N^* = \{j, k\}$. Let $\boldsymbol{y}_k = (\epsilon + x'_k/\pi_{s_1}, 0, \cdots, 0)$. By the construction of R_0, if ϵ is sufficiently small, there exists $\boldsymbol{y}_j \in \mathbb{R}^S_{++} \cap \mathring{U}(\boldsymbol{x}'_j, R_0)$ such that $\boldsymbol{y}_j \gg \boldsymbol{y}_k$ and $\hat{\boldsymbol{y}} = \boldsymbol{y}_k + (\boldsymbol{y}_j - \boldsymbol{y}_k)/2 \in \mathring{L}(\boldsymbol{x}_k, R_0)$ ($\boldsymbol{y}_j$ can be chosen sufficiently close to $(c, 0, \cdots, 0)$).

[10]Remember that m is the cardinality of S.

Define $\boldsymbol{y}_{N^*} = (\boldsymbol{y}_j, \boldsymbol{y}_k)$, and $\boldsymbol{y}'_{N^*} = (\hat{\boldsymbol{y}}, \hat{\boldsymbol{y}})$. By Lemma 1, we can apply *XAP* to obtain $\boldsymbol{y}_{N^*}\boldsymbol{P}(R_{N^*})(\boldsymbol{x}'_j, \boldsymbol{x}'_k)$. *TP* implies $\boldsymbol{y}'_{N^*}\boldsymbol{R}(R_{N^*})\boldsymbol{y}_{N^*}$. Again by *XAP*, $(\boldsymbol{x}_j, \boldsymbol{x}_k)\boldsymbol{P}(R_{N^*})\boldsymbol{y}'_{N^*}$. It follows from transitivity that $(\boldsymbol{x}_j, \boldsymbol{x}_k)\boldsymbol{P}(R_{N^*})(\boldsymbol{x}'_j, \boldsymbol{x}'_k)$.

Since $\boldsymbol{x}_i = \boldsymbol{x}'_i$ for all $i \neq j, k$, *WS* implies $\boldsymbol{x}_N\boldsymbol{R}(R_N)\boldsymbol{x}'_N$. Applying *IRBC* to adjust the preference profile, we have the desired result. □

We derive the conclusion of Theorem 1 from *CIA* and *XAP* applying the lemmas above.

Proof of Theorem 1. Let $\boldsymbol{x}_N$ and $\boldsymbol{x}'_N$ be allocations such that $\min_{i\in N} C(\boldsymbol{x}_i, R_i) > \min_{i\in N} C(\boldsymbol{x}'_i, R_i)$. Without loss of generality, suppose $C(\boldsymbol{x}'_1, R_1) = \min_{i\in N} C(\boldsymbol{x}'_i, R_i)$. Define $\boldsymbol{y}_N \in \bar{X}^N$ as $\boldsymbol{y}_i = \big(C(\boldsymbol{x}'_i, R_i) + \epsilon\big)\mathbf{1}$ for all $i \in N$, where $\epsilon > 0$ is small enough that

$$C(\boldsymbol{x}'_1, R_1) + 3\epsilon < \min\left\{\min_{i\neq 1} C(\boldsymbol{x}'_i, R_i),\ \min_{i\in N} C(\boldsymbol{x}_i, R_i)\right\}.$$

From Lemma 1, we can apply *XAP* to obtain $\boldsymbol{y}_N\boldsymbol{P}(R_N)\boldsymbol{x}'_N$.

Next, let $\boldsymbol{y}'_N \in X^N$ be such that $\boldsymbol{y}'_1 = \big(C(\boldsymbol{x}'_1, R_1) + 2\epsilon\big)\mathbf{1}$ and $\boldsymbol{y}'_i = \big(C(\boldsymbol{x}'_1, R_1) + 3\epsilon\big)\mathbf{1}$. Applying *CIA* (Lemma 2) repeatedly, we have $\boldsymbol{y}'_N\boldsymbol{R}(R_N)\boldsymbol{y}_N$. It follows from *XAP* (Lemma 1) that $\boldsymbol{x}_N\boldsymbol{P}(R_N)\boldsymbol{y}'_N$. Transitivity implies $\boldsymbol{x}_N\boldsymbol{P}(R_N)\boldsymbol{x}'_N$ as sought. □

If we require *SEP* instead of *WS*, the following result is obtained.

Lemma 3. If an SOF $\boldsymbol{R}$ satisfies *WPER, TP,* and *SEP*, it also satisfies *SM*.

Proof. Let $\boldsymbol{x}_N, \boldsymbol{x}'_N$ be such that $\boldsymbol{x}_N \gg \boldsymbol{x}'_N$. Let $N = \{1, 2, \cdots, n\}$. First, we consider, for each $i \in N$, $\{i, i_d\}$ such that $i_d \notin N$ and $R_i = R_{i_d}$. Let $\boldsymbol{x}_{i_d} = \boldsymbol{x}_i$ and $\boldsymbol{y}_i = \boldsymbol{y}_{i_d} = \boldsymbol{x}'_i + \frac{1}{2}(\boldsymbol{x}_i - \boldsymbol{x}_{i_d})$. By *TP*, $(\boldsymbol{y}_i, \boldsymbol{y}_{i_d})\boldsymbol{R}(R_{\{i,i_d\}})(\boldsymbol{x}'_i, \boldsymbol{x}_{i_d})$. *WPER* implies $(\boldsymbol{x}_i, \boldsymbol{x}_{i_d})\boldsymbol{P}(R_{\{i,i'\}})(\boldsymbol{y}_i, \boldsymbol{y}_{i_d})$. From transitivity, we obtain

$$(\boldsymbol{x}_i, \boldsymbol{x}_{i_d})\boldsymbol{P}(R_{\{i,i_d\}})(\boldsymbol{x}'_i, \boldsymbol{x}_{i_d}) \text{ for each } i \in N. \tag{4}$$

Next, we show $(\boldsymbol{x}_1, \boldsymbol{x}_2)\boldsymbol{P}(R_{\{1,2\}})(\boldsymbol{x}'_1, \boldsymbol{x}'_2)$. Since $(\boldsymbol{x}_1, \boldsymbol{x}_{1_d})\boldsymbol{P}(R_{\{1,1_d\}})(\boldsymbol{x}'_1, \boldsymbol{x}_{1_d})$ by (1), repeated applications of *SEP* imply $(\boldsymbol{x}_1, \boldsymbol{x}_{1_d}, \boldsymbol{x}'_2, \boldsymbol{x}_{2_d})\boldsymbol{P}(R_{\{1,1'\}})(\boldsymbol{x}'_1, \boldsymbol{x}_{1_d}, \boldsymbol{x}'_2, \boldsymbol{x}_{2_d})$, and thus

$$(\boldsymbol{x}_1, \boldsymbol{x}'_2, \boldsymbol{x}_{2_d})\boldsymbol{P}(R_{\{1,2,2_d\}})(\boldsymbol{x}'_1, \boldsymbol{x}'_2, \boldsymbol{x}_{2_d}). \tag{5}$$

Moreover, (1) for $i = 2$ and *SEP* imply

$$(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_{2_d})\boldsymbol{R}(R_{\{1,2,2_d\}})(\boldsymbol{x}_1, \boldsymbol{x}_2', \boldsymbol{x}_{2_d}). \tag{6}$$

It follows from (2), (3), transitivity and *SEP* that

$$(\boldsymbol{x}_1, \boldsymbol{x}_2)\boldsymbol{P}(R_{\{1,2\}})(\boldsymbol{x}_1', \boldsymbol{x}_2'). \tag{7}$$

Next, we prove $(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)\boldsymbol{P}(R_{\{1,2,3\}})(\boldsymbol{x}_1', \boldsymbol{x}_2', \boldsymbol{x}_3')$. By (4) and *SEP*, we have

$$(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3', \boldsymbol{x}_{3_d})\boldsymbol{P}(R_{\{1,2,3,3_d\}})(\boldsymbol{x}_1', \boldsymbol{x}_2', \boldsymbol{x}_3', \boldsymbol{x}_{3_d}).$$

Moreover, (1) for $i = 3$ and *SEP* implies

$$(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_{3_d})\boldsymbol{P}(R_{\{1,2,,3,3_d\}})(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3', \boldsymbol{x}_{3_d}).$$

It follows from transitivity and *SEP* that $(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)\boldsymbol{P}(R_{\{1,2,3\}})(\boldsymbol{x}_1', \boldsymbol{x}_2', \boldsymbol{x}_3')$. By repeating this procedure, we have the desired result. □

Then, since *SEP* implies *WS*, the conclusions of Lemmas 1 and 2 are obtained by replacing *SM* and *WS* with *SEP*. Hence, we have the following theorem by the same discussion as above.

Theorem 2. Suppose that an SOF $\boldsymbol{R}$ satisfies *WPER, TP, IRBC*, and *SEP*. Then, for all $R_N \in \mathcal{R}^N$ and all $\boldsymbol{x}_N, \boldsymbol{x}_N' \in X^N$,

$$\min_{i \in N} C(\boldsymbol{x}_i, R_i) > \min_{i \in N} C(\boldsymbol{x}_i', R_i) \Longrightarrow \boldsymbol{x}_N \boldsymbol{P}(R_N)\boldsymbol{x}_N'.$$

Proof. From Lemma 3, the axioms imply *SM*. Since *SEP* is stronger than *WS*, the desired result follows from Theorem 1. □

All of the axioms in Theorem 2 are satisfied by the certainty equivalence leximin ordering $\boldsymbol{R}_{LC}$ defined below. For all $R_N \in \mathcal{D}$, all $\boldsymbol{x}_N$, $\boldsymbol{x}_N' \in X^N$,

$$\boldsymbol{x}_N \boldsymbol{R}_{LC}(R_N)\boldsymbol{x}_N' \Longleftrightarrow \big(C(\boldsymbol{x}_i, R_i)\big)_{i \in N} \geq_{lex} \big(C(\boldsymbol{x}_i', R_i)\big)_{i \in N},$$

where $\geq_{lex}$ is the usual lexicographic ordering over X^N. Note that R_C does not satisfy *SEP*.

As shown above, the separability principles and the quite weak conditions of equity and efficiency imply *XAP*, which is incompatible with *DAT*. However, if *DAT* is considered to be more reasonable than *XAP* in the society, Lemma 1 is an undesirable result. Then, our next problem is to consider social orderings satisfying *DAT*. In the next section, we relax the separability principle, and derive a certain social ordering from the stronger equity conditions including *DAT* and another two Pareto conditions.

4 Dominance Averse Transfer and Relaxing Separability

In this section, we introduce a weaker separability condition and two Pareto conditions, and derive a social ordering from the stronger equity conditions. First, we introduce the weaker separability condition.

Well-off Separability (WOS). For all $R_N \in \mathcal{D}$ such that $|N| \geq 3$, and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, if $R_i = R_j$, $\boldsymbol{x}_i P_i \boldsymbol{x}_j$, $\boldsymbol{x}'_i P_i \boldsymbol{x}'_j$, and $\boldsymbol{x}_i = \boldsymbol{x}'_i$ for some $i, j \in N$, then

$$\boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N \Longrightarrow \boldsymbol{x}_{N\setminus\{i\}} \boldsymbol{P}(R_{N\setminus\{i\}}) \boldsymbol{x}'_{N\setminus\{i\}}.$$

In an egalitarian society, information about disadvantaged individuals would be important for social decision making. This axiom captures the idea and requires that an irrelevant agent should not affect the social ordering only when s/he is unambiguously better off than another.

Next, we introduce two efficiency conditions. The first Pareto axiom requires that agents' preferences for riskless situations be respected.

Pareto for Riskless Acts (PRA). For all $R_N \in \mathcal{D}$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$ such that there exists $M \subseteq N$ with $\boldsymbol{x}_M \in \bar{X}^M$, if $\boldsymbol{x}_i P_i \boldsymbol{x}'_i$ for all $i \in M$ and $\boldsymbol{x}_j = \boldsymbol{x}'_j$ for all $j \in N \setminus M$, then $\boldsymbol{x}_N \boldsymbol{R}(R_N) \boldsymbol{x}'_N$.

This axiom insists that if a subgroup of agents prefer riskless acts to risky acts, such preferences should be socially supported. In other words, this requirement says that agents can avoid risks if they want to. Note that if the agents in M have constant acts also in $\boldsymbol{x}'_M$, the axiom still makes sense because $\boldsymbol{x}_M \gg \boldsymbol{x}'_M$. This axiom avoids the problem of spurious unanimity caused by differences in beliefs. For instance, suppose that Ann does not want to buy a stock because she expects the price to decrease, whereas Bob does not want to sell short because he thinks that the price will increase. Then, it would be unreasonable to force Ann to buy the stock and make Bob sell short, even if one of them is eventually right.

We also introduce the second Pareto condition.

Pareto for Consensual Risk-taking (PCR). For all $R_N \in \mathcal{D}$ such that $E_{p_i}u_i = E_{p_j}u_j$ for all $i, j \in N$, and all $\boldsymbol{x}_N, \in X^N$, $\boldsymbol{x}'_N \in \bar{X}^N$, if $\boldsymbol{x}_i P_j \boldsymbol{x}'_i$ for all $i, j \in N$, then $\boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N$.

This axiom requires that if each agent's (potential) risk-taking behavior is supported by all agents' preferences, such a risky situation should be socially preferred. Note that if $\boldsymbol{x}_N$ is also a riskless allocation, the axiom remains reasonable because $\boldsymbol{x}_N \gg \boldsymbol{x}'_N$. Note also that this axiom is weaker than Sprumont's (2012) *Consensus*[11] and *Unanimity Pareto Principle*[12] advocated by Gayer et al. (2014). *PCR* avoids the problem of spurious unanimity caused by different beliefs, because each agent thinks that all individuals' risk-takings are beneficial according to his/her preference. This axiom may be criticized that an allocation with unequal outcomes can be socially preferred to another allocation with equal outcomes. However, we subsequently show that the axiom is consistent with strong equity conditions such as *DAT* and *CIA*.

From the axioms described above, we derive a social choice criterion. For each $\boldsymbol{z} \in X$ and $R_N \in \mathcal{D}$, let $\bar{U}(\boldsymbol{z}, R_N) = \bigcap_{i \in N} U(\boldsymbol{z}, R_i)$, and let us define

$$\kappa(\boldsymbol{x}_i, R_N) = \inf\{z \in \mathbb{R}_+ | \boldsymbol{x}_i \notin \bar{U}(\boldsymbol{z}, R_N),\ \boldsymbol{z} \in \bar{X}\}.$$

[11] *Consensus* says that for all $R_N \in \mathcal{D}$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, if $\boldsymbol{x}_i P_j \boldsymbol{x}'_i$ for all $i, j \in N$, then $\boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N$.

[12] *Unanimity Pareto Principle* requires that for all $R_N \in \mathcal{D}$ represented by $(E_{p_i}u_i)_{i \in N}$ such that $p_i = p_j$ for all $i, j \in N$, and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, if $\boldsymbol{x}_i P_j \boldsymbol{x}'_i$ for all $i, j \in N$, then $\boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N$.

By assumption on $\mathcal{R}$, $\kappa(\boldsymbol{x}_i, R_N)$ is unique for each $\boldsymbol{x}_i \in X$ and $R_N \in \mathcal{R}^N$. Then, we obtain the next theorem.

Theorem 3. Suppose that an SOF $\boldsymbol{R}$ satisfies *PCR, PRA, IRBC, WOS*, and *DAT*. Then, for all $R_N \in \mathcal{R}^N$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$,

$$\min_{i \in N} \kappa(\boldsymbol{x}_i, R_N) > \min_{i \in N} \kappa(\boldsymbol{x}'_i, R_N) \Longrightarrow \boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N.$$ [13]

To prove the theorem, we use the lemma below, which has a similar implication to *CIA*.

Lemma 4. Suppose that an SOF $\boldsymbol{R}$ satisfies *PRA, PCR, DAT*, and *IRBC*. Then, for all $R_N \in \mathcal{D}$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in \bar{X}^N$, if there exist j, k such that $\boldsymbol{x}_i \gg \boldsymbol{x}'_i$ for all $i \neq j, k$,

$$[x'_j > x_j > x_k > x'_k] \Rightarrow \boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N.$$ [14]

Proof. The proof is similar to that of Lemma 2. Suppose that $x'_j > x_j > x_k > x'_k$ and $x_i > x'_i$ for all $i \neq j, k$. Since $\boldsymbol{x}_N$ and $\boldsymbol{x}'_N$ are riskless allocations, R_N can be arbitrarily modified using *IRBC*. Then, assume that R_N is such that $R_i = R_0$ for all $i \in N$, where R_0 is the same as defined in the proof of Lemma 2. Let $\boldsymbol{y}_N, \boldsymbol{y}'_N \in X^N$ be such that $\boldsymbol{y}_j, \boldsymbol{y}'_j, \boldsymbol{y}_k, \boldsymbol{y}'_k$ are as introduced in the proof of Lemma 2, and for all $i \neq j, k$,

$$\boldsymbol{y}_i, \boldsymbol{y}'_i \in \bar{X}, \text{ and } y_i = y'_i = x_i + \epsilon,$$

where $\epsilon > 0$ is small enough that $x_i > x'_i + \epsilon$ and $\boldsymbol{x}_k - (n-2)\epsilon\mathbf{1} P_k \boldsymbol{y}'_k$. By similar arguments to Lemma 2, we can obtain $\boldsymbol{y}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N$ from *PCR* and $\boldsymbol{y}'_N \boldsymbol{P}(R_N) \boldsymbol{y}_N$ from *DAT*. It follows from transitivity that $\boldsymbol{y}'_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N$.

Let $\boldsymbol{y}''_N \in \bar{X}^N$ be such that

$$y''_k = x_k - (n-2)\epsilon, \; y''_j = x_j, \; y''_i = y'_i \text{ for all } i \neq j, k.$$

[13] It is not difficult to show another characterization replacing *WOS* with *Replication Invariance*, which requires that replicating the economy should not affect the social rankings.

[14] It is straightforward to obtain a stronger result using *TP* rather than *DAT*.

Noting that $\boldsymbol{y}''_k P_k \boldsymbol{y}'_k$, we obtain $\boldsymbol{y}''_N \boldsymbol{R}(R_N) \boldsymbol{y}'_N$ from *PRA*. Repeated applications of *DAT* (transfers from all $i \neq j, k$ to k) imply $\boldsymbol{x}_N \boldsymbol{R}(R_N) \boldsymbol{y}''_N$. $\boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N$ follows from transitivity. By modifying R_N appropriately using *IRBC*, we obtain the desired result. □

Proof of Theorem 3. Let $\boldsymbol{x}_N$ and $\boldsymbol{x}'_N$ be allocations such that $\boldsymbol{x}_N \boldsymbol{P}_\kappa(R_N) \boldsymbol{x}'_N$. We show $\boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N$.

Without loss of generality, let $1 \in N$ be such that $\kappa(\boldsymbol{x}'_1, R_N) = \min_{i \in N} \kappa(\boldsymbol{x}'_i, R_N)$. The proof is divided into two cases depending on whether or not another agent i has $C(\boldsymbol{x}'_1, R_i) = \kappa(\boldsymbol{x}'_1, R_N)$.

Case 1. Suppose that there exists agent i^* ($i^* \neq 1$) such that $C(\boldsymbol{x}'_1, R_{i^*}) = \kappa(\boldsymbol{x}'_1, R_N)$. Let $\boldsymbol{y}_N \in X^N$ be such that, $\boldsymbol{y}_i = \boldsymbol{y}_k$ for all $i, k \in N \backslash \{1\}$, and for all $j \neq 1$,

$$\boldsymbol{y}_j \in \bar{X} \cap \mathring{U}(\boldsymbol{x}_j, R_j), \ \boldsymbol{y}_j \gg \boldsymbol{y}_1 = \boldsymbol{x}'_1, \text{ and } y_j > \min_{i \in N} \kappa(\boldsymbol{x}_i, R_N).$$

By *PRA*, we have $\boldsymbol{y}_N \boldsymbol{R}(R_N) \boldsymbol{x}'_N$.

Define $\Delta^* = \boldsymbol{y}_{i^*} - \boldsymbol{y}_1$. Let $\epsilon > 0$ and $Q \subset \bar{N} \backslash N$ be as follows:

(i) $\boldsymbol{y}_{i^*} - \epsilon|Q|\Delta^* = \boldsymbol{y}_1 + 12\epsilon\Delta^*$;

(ii) $\kappa(\boldsymbol{y}_1 + 12\epsilon\Delta^*, R_N) < \min_{i \in N} \kappa(\boldsymbol{x}_i, R_N)$;

(iii) $R_j = R_1$ for all $j \in Q$.

Without loss of generality, suppose that $C(\boldsymbol{y}_1 + 12\epsilon\Delta^*, R_2) = \kappa(\boldsymbol{x}'_1, R_N)$. Denote $\boldsymbol{x}^* = \boldsymbol{y}_1 + 11\epsilon\Delta^*$ and $M = N \cup Q$. Define $\hat{\boldsymbol{y}}_M = (\boldsymbol{y}_N, \boldsymbol{x}^*_Q)$ and $\hat{\boldsymbol{x}}'_M = (\boldsymbol{x}'_N, \boldsymbol{x}^*_Q)$, where $\boldsymbol{x}^*_Q = (\boldsymbol{x}^*, \cdots, \boldsymbol{x}^*) \in X^Q$. By *WOS*, we have $\hat{\boldsymbol{y}}_M \boldsymbol{R}(R_M) \hat{\boldsymbol{x}}'_M$.

Let us introduce $\boldsymbol{y}'_M, \boldsymbol{y}''_M \in X^M$ such that

$$\boldsymbol{y}'_2 = \boldsymbol{y}'_i = \boldsymbol{y}_1 + 12\epsilon\Delta^* \text{ for all } i \in Q, \ \boldsymbol{y}'_k = \boldsymbol{y}_k \text{ for all other } k.$$

$$\boldsymbol{y}''_1 = \boldsymbol{y}''_2 = \boldsymbol{y}_1 + 6\epsilon\Delta^*, \ \boldsymbol{y}''_k = \boldsymbol{y}''_k \text{ for all other } k.$$

By repeated applications of *DAT* (transferring from agent 2 to all $i \in Q$), we have $\boldsymbol{y}'_M \boldsymbol{R}(R_M) \hat{\boldsymbol{y}}_M$. Again by *DAT* (transferring from agent 2 to 1), we obtain $\boldsymbol{y}''_M \boldsymbol{R}(R_M) \boldsymbol{y}'_M$. Then, $\boldsymbol{y}''_M \boldsymbol{R}(R_M) \hat{\boldsymbol{x}}'_M$ by transitivity.

Define $\boldsymbol{z}_M, \boldsymbol{z}'_M \in \bar{X}^M$ as follows.

$$\boldsymbol{z}_i = \boldsymbol{y}_2 \text{ for all } i \in Q \cup \{1\},\ \boldsymbol{z}_2 = \kappa(\boldsymbol{y}_1 + 7\epsilon\Delta^*, R_N)\mathbf{1},\ \boldsymbol{z}_k = \boldsymbol{y}'_k \text{ for all other } k.$$

$$\boldsymbol{z}'_2 = \kappa(\boldsymbol{y}_1 + 8\epsilon\Delta^*, R_N)\mathbf{1},\ \boldsymbol{z}'_k = \kappa(\boldsymbol{y}_1 + 9\epsilon\Delta^*, R_N)\mathbf{1} \text{ for all } k \neq 2.$$

PRA implies $\boldsymbol{z}_M \boldsymbol{R}(R_M)\boldsymbol{y}''_M$. Applying Lemma 3 repeatedly, $\boldsymbol{z}'_M \boldsymbol{P}(R_M)\boldsymbol{z}_M$. From transitivity, we obtain $\boldsymbol{z}'_M \boldsymbol{P}(R_M)\hat{\boldsymbol{x}}'_M$.

Let $\boldsymbol{z}''_M \in X^M$ be such that

$$\boldsymbol{z}''_j = \kappa(\boldsymbol{y}_1 + 10\epsilon\Delta^*, R_N)\mathbf{1} \text{ for all } j \in N,\ \boldsymbol{z}''_i = \boldsymbol{x}^* = \boldsymbol{y}_1 + 11\epsilon\Delta^* \text{ for all } i \in Q.$$

Then, by the definition of $\kappa(\cdot,\cdot)$ and condition (ii) above, we can see $\boldsymbol{z}''_i P_j \boldsymbol{z}'_i$ for all $i, j \in M$. Thus, by *PCR*, we have $\boldsymbol{z}''_M \boldsymbol{P}(R_N)\boldsymbol{z}'_M$. By transitivity, we obtain $\boldsymbol{z}''_M \boldsymbol{P}(R_M)\hat{\boldsymbol{x}}'_M$. From $R_i = R_1$ for all $i \in Q$ and $\boldsymbol{x}^* P_1 \boldsymbol{x}'_1$, *WOS* yields $\boldsymbol{z}''_N \boldsymbol{P}(R_M)\boldsymbol{x}'_N$. Similarly, by the definition of $\kappa(\cdot,\cdot)$ and condition (ii), we can see $\boldsymbol{x}_i P_j \boldsymbol{z}''_i$ for all $i, j \in M$, and thus $\boldsymbol{x}_N \boldsymbol{P}(R_N)\boldsymbol{z}''_N$ from *PCR*. Transitivity implies $\boldsymbol{x}_N \boldsymbol{P}(R_N)\boldsymbol{x}'_N$ as sought.

Case 2. Assume that agent 1 is a unique individual such that $C(\boldsymbol{x}'_1, R_1) = \kappa(\boldsymbol{x}'_1, R_N) = \min_{i \in N} \kappa(\boldsymbol{x}'_i, R_N)$. We introduce $\hat{\boldsymbol{z}}_N, \hat{\boldsymbol{z}}'_N \in \bar{X}^N$ as follows.

$$\kappa(\boldsymbol{x}'_1, R_N) < \hat{z}_1 < \min_{i \in N} \kappa(\boldsymbol{x}_i, R_N) < \hat{z}_k \text{ and } \hat{\boldsymbol{z}}_k R_k \boldsymbol{x}'_k \text{ for all } k \neq 1,$$

$$\kappa(\hat{\boldsymbol{z}}_1, R_N) < \hat{z}'_1 < \hat{z}'_k < \min_{i \in N} \kappa(\boldsymbol{x}_i, R_N) \text{ for all } k \neq 1.$$

By *PRA*, we have $\hat{\boldsymbol{z}}_N \boldsymbol{R}(R_M)\boldsymbol{x}'_N$. Applying Lemma A2 repeatedly, we can show $\hat{\boldsymbol{z}}'_N \boldsymbol{P}(R_M)\hat{\boldsymbol{z}}_N$. Transitivity implies $\hat{\boldsymbol{z}}'_N \boldsymbol{P}(R_M)\boldsymbol{x}'_N$.

By the definition of $\kappa(\cdot,\cdot)$, we can see $\boldsymbol{x}_i P_j \hat{\boldsymbol{z}}'_i$ for all $i, j \in N$. Thus, by *PCR*, we have $\boldsymbol{x}_N \boldsymbol{P}(R_N)\hat{\boldsymbol{z}}'_N$. By transitivity, we obtain $\boldsymbol{x}_N \boldsymbol{P}(R_M)\boldsymbol{x}'_N$ as desired. $\square$

Next, we consider a stronger equity condition, which requires that if there is income inequality in every state between two agents, reducing such inequality should be socially weakly preferable.

Dominance Aversion (DA). For all $R_N \in \mathcal{D}$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, if there exist $j, k \in N$ such that $\boldsymbol{x}_i = \boldsymbol{x}'_i$ for all $i \neq j, k$,

$$\left[\boldsymbol{x}'_j \gg \boldsymbol{x}_j \geq \boldsymbol{x}_k \gg \boldsymbol{x}'_k\right] \Rightarrow \boldsymbol{x}_N \boldsymbol{R}(R_N) \boldsymbol{x}'_N.$$

Then, we obtain the following result, which is remarkable since a few axioms of equity and efficiency lead to a characterization of $\boldsymbol{R}_\kappa$.

Theorem 4. Suppose that an SOF $\boldsymbol{R}$ satisfies *PCR, PRA*, and *DA*. Then, for all $R_N \in \mathcal{R}^N$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$,

$$\min_{i \in N} \kappa(\boldsymbol{x}_i, R_N) > \min_{i \in N} \kappa(\boldsymbol{x}'_i, R_N) \Longrightarrow \boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N.$$

Proof. Let $\boldsymbol{x}_N$ and $\boldsymbol{x}'_N$ be allocations such that $\boldsymbol{x}_N \boldsymbol{P}_\kappa(R_N) \boldsymbol{x}'_N$. The goal is to show $\boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N$. As in the proof of Theorem 3, let $1 \in N$ be such that $\kappa(\boldsymbol{x}'_1, R_N) = \min_{i \in N} \kappa(\boldsymbol{x}'_i, R_N)$, and the proof is divided into two cases depending on whether or not another agent i has $C(\boldsymbol{x}'_1, R_i) = \kappa(\boldsymbol{x}'_1, R_N)$.

Case 1. Without loss of generality, let $2 \in N$ be such that $C(\boldsymbol{x}'_1, R_2) = \min_{i \in N} \kappa(\boldsymbol{x}'_i, R_N)$. Define $\boldsymbol{y}_N \in X^N$ as $\boldsymbol{y}_1 = \boldsymbol{x}'_1$, and $\boldsymbol{y}_j \in \bar{X} \cap \mathring{U}(\boldsymbol{x}'_j, R_j)$ and $\boldsymbol{y}_j \gg \boldsymbol{x}'_1$ for all $j \neq 1$. By *PRA*, we have $\boldsymbol{y}_N \boldsymbol{R}(R_N) \boldsymbol{x}'_N$.

Let us also introduce $\boldsymbol{z}_N, \boldsymbol{z}'_N, \boldsymbol{z}''_N \in X^N$ as follows.

$$\boldsymbol{y}_1 \ll \boldsymbol{z}_1 \ll \boldsymbol{z}_2 \ll \boldsymbol{y}_2,\ \kappa(\boldsymbol{z}_2, R_N) < \min_{i \in N} \kappa(\boldsymbol{x}_i, R_N),\ \boldsymbol{z}_k = \boldsymbol{y}_k \text{ for all } k \neq 1, 2.$$

$$\boldsymbol{z}'_1 = \boldsymbol{y}_2,\ \boldsymbol{z}'_2 \in \bar{X},\ \kappa(\boldsymbol{z}_2, R_N) < \kappa(\boldsymbol{z}'_2, R_N) < \min_{i \in N} \kappa(\boldsymbol{x}_i, R_N),\ \boldsymbol{z}'_k = \boldsymbol{z}_k \text{ for all } k \neq 1, 2.$$

$$\boldsymbol{z}''_N \in \bar{X}^N,\ \kappa(\boldsymbol{z}'_2, R_N) < \kappa(\boldsymbol{z}''_2, R_N) < \kappa(\boldsymbol{z}''_k, R_N) < \min_{i \in N} \kappa(\boldsymbol{x}_i, R_N) \text{ for all } k \neq 2.$$

By *DA*, we have $\boldsymbol{z}_N \boldsymbol{R}(R_N) \boldsymbol{y}_N$. *PRA* implies $\boldsymbol{z}'_N \boldsymbol{R}(R_N) \boldsymbol{z}_N$. By applying *DA* repeatedly, $\boldsymbol{z}''_N \boldsymbol{R}(R_N) \boldsymbol{z}'_N$. From transitivity, we obtain $\boldsymbol{z}''_N \boldsymbol{R}(R_N) \boldsymbol{x}'_N$.

Here, by the definition of $\kappa(\cdot, \cdot)$ and

$$\min_{i \in N} \kappa(\boldsymbol{x}_i, R_N) > \kappa(\boldsymbol{z}''_j, R_N) \text{ for all } j \in N$$

we can see $\boldsymbol{x}_i P_j \boldsymbol{z}''_i$ for all $i, j \in N$. Thus, by *PCR*, we have $\boldsymbol{x}_N \boldsymbol{P}(R_N)\boldsymbol{z}''_N$. By transitivity, we obtain $\boldsymbol{x}_N \boldsymbol{P}(R_N)\boldsymbol{x}'_N$ as sought.

Case 2. This case exactly resembles *Case 2* in the proof of Theorem 3, and thus we can safely omit it. $\square$

If we additionally require a continuity property in Theorems 3 and 4, we can characterize the intersection maximin ordering $\boldsymbol{R}_\kappa$ defined below. For all $R_N \in \mathcal{D}$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$,

$$\boldsymbol{x}_N \boldsymbol{R}_\kappa(R_N)\boldsymbol{x}'_N \Longleftrightarrow \min_{i\in N} \kappa(\boldsymbol{x}_i, R_N) \geq \min_{i\in N} \kappa(\boldsymbol{x}'_i, R_N).$$

This social criterion first compares agents' acts based on $\bar{U}(\boldsymbol{x}_i, R_N)$, which is constructed from all agents' preferences. $\bar{U}(\boldsymbol{x}_i, R_N)$ could be interpreted as an upper-contour set of a social planner's preference for evaluating acts. Then, R_κ compares allocations using the certainty equivalence of the worst acts with respect to the planner's preference, described by $\min_{i\in N} \kappa(\boldsymbol{x}_i, R_N)$.

If the domain is restricted as described below, we can use a simpler criterion. Let $\mathcal{R}^C$ denote the set of monotonic, continuous, and *convex* preference orderings. Define $\mathcal{D}^C = \cup_{N\in\mathcal{N}}(\mathcal{R}^C)^N$. Then, noting that $\min_{i\in N} \kappa(\boldsymbol{x}_i, R_N) = \min_{i,j\in N} C(\boldsymbol{x}_i, R_j)$ on $\mathcal{D}^C$, we have corollaries to Theorems 3 and 4. These are that if $\boldsymbol{R}$ satisfies the sets of axioms in the theorems, then for all $R_N \in \mathcal{D}^C$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$,

$$\min_{i,j\in N} C(\boldsymbol{x}_i, R_j) > \min_{i,j\in N} C(\boldsymbol{x}'_i, R_j) \Longrightarrow \boldsymbol{x}_N \boldsymbol{P}(R_N)\boldsymbol{x}'_N.$$

According to this criterion, allocations are evaluated based on the minimum certainty equivalences of the most risk averse preferences. This property comes from *PCR*, which requires that risk-takings should be considered socially desirable only when those are supported by all agents in the society. Convexity is often considered as risk and uncertainty aversion (e.g., Yaari, 1969; Gilboa and Schmeidler, 1989; Rigotti et al., 2008).

Next, we explain how the results in this section are related to those of Sprumont (2012). In a deterministic environment, Sprumont (2012, Theorem 1) characterized a class of social orderings called *consensual Rawlsian orderings* using *DA* and *Consensus* introduced above. Each consensual Rawlsian ordering evaluates allocations based on worst bundles with respect to a social evaluation ordering R^* over commodity bundles. The ordering R^* is assumed to agree with unanimous judgments in the sense that for any two bundles $\boldsymbol{x}$ and $\boldsymbol{y}$, $\boldsymbol{x}P^*\boldsymbol{y}$ if $\boldsymbol{x}P_i\boldsymbol{y}$ for all $i \in N$. Note that there are as many consensual Rawlsian orderings as the number of social evaluation orderings over commodity bundles. Although Sprumont's results are remarkable, it is difficult to determine which social evaluation ordering should be adopted. Our social ordering uses simple criteria for interpersonal comparison based on $\kappa(\boldsymbol{x}_i, R_N)$ and certainty equivalence. Another difference between Sprumont's (2012) results and ours is that he used a single-profile framework, while we have provided a multi-profile theorem (Theorem 3) using the weaker equity condition (*DAT*) and independence axiom (*IRBC*).

5 Statewise Dominance

So far we have only required that social criteria should satisfy completeness and transitivity as social rationality. In this section, we consider a well-known social rationality condition called *Statewise Dominance*, and show that this axiom conflicts with ex post equity, efficiency, and separability in our environment. The argument is not new, but it is worth discussing inconsistency between *Statewise Dominance* and these three principles in the context of our environment.

First, we formally introduce a weak form of *Statewise Dominance*. For each $\boldsymbol{x} \in X$ and each $s \in S$, let $\boldsymbol{x}(s) \in \bar{X}$ be such that $x_{s'}(s) = x_{s''}(s) = x_s$ for all $s', s'' \in S$. $\boldsymbol{x}_N(s) \in \bar{X}^N$ is similarly defined.

Weak Dominance (WD). For all $R_N \in \mathcal{D}$ and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, $\boldsymbol{x}_N(s)\boldsymbol{R}(R_N)\boldsymbol{x}'_N(s)$ for all $s \in S$, then $\boldsymbol{x}_N\boldsymbol{R}(R_N)\boldsymbol{x}'_N$.

This axiom states that if every consequence of an allocation is weakly socially better than that of another allocation, the former allocation is socially weakly preferred to the latter. If the axiom is violated, society may choose an allocation that could result in a worse outcome.

Below, we provide an ex post equity condition.

Certainty Poverty Aversion (CPA). For all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in \bar{X}^N$, if there exist j, k such that $\boldsymbol{x}_i = \boldsymbol{x}'_i$ for all $i \neq j, k \in N$,

$$[x'_j > x_j \geq x_k > x'_k = 0] \Longrightarrow \boldsymbol{x}_N \boldsymbol{R}(R_N) \boldsymbol{x}'_N.$$

This axiom applies to two agents, one of whom has no money (i.e., is poor) and the other has some money, both with certainty. The axiom requires that redistribution to reduce inequality and poverty should be weakly preferred. This axiom is clearly weaker than *CIA*.

We also consider an efficiency requirement.

Pareto for Risk with Equivalent Value (PREV). For all $R_N \in \mathcal{D}$, and all $\boldsymbol{x}_N \in X^N$ and $\boldsymbol{x}'_N \in \bar{X}^N$ such that $R_i = R_j$, $\boldsymbol{x}_i I_i \boldsymbol{x}_j$, and $x'_i = x'_j$ for all $i, j \in N$, if $\boldsymbol{x}_i P_i \boldsymbol{x}'_i$ for all $i \in N$, then $\boldsymbol{x}_N \boldsymbol{P}(R_N) \boldsymbol{x}'_N$.

This axiom requires that if all agents with the same preference prefer possibly uncertain acts to riskless acts and the acts in each allocation are equally valuable to the agents, then the possibly uncertain allocation should be socially preferred to the riskless allocation. This requirement is weaker than *PCR*, and seems reasonable in terms of compatibility with equity. This condition also avoids the problem of spurious unanimity to some degree for the same reason as does *PCR*.

We now obtain the following impossibility theorem.[15]

Theorem 5. There exists no SOF satisfying *PREV, CPA*, and *WD*.

Proof. Let us consider $S = \{1, 2\}$, $\{1, 2\} \in \mathcal{N}$ and $R_1 = R_2 = R^* \in \mathcal{R}$ such that $(30, 0) I^* (0, 30) P^* (10, 10)$.

[15]The example in the proof is quite similar to the one provided by Fleurbaey and Voorhoeve (2013) to show that *SD* and *XAP* conflict with ex post equity. Our discussion is intended to clarify that in our environment, the conflict remains even if *XAP* is relaxed to *PREV*.

On the one hand, it follows from *PREV* that

$$\big((30,0),(0,30)\big)\boldsymbol{P}(R_{\{1,2\}})\big((10,10),(10,10)\big). \tag{8}$$

On the other hand, *CPA* implies

$$\big((5,5),(5,5)\big)\boldsymbol{R}(R_{\{1,2\}})\big((30,30),(0,0)\big)$$

and *PREV* implies

$$\big((10,10),(10,10)\big)\boldsymbol{P}(R_{\{1,2\}})\big((5,5),(5,5)\big).$$

From transitivity, we obtain

$$\big((10,10),(10,10)\big)\boldsymbol{P}(R_{\{1,2\}})\big((30,30),(0,0)\big).$$

By the same argument, we can have

$$\big((10,10),(10,10)\big)\boldsymbol{P}(R_{\{1,2\}})\big((0,0),(30,30)\big).$$

Thus, *WD* implies

$$\big((10,10),(10,10)\big)\boldsymbol{R}(R_{\{1,2\}})\big((30,0),(0,30)\big). \tag{9}$$

(5) and (6) together imply a contradiction. □

It is well known in the literature that there is a tension between *SD* and ex ante equity.[16] This fact and Theorem 5 mean that it is difficult to find equitable social criteria satisfying *SD*. If agents are risk averse and have nonconvex preferences, it is straightforward to obtain a stronger impossibility result by replacing *CPA* with a weaker ex post Pigou-Dalton transfer principle.

A crucial point is that, if an SOF satisfies *PREV*, an uncertain allocation in which agents have different outcomes is preferred to a riskless allocation where agents have equal outcomes, which leads to incompatibility with *SD* and *CPA*. One way to avoid this problem

[16] See Mongin and Pivato (2015b) .

is to restrict the Pareto principle to the case in which all agents have equal risk (*Pareto for Equal Risk*). This is examined by Fleurbaey and Zuber (2015) in an environment where agents' preferences are expected utility. They also study several combinations of efficiency and social rationality while maintaining consistency with ex post equity.

There is another problem if we require SOFs to be separable with respect to irrelevant agents.[17]

Theorem 6. There exists no SOF satisfying *WPER, CPA, WOS*, and *WD*.

Proof. Let us consider $S = \{1, 2\}$, $N = \{1, 2, 3\} \in \mathcal{N}$ and $R_1 = R_2 = R_3 = R^* \in \mathcal{R}$ such that $(0, 40)P^*(30, 0)I^*(0, 30)P^*(10, 10)$.

We first introduce allocations $\boldsymbol{v}_N, \boldsymbol{w}_N, \boldsymbol{x}_N, \boldsymbol{y}_N, \boldsymbol{z}_N$ such that

$$\boldsymbol{v}_1 = \boldsymbol{v}_2 = (30, 0), \boldsymbol{v}_3 = (0, 40);$$

$$\boldsymbol{w}_1 = \boldsymbol{w}_2 = (0, 30), \boldsymbol{w}_3 = (0, 40);$$

$$\boldsymbol{x}_1 = \boldsymbol{x}_2 = (30, 0), \boldsymbol{x}_3 = (40, 0);$$

$$\boldsymbol{y}_1 = \boldsymbol{y}_2 = (0, 30), \boldsymbol{y}_3 = (40, 0);$$

$$\boldsymbol{z}_1 = \boldsymbol{z}_2 = \boldsymbol{z}_3 = (10, 10).$$

By completeness, we consider two cases: (i) $\boldsymbol{y}_N \boldsymbol{R}(R_N)\boldsymbol{x}_N$; (ii) $\boldsymbol{x}_N \boldsymbol{P}(R_N)\boldsymbol{y}_N$.

(i) Suppose $\boldsymbol{y}_N \boldsymbol{R}(R_N)\boldsymbol{x}_N$. Since *WPER* implies $\boldsymbol{x}_N \boldsymbol{P}(R_N)\boldsymbol{z}_N$, we have $\boldsymbol{y}_N \boldsymbol{P}(R_N)\boldsymbol{z}_N$ by transitivity. It is straightforward, however, to obtain $\boldsymbol{z}_N \boldsymbol{P}(R_N)\boldsymbol{y}_N$ using *WD* and *CPA*, by the same argument as in Theorem 5.

(2) Suppose $\boldsymbol{x}_N \boldsymbol{P}(R_N)\boldsymbol{y}_N$. We have $\boldsymbol{x}_{\{1,2\}} \boldsymbol{P}(R_{\{1,2\}})\boldsymbol{y}_{\{1,2\}}$ by *WOS*. By *WS* again, we obtain $\boldsymbol{v}\boldsymbol{R}(R_N)\boldsymbol{w}$. Since *WPER* yields $\boldsymbol{w}_N \boldsymbol{P}(R_N)\boldsymbol{z}$, we have $\boldsymbol{v}_N \boldsymbol{P}(R_N)\boldsymbol{z}_N$ by transitivity. Again by the same argument as in Theorem 5, we can show $\boldsymbol{z}_N \boldsymbol{P}(R_N)\boldsymbol{v}_N$, which is a contradiction. □

[17] The discussion of Theorem 6 would be similar to that of Fleurbaey (2010, p. 666), but because our environment is different, we provide a proof for completeness.

In Theorem 6 we only require *WOS*, which is weaker than *SEP* and compatible with equity principles such as *DA*. The proof shows that if *WOS* is combined with *WPER*, we have an implication similar to *PREV* discussed above (after the proof of Theorem 5). Thus, we should relax *WS* if dynamic consistency is more compelling than separability. Fleurbaey and Zuber (2013) studied this issue in an environment. They characterized classes of social welfare functions using *WD, Pareto for Equal Risk*, and weaker separability conditions. It remains for future research to explore social welfare criteria satisfying *WD* and weaker separability requirements for our environment.

Although $\boldsymbol{R}_C$ and $\boldsymbol{R}_\kappa$ violate *WD*, those criteria satisfies the following ex ante equity condition.

Ex Ante Inequality Reduction. For all $R_N \in \mathcal{D}$, and all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, if there exist j, k such that $\boldsymbol{x}_i = \boldsymbol{x}'_i$ for all $i \neq j, k \in N$,

$$\left[U(\boldsymbol{x}'_i, R_i) \subset U(\boldsymbol{x}_i, R_i) \subseteq U(\boldsymbol{x}_j, R_j) \subset U(\boldsymbol{x}'_j, R_j)\right] \Longrightarrow \boldsymbol{x}_N \boldsymbol{R}(R_N) \boldsymbol{x}'_N.$$

This axiom requires that reducing inequality in terms of ex ante preferences should be socially acceptable. It can be checked, using the usual example (Diamond, 1967), that this requirement is not compatible with *WD*. Thus, the criteria derived from our axioms respect ex ante equity rather than dynamic consistency.

6 Concluding Remarks

In this paper, we studied welfare criteria for social decisions under uncertainty. Two criteria have been derived from the principles of equity, efficiency and separability. Our results show that separability principles (*SEP* and *WS*) have the strong implication that when those are combined with requirements of equity and efficiency (*WPER*, *SM*, and *TP*), we must have *XAP* and thus give up *DAT*. Hence, to obtain a social ordering satisfying *DAT*, we have introduced *WOS* and derived another maximin criterion. Our interpersonal comparison is based on certainty equivalence, which is reasonable in the context of risk and uncertainty.

We have also argued that our social criteria are not consistent with *Statewise Dominance*, but satisfy ex ante equity such as *Ex Ante Inequality Reduction*.

Our results cast doubt on separability in the form of *SEP* or *WOS*, as shown by Lemma 1 and Theorem 6. If we place more importance on principles of equity and dynamic consistency such as *DAT* and *WD*, other forms of separability are required. This issue has been considered in the environment of expected utility (Fleurbaey and Zuber, 2013; Fleurbaey et al., 2015), but as far as we know, there is no research investigating this issue in our environment.

In this paper, we considered individual preferences over state-contingent monetary outcomes. In general, however, state-contingent outcomes are multidimensional (e.g., Fleurbaey and Zuber, 2015). On this point, it would be straightforward to extend our results to the general case with minor modifications. We also emphasize that our results do not depend on the domain restriction to the subjective expected utility functions: The same results can be obtained in the domains of various preferences including non-expected utility functions.

We focused on the implications of the axioms rather than optimal allocations or public policies to implement the allocations. In future research, we intend to model social insurance systems, such as unemployment and disability insurance using modified versions of our social welfare criteria in the relevant models.

7 Appendix: Independence of Axioms

The following examples show none of the axioms in Theorem 1 is redundant.

Dropping WPER. Consider social ordering $\boldsymbol{R}^1$ defined as follows: For all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, $\boldsymbol{x}_N \boldsymbol{R}^1(R_N)\boldsymbol{x}'_N$ if and only if $\min_{i\in N} \min_{s\in S} x_{is} \geq \min_{i\in N} \min_{s\in S} x'_{is}$.

Dropping SM. Consider social ordering $\boldsymbol{R}^2$ defined as follows: For all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, (1) if $|N| \leq 3$, $\boldsymbol{x}_N \boldsymbol{R}^2(R_N)\boldsymbol{x}'_N$ if and only if $\min_{i\in N} C(\boldsymbol{x}_i, R_i) \geq \min_{i\in N} C(x'_i, R_i)$; (2) if $|N| > 3$, $\boldsymbol{x}_N \boldsymbol{R}^2(R_N)\boldsymbol{x}'_N$ if and only if $\min_{i\in N} C(x_i, R_i) \geq \min_{i\in N} C(\boldsymbol{x}'_i, R_i)$ when $R_i = R_j$ for all

$i, j \in N$; and $\boldsymbol{x}_N \boldsymbol{I}^2(R_N)\boldsymbol{x}'_N$ otherwise.

Dropping TP. Consider social ordering $\boldsymbol{R}^3$ defined as follows: For all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, $\boldsymbol{x}_N \boldsymbol{R}^3(R_N)\boldsymbol{x}'_N$ if and only if $\sum_{i\in N} C(\boldsymbol{x}_i, R_i) \geq \sum_{i\in N} C(x'_i, R_i)$.

Dropping WS. Consider $\boldsymbol{R}_\kappa$.

Dropping IRBC. Let $A(\boldsymbol{x}, R_i) = \inf\{c \in \mathbb{R}_+ | (2c, c, \cdots, c)R_i\boldsymbol{x}\}$ for each $\boldsymbol{x} \in X$ and $R_i \in \mathcal{R}$. Consider social ordering $\boldsymbol{R}^4$ defined as follows: Then, for all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, $\boldsymbol{x}_N \boldsymbol{R}^4(R_N)\boldsymbol{x}'_N$ if and only if $\min_{i\in N} A(\boldsymbol{x}_i, R_i) \geq \min_{i\in N} A(\boldsymbol{x}'_i, R_i)$

Now we show the independence of the axioms in Theorem 2. To drop *TP*, *SEP*, and *IRBC*, we can use $\boldsymbol{R}^3$, $\boldsymbol{R}_\kappa$, and $\boldsymbol{R}^4$ defined above, respectively.

Dropping WPER. Consider social ordering $\boldsymbol{R}^5$ defined as follows: For all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, $\boldsymbol{x}_N \boldsymbol{I}^5(R_N)\boldsymbol{x}'_N$.

The following examples show none of the axioms in Theorem 3 is redundant.

Dropping PRA. Consider social ordering $\boldsymbol{R}^1$ above.

Dropping PCR. Consider social ordering $\boldsymbol{R}^6$ defined as follows: For all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$, $x_N \boldsymbol{R}^6(R_N)x'_N$ if and only if $\min_{i\in N} \max_{j\in N} C(x_i, R_j) \geq \min_{i\in N} \max_{j\in N} C(x'_i, R_j)$.

Dropping DAT. Consider $\boldsymbol{R}_C$.

Dropping IRBC. Remember $A(\boldsymbol{x}, R_i) = \inf\{c \in \mathbb{R}_+ | (2c, c, \cdots, c)R_i\boldsymbol{x}\}$ for each $\boldsymbol{x} \in X$ and $R_i \in \mathcal{R}$. Consider social ordering $\boldsymbol{R}^7$ defined as follows: Then, for all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$,

(1) $x_N \boldsymbol{P}^7(R_N)x'_N$ if $\big(\kappa(\boldsymbol{x}_i, R_N)\big)_{i\in N} >_{lex} \big(\kappa(\boldsymbol{x}'_i, R_N)\big)_{i\in N}$, where $\geq_{lex}$ is the usual lexicographic ordering; (2) if $\big(\kappa(\boldsymbol{x}_i, R_N)\big)_{i\in N} =_{lex} \big(\kappa(\boldsymbol{x}'_i, R_N)\big)_{i\in N}$, then $\boldsymbol{x}_N \boldsymbol{R}^7(R_N)\boldsymbol{x}'_N$ if and only if $\min_{i\in N} A(\boldsymbol{x}_i, R_i) \geq \min_{i\in N} A(\boldsymbol{x}'_i, R_i)$.

Dropping WOS. Consider social ordering $\boldsymbol{R}^8$ defined as follows: For all $\boldsymbol{x}_N, \boldsymbol{x}'_N \in X^N$,

(1) If $\min_{i\in N} \kappa(\boldsymbol{x}_i, R_N) > \min_{i\in N} \kappa(\boldsymbol{x}'_i, R_N)$, then $\boldsymbol{x}_N \boldsymbol{P}^8(R_N)\boldsymbol{x}'_N$; (2) if $\min_{i\in N} \kappa(\boldsymbol{x}_i, R_N) = \min_{i\in N} \kappa(\boldsymbol{x}'_i, R_N)$, then $\boldsymbol{x}_N \boldsymbol{R}^8(R_N)\boldsymbol{x}'_N$ if and only if $|\{i | C(\boldsymbol{x}_i, R_i) = \min_{j\in N} \kappa(\boldsymbol{x}_j, R_N)\}| \geq |\{i | C(\boldsymbol{x}'_i, R_i) = \min_{j\in N} \kappa(\boldsymbol{x}'_j, R_N)\}|$.

The axioms in Theorem 4 are obviously independent.

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