Differential income taxation and excludable, congested public goods

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July 5, 2016

Abstract

This paper examines optimal nonlinear income taxes under the provision of excludable public goods with congestion when individuals differ in public goods preferences and earning abilities. We consider that the government implement the group-specific income tax schedules on two groups: one group benefits from a public good and the other does not. Our main argument is that the government redistributive tastes, the correlation of public goods preferences and earning abilities, and congestion effect are especially crucial in differentiating marginal income tax rates. In numerical simulations, we present how these factors affect the shape of the optimal differentiated marginal income tax rates using some social welfare functions.

JEL Classification: H20, H41
Keywords: Extensive margin, Optimal nonlinear income taxation, Excludable public goods, Congestion, Tagging

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1 Introduction

Imposing taxes on labor incomes plays an important role to not only improve income inequalities but also provide public goods. From this viewpoint, optimal income tax system have been examined in the presence of the provision of public goods (Boadway and Keen (1993)). Previous papers mainly focus on pure public goods that satisfy both non-excludable (everyone can access without charge) and non-rival (one individual does not affect availability to others). However, public goods are, more or less to some extent, excludable and/or rival. For example, the exclusion property of public goods are relevant for libraries, universities, health services, swimming pools, and highways, moreover, they are also subject to congestion. In this case, how should the government finance excludable public goods with congestion imposing user fees (payment to use public services and facilities) and labor income taxes?

The objective of our papers is to investigate how the difference of benefits from public goods should be reflected in the income tax schedule. We consider the economy in which consists of individuals who differ in public goods preferences and earning abilities, and make a labor supply decision and a binary decision whether to obtain benefits from services paying user fees. The exclusion property separates the population into two groups: one group benefits from a public good and the other does not. The government implements differential income tax schemes on two groups to maximize the social welfare while considering the labor supply decision at the intensive margin and the voluntary participation at the extensive margin.

The shape of the optimal differentiated marginal income tax rates dramatically changes depending on the government redistributive tastes and the correlation between public goods preferences and earning abilities. We analytically demonstrate that, if public goods preferences and earning abilities are independently distributed and the first derivative of the social welfare function is strictly convex, the marginal income tax rates on individuals who enjoy the public good are lower than the marginal income tax rates on individuals who do not obtain benefits from the consumption in the public good. In addition, even though the social welfare function is Rawlsian, this result holds. The reason why it is optimal is that, under such an assumption, the participation effect are always greater than the mechanical effect, which are factors of the total welfare effect due to a small tax reform. This implies that the government intend to make individuals to migrate into a group enjoying a public good by decreasing marginal income tax rates on the group since the tax burden on the group are heavier. Moreover, we numerically present the implication obtained by introducing the positive or negative correlation between public goods preferences and earning abilities in terms of some social welfare criteria.[1]

[1]It is observed that public goods preferences and income levels are correlated. According to empirical studies, there are some evidences which preferences for public goods are correlated with income levels. For example, Banzhaf and Walsh (2008) show that households migrate across regions based on environmental quality, and high income earner migrate into regions where air quality is good. Also, Pritchett and Summers(1997) find the positive relationship between income and health, which implies individuals with high income invest more

This paper draws from the growing body of literature examining separated income tax schedules on groups divided by observable characters, which is so-called ”tagging” (Akerlof (1978), Boadway and Pestieau (2007), Cremer et.al (2010), Immonen et al. (1998), Mankiew et al. and, Viard (2001)). In particular, Boadway and Pestieau (2007) and Cremer et.al (2010) analytically examine the optimal income taxation with tagging in the economy which consists of two groups, one of which has a higher proportion of high-ability individuals than the other, and conclude that, under the situation, the tax system with inter-group transfer will be more redistributive compared to the standard optimal taxation model of Mirrlees (1971) and Saez (2001).[2] The crucial difference is that we consider a variable category as the tag, which means that individuals have the decision making at the extensive margin as well as the intensive margin. In other words, the government need to pay attention to two kinds of distortions on individual's labor supply when implementing income taxation. In contrast, much of previous literatures has supposed that a tagged group are immutable, that is, individuals response along the intensive margin only.[3] Therefore, we aim to explore how responses along the extensive margin affect the differentiation of income taxation.

Our paper is part of large amount of papers dealing with the optimal nonlinear income taxes in random participation models with the multidimensional heterogeneity (Gomes et al. (2014), Kleven, Kreiner, and Saez (2009), Jacquet, Lehmann, and Van der Linden (2013), Kessing, Lipatov, and Zoubek (2014), Lehmann, Simula, and Trannoy (2014)). [4] The closest paper to ours in the setting is Kleven et. al (2009) who examine how the government should differentiate income tax schedules depending on whether the spouse works or not. They conclude that, if an ability and a work cost are independently distributed, the marginal income tax rates on households the spouse work are lower. Moreover, they numerically show that such a tax reform is the robust result which is not affected by the correlation between an ability and a work cost. Our paper differs in main two ways from their framework. First, applications of the findings in the present paper pertain to the design of optimal transfer program related to excludable public goods. In practical, differential income taxation on the basis of public goods is used as income tax deductions for higher educations or health care expenses. As a result, we suggest potentially implications for tax policy involved with public goods. Second, we assume the presence of crowding cost which leads to the restriction of group size. Under the circumstance, the optimal income tax schemes are influenced because the crowding cost weakens the incentive for the government to make individuals migrate into groups enjoying public goods. As a result, in the presence of crowding, we numerically show that the magnitude relation of separated marginal income tax rates can change. This

in public health compared to individuals with low income.

[2]Boadway and Pestieau (2007) assume two types of individuals, and then Cremer et.al (2010) extend Boadway and Pestieau (2007) to a model with a continuum of individuals.

[3]Indeed, previous literatures mainly consider demographic characters such as age or gender, or health condition like an illness or a disability as observable characters. In this case, individuals do not have the decision making along the extensive margin since they cannot change groups.

[4]These papers mainly consider that individuals differ in an ability and an participation cost such as migration cost or work cost. On the other hand, in our paper, individuals differ in an ability and public goods preferences.

is the main different theoretical contribution between Kleven et al. (2009) and the present paper. Moreover, we check whether the findings are robust using a variety of social welfare functions.

Using the mechanism design approach, several papers investigate how a public good should be financed from the viewpoint of distributive concerns under the provision of excludable public goods (see, e.g., Cremer and Laffont (2003); Hellwig (2004, 2005)). However, these studies restrict only to the lump-sum transfer. In contrast, introducing the notion of ”tagging” into a Mirrleesian framework of income taxation in the presence of public goods provision, the present study allows the government to differentiate income tax schedules. Moreover, our model is the extension of Hellwig (2004), however, we analyze the optimal level of admission fees with congestion. In this case, admission fees are always positive in the presence of congestion compared to Hellwig (2004). Given this, we show that the level of admission fees, that is, whether admission fees are greater than congested effect, crucially depends on redistibutive tastes and correlation.

This paper is organized as follows. In section 2, we describe the framework of the basic model. In section 3, we characterize income tax schedules with the lump-sum differentiation which is benchmark result in our papers. In section 4, we show that it is desirable to introduce differential income tax schemes, and we suggest numerical results in section 5. Finally, we present the conclusion in section 6.

2 The Model

Consider the economy which consists of individuals with their types (θ, w), where let θ and w be preferences for public goods and earnings abilities respectively according to $F(\theta, w)$ with strictly positive and continuously differentiable density function $f(\theta, w)$ over $[\underline{\theta}, \overline{\theta}] \times [\underline{w}, \overline{w}]$. We assume that $0 = \underline{\theta} < \overline{\theta} < \infty$ and $0 < \underline{w} < \overline{w} < \infty$. Indicator A represents a group in which people benefit from public goods and indicator B represents otherwise. As used in Diamond (1998), the utility function is described by the quasi-linear:

$$U_i = \theta G \cdot \mathbf{1}(i) + x_i - v(\ell_i) \qquad\qquad i = A, B \tag{1}$$

where let G, x and, ℓ denote public goods, private consumption, and labor supply. For the simplicity, we consider the disutility of labor v as $v(\ell) = \ell^{1+1/e}/(1 + 1/e)$, where e is the constant elasticity of labor supply with respect to the net-of-tax wage rate. Furthermore, budget constraints faced by individuals are given by $x_i = z_i - T_i(z_i)$, where z is labor income which is equal to $w\ell$, and $T_i(\cdot)$ is income tax function imposed on people who belong to group i.

2.1 Intensive Margin

Individuals in group i facing income tax $T_i(\cdot)$ maximize their utilities subject to their budget constraints. So, this problem is expressed by:

$$max\ U_i = \theta G \cdot \mathbf{1}(i) + x_i - v(z_i/w) \qquad s.t\ x_i = z_i - T_i(z_i)$$

Then, we get the following first order condition with respect to labor income z_i:

$$\frac{v'(z_i/w)}{w} = 1 - T_i'(z_i) \qquad \forall w\ i = A, B \tag{2}$$

As a result, we obtain the indirect utility function $\theta G + V_A(w) \equiv \theta G + x_A(w) - v(\ell_A(w))$ and $V_B(w) \equiv x_B(w) - v(\ell_B(w))$, where we let $x_i(w)$ and $\ell_i(w)$ be optimal value for any w.

2.2 Extensive Margin

Each group is endogenously determined since individuals can select to which a group to belong, which corresponds to the extensive margin. More specifically, the choice is made on the basis of the following constraint.

$$\hat{\theta}(w)G + V_A(w) = V_B(w) \qquad\qquad \forall w \tag{3}$$

where, let $\hat{\theta}(w)$ be continuously differentiable. This equation (3) is interpreted as follows. If preferences are higher than $\hat{\theta}(w)$, the persons with skill w participate into group A since they can obtain the higher utility than it in group B, that is, $\theta G + V_A(w) > V_B(w)$ holds. On the other hand, if preferences are lower than $\hat{\theta}(w)$, the people belong to group B because $\theta G + V_A(w) < V_B(w)$. Also, $\hat{\theta}(w)$ is an endogenous variable for the government.

2.3 The Government

We assume that the government can observe individuals income but not skill levels. Therefore, the government levies nonlinear differentiated income tax on each group to redistribute income and finance a public good. Therefore, budget constraint of the government is:

$$\int_{\underline{w}}^{\overline{w}} T_A(w) f_A(w) dw + \int_{\underline{w}}^{\overline{w}} T_B(w) f_B(w) dw = \phi(G, \left[\int_{\underline{w}}^{\overline{w}} f_A(w) dw\right]^{\alpha}) \tag{4}$$

where $f_A(w) := \int_{\hat{\theta}(w)}^{\overline{\theta}} f(\theta, w) d\theta$ and $f_B(w) := \int_{\underline{\theta}}^{\hat{\theta}(w)} f(\theta, w) d\theta$, and ϕ is the strictly increasing, strictly convex, and continuously differentiable cost function for public goods. Also, α is the congestion parameter and we assume $\alpha = 1$ indicating a public good is equivalent to a private good.

We mainly focus on the Bergson-Samuelson criterion which is represented as follows.

$$\mathcal{W} := \int_{\underline{w}}^{\overline{w}} \left[\int_{\hat{\theta}(w)}^{\overline{\theta}} W(\theta G + V_A) f(\theta, w) d\theta + \int_{\underline{\theta}}^{\hat{\theta}(w)} W(V_B) f(\theta, w) d\theta \right] dw \tag{5}$$

where W is concave function, that is, $W' > 0$ and $W'' \leq 0$.

Moreover, we assume the following social criteria to examine the special case in which our results are clarified more. First of all, Benthamite social preferences reflecting that the government maximize the sum of utilities is expressed as follows:

$$\mathcal{W}^{\mathcal{B}} := \int_{\underline{w}}^{\overline{w}} \left[\int_{\hat{\theta}(w)}^{\overline{\theta}} (\theta G + V_A) f(\theta, w) d\theta + \int_{\underline{\theta}}^{\hat{\theta}(w)} V_B f(\theta, w) d\theta \right] dw \tag{6}$$

Second, Rawlsian social preferences that the goal of the government is to maximize the worst-off individuals utility is as follows:

$$\mathcal{W}^{\mathcal{R}} := min[\hat{\theta}(\underline{w})G + V_A(\underline{w}),\ V_B(\underline{w})] \tag{7}$$

Finally, we consider weighted utilitarian preferences indicating a weighted sum of utilities. The social criterion is as follows:

$$\mathcal{W}^{\mathcal{W}} := \int_{\underline{w}}^{\overline{w}} \left[\int_{\hat{\theta}(w)}^{\overline{\theta}} \beta(w)(\theta G + V_A) f(\theta, w) d\theta + \int_{\underline{\theta}}^{\hat{\theta}(w)} \beta(w) V_B f(\theta, w) d\theta \right] dw \tag{8}$$

where, let $\beta(w)$ denote the welfare weight, and it decreases for w.[5] In the second best environment, by the revelation principle, it suffices to make people reveal their true types for earning ability to maximize the objective of the government. As shown by Mirrlees (1971), the first-order incentive compatibility constraint is given by[6]:

$$V_i'(w) = \frac{\ell_i(w)}{w} v'(\ell_i(w)) \qquad \forall w\ i = A, B \tag{9}$$

This is the necessary condition to meet the incentive compatibility constraint. Hereafter, we assume that the sufficient condition is satisfied, that is, the Spence-Mirrlees condition and monotonicity conditions hold[7].

[5] In the case of weighted utilitarian preferences, the welfare weight depends on not only earning abilities but also preferences for public goods, that is, it may be expressed by $\beta(\theta, w)$. However, following by the ethical consideration in Fleurbaey and Maniquet (2006), we assume that the welfare weight depends only on earning abilities. For example, Boadway, Marchand, Pestieau, and del Mar Racionero (2002), and Lockwood and Weinzierl (2014) investigate the optimal income taxation problem under such a condition in multidimensional heterogeneity setting.

[6] Since the government can observe groups to which individuals belong, they cannot mimic others in the other group. Therefore, incentive compatibility constraints are applied only within groups.

[7] In our models, the Spence-Mirrlees condition is satisfied because the marginal rate of substitution $\frac{v'(\ell_i)}{w}$ is decreasing for w.

3 Benchmark analysis: The lump-sum differentiation

First, as a benchmark case, we analyze the case in which the government implement the tax system with lump-sum differentiation, i.e. a tax system where the marginal income tax rates are carried out so that $T'_A = T'_B$ for all w holds. Thus, under the tax system, the incentive constraint in each group coincides because the labor supply is same from the assumption of the quasilinear preferences. Furthermore, equation (3) is transformed as follows.

$$T_A - T_B = \hat{\theta}(w)G + \int_{z_B}^{z_A} (1 - v'(\frac{z}{w})/w)dz \ \forall w \tag{10}$$

This equation describes the difference of tax paid to satisfy equation (3). If $T'_A = T'_B$ holds, the second term in the right hand side vanishes. Additionally, $\hat{\theta}(w)$ is constant on w because we can get the following by differentiating equation (3) with respect to w and substituting equation (2) and (9) for it.

$$\hat{\theta}'(w) = \frac{w^e\Big[(1 - T'_B)^{e+1} - (1 - T'_A)^{e+1}\Big]}{G} \tag{11}$$

Thus, if $T'_A = T'_B$ holds, $\hat{\theta}(w)$ is constant on w, denoted by $\hat{\theta}$. As a result, the difference of T_A and T_B is constant, and equation (10) reduces to $T_A - T_B = \hat{\theta}G$. Substituting this into budget constraint (4), the following is obtained.

$$\int_{\hat{\theta}}^{\bar{\theta}} \hat{\theta}Gf(\theta)d\theta + \int_{\underline{w}}^{\overline{w}} T_B(w)f(w)dw = \phi(G, \int_{\hat{\theta}}^{\bar{\theta}} f(\theta)d\theta) \tag{12}$$

Moreover, substituting this into the social welfare function (5), the following denoted by $\hat{\mathcal{W}}$ is obtained.

$$\hat{\mathcal{W}} := \int_{\underline{w}}^{\overline{w}} \Big[\int_{\hat{\theta}}^{\bar{\theta}} W(\theta G - \hat{\theta}G + V_B)f(\theta, w)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W(V_B)f(\theta, w)d\theta \Big] dw \tag{13}$$

If the government cannot use tagging, the problem for the government is to choose $V_B(w)$, $\ell_B(w)$, G and $\hat{\theta}$ to maximize social welfare function (5) subject to the budget constraint (12) and incentive constraints (9) for group B. Therefore, the optimization problem is formulated as follows:

$$\begin{gathered} max_{V_B(w), \ell_B(w), G, \hat{\theta}} \quad \hat{\mathcal{W}} \quad s.t. \quad V'_B(w) = \frac{\ell_B(w)}{w} v'(\ell_B(w)) \quad and \\ \int_{\hat{\theta}}^{\bar{\theta}} \hat{\theta}Gf(\theta)d\theta + \int_{\underline{w}}^{\overline{w}} T_B(w)f(w)dw = \phi(G, \int_{\hat{\theta}}^{\bar{\theta}} f(\theta)d\theta) \end{gathered} \tag{14}$$

The corresponding Lagrangian is:

$$
\begin{aligned}
\mathcal{L} = \hat{\mathcal{W}} + \gamma\Big[\int_{\hat{\theta}}^{\bar{\theta}} \hat{\theta} G f(\theta) d\theta + \int_{\underline{w}}^{\overline{w}} T_B(w) f(w) dw - \phi(G, \int_{\hat{\theta}}^{\bar{\theta}} f(\theta) d\theta)\Big] \\
+ \int_{\underline{w}}^{\overline{w}} \lambda_B(w)\Big[\frac{\ell_B(w)}{w} v'(\ell_B(w)) - V_B'(w)\Big] dw
\end{aligned}
\tag{15}
$$

where let γ be the Lagrangian multiplier on the resource constraint, $\lambda_B(w)$ be the co-state variable on the incentive constraint. Before investigating results, it is useful to define the following in advance: g_A and g_B. These are the average social marginal welfare weight for people with skill w in each group that are given by:

$$
g_A := \frac{\int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G + V_A) f(\theta|w) d\theta}{\gamma f_A^c}, \; g_B := \frac{\int_{\underline{\theta}}^{\hat{\theta}} W'(V_B) f(\theta|w) d\theta}{\gamma f_B^c}.
$$

These reflect preferences for redistribution between group A and group B, as shown later. Put it differently, the optimal redistribution between groups crucially depends on the degree of g_A and g_B.

The first-order conditions are shown in the Appendix, and the following proposition provides conditions for the optimal marginal income tax rates, provision of public goods and the difference of tax paid.

Proposition 1. *Under lump-sum differentiation, the optimal rules for marginal income tax rates and the difference of tax paid, and the modified Samuelson condition are characterized by:*

$$
\frac{T'}{1-T'} = \Big[1 + \frac{1}{e}\Big] \frac{1}{w f(w)} \int_{w}^{\overline{w}} [1 - \overline{g}(x)] f(x) dx \tag{16}
$$

$$
\underbrace{\frac{T_A - T_B - \phi_{f_A}}{G} f(\hat{\theta})}_{The\ efficiency\ term} = \underbrace{(1 - F(\hat{\theta})) \int_{\underline{w}}^{\overline{w}} g_B f_B^c(w) f(w) dw - F(\hat{\theta}) \int_{\underline{w}}^{\overline{w}} g_A f_A^c(w) f(w) dw}_{The\ equity\ term} \tag{17}
$$

$$
\frac{\int_{\underline{w}}^{\overline{w}} \int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G + V_A)(\theta - \hat{\theta}) f(\theta, w) d\theta dw}{\gamma} + \hat{\theta}(1 - F(\hat{\theta})) = \phi_G \tag{18}
$$

where, $T' := T_A' = T_B'$, $\overline{g} := f_A^c \cdot g_A + f_B^c \cdot g_B$ and $f_A^c := \int_{\hat{\theta}(w)}^{\bar{\theta}} f(\theta|w) d\theta$ and $f_B^c := \int_{\underline{\theta}}^{\hat{\theta}(w)} f(\theta|w) d\theta$, that is, these are the conditional cumulative distribution of preferences for public goods at a given skill level for each group.

Equation (16) is the traditional optimal marginal income tax rate formula obtained by Mirrlees (1971). Equation (17) is the equity-efficiency tradeoff in implementing lump-sum transfer, which means that this equation expresses how the level of lump-sum differentiation is determined. Subsequently, we demonstrate that this level depends on the government redistributive tastes and the correlation of public goods preferences and earning abilities. Equation (18) is the modified Samuelson rule for the public good. The second term in the left hand side in equation (18) expresses the effect that appears due to the government's redistributional concerns, and the sign of this term is determined by equation (17). If this term is positive, the original Samuelson public provision level is distorted downward. On the other hand, if this term is zero, the original Samuelson rule for the public good is achieved.

To help intuition why tax paid is differentiated, we provide the heuristic derivation for equation (17). We suppose the situation in which the government uniformly increase the tax on people in group A and decrease the tax on people in group B. Let dT_A and dT_B be a small tax reform for each group, and these reform are inversely proportional to population with public goods preferences in each group. That is, $dT_A = \frac{dT}{1-F(\hat{\theta})}$ and $dT_B = -\frac{dT}{F(\hat{\theta})}$. First, a small reform so that tax paid in group A increases and tax paid in group B decreases distorts the migration decision making. This leads to results that people in group A emigrate into group B, which amounts to the size of $f(\hat{\theta})d\hat{\theta}$. Since these movers additionally pay the amount of $T_A - T_B$, the decrease of tax revenue occurs. On the other hand, the migration from group A decreases the marginal cost of externalities ϕ_{f_A}. As a result, we can express this effect as follows.

$$dP := -(T_A - T_B - \phi_{f_A})f(\hat{\theta})d\hat{\theta}$$

Moreover, because $d\hat{\theta} \cdot G = dT_A - dT_B$ holds, by rearranging, we can get the following.

$$dP := -\frac{T_A - T_B - \phi_{f_A}}{G}\frac{f(\hat{\theta})}{F(\hat{\theta})(1-F(\hat{\theta}))}dT$$

Second, a small perturbation that changes tax burden in each group purely affects tax revenues, and the net mechanical effect is measured as follows.

$$dM^A := \int_{\underline{w}}^{\overline{w}}(1-g_A)f_A(x)dx \times dT_A = \frac{1}{1-F(\hat{\theta})}\int_{\underline{w}}^{\overline{w}}(1-g_A)f_A(x)dx \times dT \tag{19}$$

$$dM^B := \int_{\underline{w}}^{\overline{w}}(1-g_B)f_B(x)dx \times dT_B = -\frac{1}{F(\hat{\theta})}\int_{\underline{w}}^{\overline{w}}(1-g_B)f_B(x)dx \times dT \tag{20}$$

To sum up, the optimal threshold must satisfy $dP + dM^A + dM^B = 0$. Thus, rearranging this, we can get the heuristic derivation in equation (17).

Equation (17) just expresses an equity-efficiency tradeoff. The left hand side corresponds to efficiency term, which means that implementing lump-sum transfer results in the reduction of externalities of consumption in a public good. The right hand side corresponds to equity term and represents the redistributive tastes for government. From equation (17), the optimal user fees are not zero because of existing externalities from public goods, which corresponds to economic rationales for pricing on a public good, that is, $T_A > T_B$ holds. Indeed, if we evaluate equation (17) at $\hat{\theta} = 0$, the efficiency term is negative while the equity term is zero. Thus, equation (17) is not satisfied.

The next interest is how the level of lump-sum differentiation is determined. The equity term in the right hand side clarifies. If the first term is larger than the second term in the right hand side, that is, the equity term is positive, the optimal level of lump-sum differentiation are levied exceeding the cost of externalities. This implies that the government prefers to redistribute tax revenues obtained by imposing on group A more than necessary to group B. Indeed, from equation (12), if the equity term is positive, $\int_{\underline{w}}^{\overline{w}} T_B(w)f(w)dw < \phi(G)$ holds, which implies that tax burden in group B reduces implementing lump-sum transfer exceeding the cost of externalities, i.e. $T_A - T_B > \phi_{f_A}$.

The correlation between θ and w and the social welfare function which are factors of the right hand side in Equation (17) crucially plays a important role in determining the level of lump-sum differentiation. Here, to understand the interpretations of the results, we consider some special cases. First, if θ and w are independently distributed and the social welfare is strictly concave function, equation (17) is transformed as follows.

$$\frac{T_A - T_B - \phi_{f_A}}{G} f(\hat{\theta}) = F(\hat{\theta})(1 - F(\hat{\theta})) \int_{\underline{w}}^{\overline{w}} (g_B - g_A) f(w)dw \tag{21}$$

In this case, the equity term is positive because $g_B > g_A$ holds due to the concavity of the social welfare function and the migration constraint (3). Therefore, we can summarize the statement above as follows.

Corollary 1. *If θ and w are independently distributed and the social welfare is strictly concave function, the level of lump-sum transfer exceeds the marginal cost of externalities from public goods.*

However, if the social welfare is described by weighted utilitarian as in equation (8), the level is zero from equation (21) since the government's redistributive taste is indifferent, i.e. $g_A = g_B$. Thus, the optimal level of lump-sum transfer is just equal to the cost of externalities.

Second, regardless the correlation between tastes of public goods and earning abilities, if the social welfare is Benthamite case as in equation (6), the equity term is zero because $g_B = g_A = 1/\gamma$. This means that the government does not intend to impose lump-sum differentiation exceeding the congestion to reinforce the redistribution. Thus, the optimal level of lump-sum transfer is just equal to the cost of externalities, i.e. $T_A - T_B = \phi_{f_A}$, which leads to $\int_{\underline{w}}^{\overline{w}} T_B(w)f(w)dw = \phi(G)$. On the other hand, under Rawlsian case as in equation (7), equation (17) is transformed as follows.

$$\frac{T_A - T_B - \phi_{f_A}}{G} f(\hat{\theta}) = (1 - F(\hat{\theta})) > 0 \tag{22}$$

Intuitively, the improvement of the welfare due to the increase of the consumption of the worst-off individuals is valued more than the efficiency loss that is caused by imposing exceeding externalities. Therefore, we can summarize the statement above as follows.

Corollary 2. *If the social welfare is Benthamite criteria, the optimal level of lump-sum transfer is just equal to the cost of externalities. On the other hand, if the social welfare is Rawlsian criteria, the optimal level of lump-sum transfer exceeds the cost of externalities.*

Consequently, whether the optimal level of lump-sum transfer exceeds the marginal cost of externalities from public goods depends on the correlation of θ and w and redistributive tastes for the government.

4 Introducing the differential tax rates

In this section, we now turn to the analysis of introducing a differentiation of income tax schemes. In this case, because the marginal income tax rates are separated, $\hat{\theta}(w)$ can depend on w from equation (11). If the government is able to carry out the differentiated income tax schedules, the problem for the government is to choose $V_i(w)$, $\ell_i(w)$ for $i = A, B, G$, and $\hat{\theta}(w)$ to maximize social welfare function (5) subject to the budget constraint (4) and incentive constraints (9) and migration constraint (3). Therefore, the optimization problem under labor mobility is formulated as follows:

$$\begin{aligned} &max_{V_i(w),\ell_i(w)} \quad \mathcal{W} \quad s.t. \quad V_i'(w) = \frac{\ell_i(w)}{w}v'(\ell_i(w)), \quad \hat{\theta}(w)G + V_A(w) = V_B(w) \quad and \\ &\int_{\underline{w}}^{\overline{w}} T_A(w)f_A(w)dw + \int_{\underline{w}}^{\overline{w}} T_B(w)f_B(w)dw = \phi(G, \int_{\underline{w}}^{\overline{w}} f_A(w)dw) \end{aligned} \tag{23}$$

The corresponding Lagrangian is:

$$\begin{aligned} \mathcal{L} = \mathcal{W} + \gamma\Bigg[&\int_{\underline{w}}^{\overline{w}} T_A(w)f_A(w)dw + \int_{\underline{w}}^{\overline{w}} T_B(w)f_B(w)dw - \phi(G, \int_{\underline{w}}^{\overline{w}} f_A(w)dw) \\ &+ \sum_{i=A,B}\int_{\underline{w}}^{\overline{w}} \lambda_i(w)\Bigg[\frac{\ell_i(w)}{w}v'(\ell_i(w)) - V_i'(w)\Bigg]dw + \int_{\underline{w}}^{\overline{w}} \mu(w)\Big[\hat{\theta}(w)G + V_A(w) - V_B(w)\Big] \end{aligned} \tag{24}$$

where let γ be the Lagrangian multiplier on the resource constraint, $\lambda_i(w)$ be the co-state variable on the incentive constraint, and $\mu(w)$ be the co-state variable on the migration constraint. The first-order conditions are given in the Appendix, and the differentiated marginal income tax rates at the optimum for each group are obtained by rearranging them.

Proposition 2. *The optimal marginal income tax rates for each group are characterized by:*

$$\frac{T_A'}{1-T_A'} = \Bigg[1+\frac{1}{e}\Bigg]\frac{1}{wf_A(w)}\int_{w}^{\overline{w}}\Bigg[(1-g_A)f_A^c(x) - \frac{T_A - T_B - \phi_{f_A}}{G}f(\hat{\theta}(x)|x)\Bigg]f(x)dx \tag{25}$$

$$\frac{T_B'}{1-T_B'} = \Bigg[1+\frac{1}{e}\Bigg]\frac{1}{wf_B(w)}\int_{w}^{\overline{w}}\Bigg[(1-g_B)f_B^c(x) + \frac{T_A - T_B - \phi_{f_A}}{G}f(\hat{\theta}(x)|x)\Bigg]f(x)dx \tag{26}$$

These formulas describe the optimal marginal nonlinear income tax rates for each group when the government apply tagging to the tax system, which is consistent with previous literatures examining the differential tax system with random participation model. The slightly difference is that the congestion effect ϕ_{f_A} enter the formula for the marginal income tax rates, which this positively (negatively) works to tax rates on group A (group B). The result is very intuitive. If the congestion exists, the government reforms the income tax schemes so that the marginal tax rates in group A increase to improve efficiency loss and makes people in group A to migrate into group B. Therefore, in contrast to previous studies, the marginal income tax rates are modified so that it includes the efficiency loss. This implies that, even if the government can extract the additional revenue $T_A - T_B$ by making people moving into group A, it must determine the optimal income tax schedules considering the efficiency loss due to the capacity of a public good.

The social welfare criteria affect the differentiation of the marginal income tax rates. If the objective function for the government is the Bergson-Samuelson criterion, marginal income tax rates in group i depends on redistributive tastes g_i for the government, which leads to make marginal income tax rates in group A to be higher compared to one in group B because redistributive tastes in group B may be higher than one in group A due to benefits from public goods. In other words, the government intend to redistribute income from group A to group B imposing more on high income earner in group A. On the other hand, if the social welfare function is weighted utilitarian preferences, such a differentiation is vanished because of the ethical consideration.

Furthermore, it is shown that, in our model, the property of the optimal differentiated marginal income tax rates obtained by Cremer et. al (2010) holds.

Corollary 3. *(i) The optimal marginal income tax rates without tagging faced by the w-type individuals are bracketed by the optimal marginal nonlinear income tax rates with Tagging faced by the w-type individuals in each groups:*

$$\frac{T'_M}{1-T'_M} := \left[1+\frac{1}{e}\right]\frac{1}{wf(w)}\int_w^{\overline{w}}[1-\overline{g}(x)]f(x)dx = \frac{T'_A}{1-T'_A}\frac{f_A(w)}{f(w)} + \frac{T'_B}{1-T'_B}\frac{f_B(w)}{f(w)} \tag{27}$$

(ii) If $T'_A > T'_B$, then $T'_A > T'_M > T'_B$, and vice versa.

The first result is that the marginal income tax rates without tagging coincides with a weighed average of the marginal income tax rates for each group. Also, the second result is that, if applying tagging, one is imposed higher marginal income tax rates compared to the marginal income tax rates without tagging, the other is levied lower marginal income tax rates compared to the marginal income tax rates without tagging. These results remain under the Bergson-Samuelson criterion.

4.1 The direct proof of the optimal marginal nonlinear income tax rates under labor mobility

We give an intuitive interpretation by characterizing the optimal marginal nonlinear income tax rates by means of direct derivation as in Saez (2001). We consider the situation in where the government conduct the slight rise of the marginal income tax rates dT'_A for individuals in group A whose income levels distribute over a small interval $[z_A, z_A + dz]$ fixing T'_B. Similarly, the derivation of the optimal marginal income tax rates in group B is also applied. This small tax reform causes the following three effects: Mechanical effect, Behavioral responses, and Participation effect.

4.1.1 *Mechanical effect net of welfare loss*

The rise of the marginal income tax rates purely raise tax revenues. Since individuals in group A whose earnings abilities are more than w_{z_A} must pay the additional payment, thus added net tax receipts amount to:

$$\varrho_M := \int_{w_{z_A}}^{\overline{w}} (1 - g_A) f_A(x) dx \times dT'_A \times dz_A \tag{28}$$

where, w_{z_A} is the ability of individuals who earn income z_A.

4.1.2 *Behavioral responses*

Behavioral responses are interpreted as substitution effect because the decision making in terms of labor supply is distorted by the increase of the marginal income tax rates. As a result, tax receipts decrease since tax base decreases due to the reduction of labor supply. To measure this effect, we rearrange the change of income levels owing to a small change of the marginal income tax rates, that is, $\frac{dz_A}{dT'_A}$ as follows:

$$dz_A = -\frac{z_A}{1 - T'_A(z_A)} \times \xi \times dT'_A \tag{29}$$

where, $\xi := \frac{1-T'_A(z_A)}{z_A} \frac{\partial z_A}{\partial 1-T'_A(z_A)}$. Substituting equation (13) for $dT(z_A) = T'(z_A) dz_A$, we can get:

$$dT(z_A) = -T'(z_A) \frac{z_A}{1 - T'_A(z_A)} \times \xi \times dT'_A \tag{30}$$

Let ϱ_B be the total reduction brought about by Behavioral responses. Thus ϱ_B is equal to $dT(z_A) \times \int_{\hat{\theta}(w_{z_A})}^{\overline{\theta}} f(\theta, w_{z_A}) d\theta d\hat{w}$ because individuals whose skill levels are within the interval $[w_{z_A}, w_{z_A} + d\hat{w}]$ that are affected by the substitution effect. Using the fact that $d\hat{w} = \frac{dz_A}{(1+e)\ell}$ and

$\xi = e$, we can obtain[8]:

$$\varrho_B = -\frac{T'(z_A)}{1 - T'_A(z_A)} \times \frac{e}{1+e} \times w_{z_A} \int_{\hat{\theta}(w_{z_A})}^{\bar{\theta}} f(\theta, w_{z_A}) d\theta \times dT'_A \times dz_A \tag{31}$$

4.1.3 *Participation effect*

Unlike the traditional literatures, our models allow individuals to select group they belong to. The increase of marginal tax rates dT'_A causes the welfare loss measured by μ since those who are on the threshold migrate to group B. μ is interpreted as the value for special benefits that individuals on the threshold can additionally pay to migrate into attractive region A. Therefore, the total cost in terms of welfare loss is $-\int_{w_{z_A}}^{\overline{w}} \mu(x)dx$ and the decrease of tax receipts due to participation effect is as follows.

$$\varrho_P = -\int_{w_{z_A}}^{\overline{w}} \mu(x)dx \times dT'_A \times dz_A \tag{32}$$

As a whole, three effects above need to be offset at the optimum, accordingly $\varrho_M + \varrho_B + \varrho_P = 0$ must hold. Rearranging this equation, we can get results in terms of the differentiated optimal marginal income tax rates in Proposition 2.

4.2 The tax perturbation method: the suboptimality of lump-sum differentiation

Starting from the tax system with lump-sum differentiation, we introduce a little bit of tax reform to examine how different tax schedules should be. Let dT_A and dT_B be a small tax reform for each group above some skill level w. Moreover, these reforms are inversely proportional to population with skill w in each group, thus, $dT_A = -\frac{dT}{f_A(w)}$, $dT_B = \frac{dT}{f_B(w)}$.

We suppose the situation in which the government applies negative jointness to tax system. We omit the derivation in the case of positive jointness since we can get by a symmetric way. An implementation of tax policy affects the extensive margin decision making, which causes the effect to the revenue which is measured by $-(T_A - T_B)$. In addition, the marginal cost of externalities ϕ_{f_A} occurs. By carrying out tax policy, the number of people who face these effects amounts to the size of $f(\hat{\theta}(x), x)d\hat{\theta}(x)$. As a result, we formulate this effect as follows.

$$dP := -(T_A - T_B - \phi_{f_A}) f(\hat{\theta}(x), x) d\hat{\theta}(x)$$

[8] By the assumption of quasi-linear preferences, equation (2) is reduced to the following:

$$\ell_A = w^e (1 - T'_A)^e$$

That is, the optimal labor supply is characterized above. Hence, we can derive $d\hat{w} = \frac{dz_A}{(1+e)\ell}$ and $\xi = e$.

Moreover, because $d\hat{\theta}(x) \cdot G = dT_A - dT_B$ holds, by rearranging, we can get the following.

$$dP := \Big(\frac{1}{f_A(w)} + \frac{1}{f_B(w)}\Big)\Big(\frac{T_A - T_B - \phi_{f_A}}{G}\Big) f(\hat{\theta}(x), x) \times dT$$

Therefore, the total effect to the extensive margin above skill w is expressed as follows.

$$dP := \Big(\frac{1}{f_A(w)} + \frac{1}{f_B(w)}\Big) \int_w^{\overline{w}} \Big(\frac{T_A - T_B - \phi_{f_A}}{G}\Big) f(\hat{\theta}(x), x) dx \times dT \tag{33}$$

Second, a small perturbation that changes tax burden in each group purely affects tax revenues, and the net mechanical effect is measured as follows.

$$dM_A := \int_w^{\overline{w}} (1 - g_A) f_A(x) dx \times dT_A = -\frac{1}{f_A(w)} \int_w^{\overline{w}} (1 - g_A) f_A(x) dx \times dT \tag{34}$$

$$dM_B := \int_w^{\overline{w}} (1 - g_B) f_B(x) dx \times dT_B = \frac{1}{f_B(w)} \int_w^{\overline{w}} (1 - g_B) f_B(x) dx \times dT \tag{35}$$

Final, a small perturbation affects the behavior of individuals with skill level around some w at the intensive margin. As with the derivation of behavioral responses in section 4.1.2, we can describe this effect around skill w.

$$dB_A := -\frac{T'_A}{1 - T'_A}\frac{e}{1+e} w f_A(w) \times dT_A = \frac{T'_A}{1 - T'_A}\frac{e}{1+e} w \times dT \tag{36}$$

$$dB_B := -\frac{T'_B}{1 - T'_B}\frac{e}{1+e} w f_B(w) \times dT_B = -\frac{T'_B}{1 - T'_B}\frac{e}{1+e} w \times dT \tag{37}$$

From starting the separable tax system with only lump-sum differentiation, which means that $T'_A = T'_B$ and $T_A - T_B = \hat{\theta}G$ is constant for all w, the effect to behavioral responses cancel out. Hence, the net welfare effect owing to negative jointness or positive jointness is represented as follows.

$$\begin{aligned} dW &= dP + dM_A + dM_B \\ &= \mathrm{sgn}(i) \cdot \Bigg[\underbrace{\Big(\frac{1}{f_A(w)} + \frac{1}{f_B(w)}\Big) \int_w^{\overline{w}} \frac{\hat{\theta}G - \phi_{f_A}}{G} f(\hat{\theta}, x) dx}_{The\ participation\ effect} \\ &\quad \underbrace{-\frac{1}{f_A(w)} \int_w^{\overline{w}} (1 - g_A) f_A(x) dx + \frac{1}{f_B(w)} \int_w^{\overline{w}} (1 - g_B) f_B(x) dx}_{The\ mechanical\ effect\ above\ w\ due\ to\ tagging}\Bigg] \times dT \end{aligned} \tag{38}$$

where, if i is negative jointness, the indicator is equal to 1, on the other hand, it is equal to -1 otherwise. In the traditional literature that considers a tagged group as exogenous one, whether to separate income tax schedules depends only on the mechanical effect term.

However, in our models, the change of tax system due to tagging distorts the decision making at extensive margin. Hence, tax revenues are influenced by the participation effect.

The sign of the direct welfare effect dW strongly depends on the government redistributive tastes and the correlation of public goods preferences and earning abilities. Because it is so difficult to determine the sign of dW in the general, we consider no correlation between θ and w as the special case, which clearly gives the magnitude relation of the marginal income tax rates. As a result, we show that, if θ and w are independently distributed and the first derivative of the social welfare function is strictly convex, the negative jointness is always desirable since the participation effect is always greater than the mechanical effect. If θ and w are independently distributed, dW is transformed as follows.

$$dW = \mathrm{sgn}(i) \cdot \frac{1}{f(w)} \left[\frac{\hat{\theta} G - \phi_{f_A}}{G} \frac{f(\hat{\theta})}{F(\hat{\theta})(1 - F(\hat{\theta}))} (1 - F(w)) - \int_{w}^{\overline{w}} (g_B - g_A) f(w) dw \right] \tag{39}$$

Moreover, substituting equation (17) into equation (39) and rearranging, we can get the following:

$$dW = \mathrm{sgn}(i) \cdot \frac{1}{f(w)} \left[(1 - F(w)) \int_{\underline{w}}^{w} (g_B - g_A) f(w) dw - F(w) \int_{w}^{\overline{w}} (g_B - g_A) f(w) dw \right] \tag{40}$$

As shown in Appendix, dW is positive under the assumption of the first derivative of the social welfare function is strictly convex. Thus, the following statement holds.

Proposition 3. *If θ and w are independently distributed and the first derivative of the social welfare function is strictly convex, starting from the tax system with lump-sum differentiation, the social welfare increases introducing negative jointness for any w.*

Our tax perturbation result is consistent with Kleven et al. (2009) who show that the negative jointness is desirable when the characteristics are independently distributed and the objective of the government is a standard social welfare preference such as CRRA form. On the other hand, if the objective function is weighted utilitarian or Benthamite preferences, dW is zero due to $g_A = g_B$, that is, the differentiation of income tax schedules is redundant. However, if it is Rawlsain preference, the negative jointness is desirable. This is because the participation effect is positive from equation (22) while the mechanical effect is zero from the property of Rawlsian preferences, that is, dW is positive.

Corollary 4. *If θ and w are independently distributed and the social welfare function is Rawlsian, starting from the tax system with lump-sum differentiation, the social welfare increases introducing negative jointness for any w.*

However, we do not know whether this result remains even if it allows for the correlation of θ and w. To confirm how the income tax schemes at the optimum are differentiated under a variety of situations, we exercise numerical simulations in the next section.

5 Numerical examples

To illustrate our results more clearly, we now exercise a simulation. The goal of this section is as follows. First, if public goods preferences and earning abilities are independently distributed, we show that the tax perturbation analysis that the negative jointness is desirable is implemented at the optimum. Second, when the correlation between public goods preferences and earning abilities is allowed, we confirm whether the negative jointness result is robust. Third, we examine how the choice of the social welfare function affect the optimal differentiated marginal income tax rates.

In the simulation, we set the following assumptions. First, we assume that the Bergson-Samuelson criterion is CRRA with coefficient of risk aversion $\pi > 0$, which is expressed by $W = V^{1-\pi}/(1-\pi)$. We consider $\pi = 2$. Second, we assume that the constant elasticity of labor supply with respect to the net-of-tax wage rate $e = 0.5$. Third, following by Kleven et al. (2009), public goods preferences θ are distributed as a power function $F(\theta) = (\theta/\overline{\theta})^{\eta}$ with the density function $\eta \cdot \theta^{\eta-1}/\overline{\theta}^{\eta}$ on the interval $[0, \overline{\theta} = 3]$ with $\eta = 0.5$. Fourth, we assume that the density function for earning abilities is $f(w) = [1/\sqrt{2} - 1][1/2\sqrt{w}]$ on the interval $[\underline{w} = 1, \overline{w} = 2]$. Finally, we assume that the cost function for the public good is the following functional form: $\phi(G, \int_{\underline{w}}^{\overline{w}} f_A(w)dw) = G\left[\int_{\underline{w}}^{\overline{w}} f_A(w)dw\right]$.

Figure 3 describes the differentiated optimal marginal income tax rates when public goods preferences and earning abilities are independently distributed. In the Bergson-Samuelson and the Rawlsian criterion case, the marginal income tax rates on individuals enjoying the public good are lower, which means this result justifies that our tax perturbation method is reasonable. Also, we can confirm that the difference of tax paid between group A and group B is greater than the congestion externalities, which is described by $\tau := (T_A - T_B - \phi_{f_A})/G$. This remains unchanged from the result at the lump-sum differentiation when public goods preferences and earning abilities are independently distributed. In the Rawlsian case, the result the marginal income tax rate in group A is zero at the bottom as with the Bergson-Samuelson case hold no longer. This is consistent with Boadway and Jaquet (2008) who investigate properties of the marginal income tax rates when the social welfare function is Rawlsian. Additionally, τ shifts upwardly compared to the Bergson-Samuelson criterion, which this reflects higher government redustributive tastes for the worst-off individuals in group B.

Here, we examine the implications of introducing positive or negative correlation between public goods preferences and earning abilities. Figure 4 – 6 plots the differentiated optimal marginal income tax rates in terms of some social welfare functions under positive and negative correlation. We introduce the correlation by considering $\overline{\theta}$ as a function of w. If they are positively correlated, $\overline{\theta}$ is a increasing function for w, and vice versa. In this setting, the marginal income tax rates in group A are higher (lower) in positive (negative) correlation than one in no correlation. This is because the mechanical effect in group A increases (decreases). This is confirmed by previous literatures as in Cremer et al (2010). Nevertheless, even if the mechanical effect in group A increases (decreases) in the positive (negative) correlation case, the marginal tax rates in group A are lower (higher) over high income levels than one in group B. This reflects that the government can obtain more tax revenues by making high income earner migrate into group A, which means that the participation effect is still greater than the mechanical effect. However, the negative jointness result no longer hold over all income levels due to the change of mechanical effect. Consequently, tax perturbation analysis following by Kleven et al. (2009) are not robust when the correlation is allowed.

6 Concluding Remarks

This paper analyzes the optimal nonlinear income taxes under the provision of public goods when individuals have two different types and determine labor supply along both intensive margin and extensive margin. The government can use tagging on the basis of the following two groups: one group benefits from a public good and the other does not. We show that the government redistributive tastes and the correlation of public goods preferences and earning abilities are especially crucial in determining the shape of the income tax schedules. In particular, if tastes of public goods and earning abilities are independently distributed, the marginal income tax rates on individuals enjoying the public good is lower. In this case, this is interpreted as follows: the participation effect, which is caused when a tagged group is variable, is greater than the mechanical effect. However, it is not necessary that this findings hold. Indeed, we numerically present that the marginal income tax rates on individuals enjoying the public good can partially be higher in the positive correlation case due to the increase of the mechanical effect and the decrease of participation effect. It reflects the increase of congested effect strengthens the restriction of group size. Therefore, compared to previous literatures examining exogenous tagging, the government need to consider both the mechanical effect and the participation effect when differentiating the income tax schemes.

Applications of the findings in the present paper pertain to the design of optimal transfer program related to excludable public goods. In practical, differential income taxation on the basis of public goods is used as income tax deductions for higher educations or health care expenses. As a result, we suggest potentially implications for tax policy involved with public goods.

7 Appendix: First-order conditions

7.1 Proof of Proposition 1

Using integration by parts, $\int_{\underline{w}}^{\overline{w}} \lambda_B(w)V_B'(w)$ is transformed into $\lambda_B(\overline{w})V_B(\overline{w}) - \lambda_B(\underline{w})V_B(\underline{w}) - \int_{\underline{w}}^{\overline{w}} \lambda_B'(w)V_B(w)$. Applying this to the optimization problem without tagging under labor mobility, the first-order conditions are as follows.

$$\frac{\partial \mathcal{L}}{\partial V_B(w)} = \left[\int_{\hat{\theta}}^{\overline{\theta}} W'(\theta G + V_A)f(\theta, w)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W'(V_B)f(\theta, w)d\theta\right] + \lambda_B'(w) - \gamma f(w) = 0 \quad (41)$$

$$\frac{\partial \mathcal{L}}{\partial \ell_B(w)} = \lambda_B(w)\left[\frac{v'(\ell_B(w))}{w} + \frac{\ell_B(w)}{w}v''(\ell_B(w))\right] + \gamma\left[w - v'(\ell_B(w))\right]f(w) = 0 \quad (42)$$

$$\frac{\partial \mathcal{L}}{\partial G} = \int_{\underline{w}}^{\overline{w}} \int_{\hat{\theta}}^{\overline{\theta}} W'(\theta G + V_A)(\theta - \hat{\theta})f(\theta, w)d\theta dw + \gamma(\hat{\theta} - 1)(1 - F(\hat{\theta})) - \gamma\phi_G = 0 \quad (43)$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\theta}} = -\gamma\left[\hat{\theta}f(\hat{\theta}) - (1 - F(\hat{\theta}))\right]G - \int_{\underline{w}}^{\overline{w}} \int_{\hat{\theta}}^{\overline{\theta}} G \cdot W'(\theta G + V_A)f(\theta, w)d\theta dw + \gamma\phi_{f_A}f(\hat{\theta}) = 0 \quad (44)$$

$$\frac{\partial \mathcal{L}}{\partial V_B(\overline{w})} = -\lambda_i(\overline{w}) = 0, \ \frac{\partial \mathcal{L}}{\partial V_B(\underline{w})} = \lambda_i(\underline{w}) = 0 \quad (45)$$

Integrating $\lambda_B'(w)$ in (41) and using the transversality condition (45), we can get the following.

$$-\frac{\lambda(w)}{\gamma} = \int_{w}^{\overline{w}} (1 - f_A^c \cdot g_A - f_B^c \cdot g_B)f(x)dx \quad (46)$$

Using $\frac{\ell v''}{v'} = \frac{1}{e}$, (42) is rewritten as follows.

$$\frac{T'}{1 - T'} = -\left[1 + \frac{1}{e}\right]\frac{\lambda(w)}{\gamma}\frac{1}{wf(w)} \quad (47)$$

Finally, combining (46) and (47), we can obtain the optimal marginal income tax rates (16). Also, (17) is derived from (44) which is transformed as follows.

$$\gamma\left[T_A - T_B - \phi_{f_A}\right]f(\hat{\theta}) = \gamma(1 - F(\hat{\theta}))G - \int_{\underline{w}}^{\overline{w}} \int_{\hat{\theta}}^{\overline{\theta}} G \cdot W'(\theta G + V_A)f(\theta, w)d\theta dw \quad (48)$$

Furthermore, $\gamma = \int_{\underline{w}}^{\overline{w}}\left[\int_{\hat{\theta}}^{\overline{\theta}} W'(\theta G + V_A)f(\theta, w)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W'(V_B)f(\theta, w)d\theta\right]dw$ holds from (41) using the transversality condition (45). Using this, the following is obtained.

$$\left[T_A - T_B - \phi_{f_A}\right]f(\hat{\theta}) = G(1 - F(\hat{\theta}))\int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\hat{\theta}} \frac{W'(V_B)}{\gamma}f(\theta, w)d\theta dw - GF(\hat{\theta})\int_{\underline{w}}^{\overline{w}} \int_{\hat{\theta}}^{\overline{\theta}} \frac{W'(\theta G + V_A)}{\gamma}f(\theta, w)d\theta dw \quad (49)$$

Therefore, we can get (17) by the definition of g_A and g_B.

7.2 Proof of Proposition 2

Using integration by parts, $\int_{\underline{w}}^{\overline{w}} \lambda_i(w)V_i'(w)$ is transformed into $\lambda_i(\overline{w})V_i(\overline{w}) - \lambda_i(\underline{w})V_i(\underline{w}) - \int_{\underline{w}}^{\overline{w}} \lambda_i'(w)V_i(w)$. Applying this to the optimization problem with tagging under labor mobility, the first-order conditions are as follows.

$$\frac{\partial \mathcal{L}}{\partial V_A(w)} = \int_{\hat{\theta}(w)}^{\overline{\theta}} W'(\theta G + V_A)f(\theta, w)d\theta + \lambda_A'(w) - \gamma f_A(w) + \mu(w) = 0 \tag{50}$$

$$\frac{\partial \mathcal{L}}{\partial V_B(w)} = \int_{\underline{\theta}}^{\hat{\theta}(w)} W'(V_B)f(\theta, w)d\theta + \lambda_B'(w) - \gamma f_B(w) - \mu(w) = 0 \tag{51}$$

$$\frac{\partial \mathcal{L}}{\partial \ell_A(w)} = \lambda_A(w)\left[\frac{v'(\ell_A(w))}{w} + \frac{\ell_A(w)}{w}v''(\ell_A(w))\right] + \gamma\left[w - v'(\ell_A(w))\right]f_A(w) = 0 \tag{52}$$

$$\frac{\partial \mathcal{L}}{\partial \ell_B(w)} = \lambda_B(w)\left[\frac{v'(\ell_B(w))}{w} + \frac{\ell_B(w)}{w}v''(\ell_B(w))\right] + \gamma\left[w - v'(\ell_B(w))\right]f_B(w) = 0 \tag{53}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\theta}(w)} = -\gamma\left[T_A(Z_A(w)) - T_B(Z_B(w)) - \phi_{f_A}\right]f(\hat{\theta}(w), w) + \mu(w)G = 0 \tag{54}$$

$$\frac{\partial \mathcal{L}}{\partial V_i(\overline{w})} = -\lambda_i(\overline{w}) = 0, \frac{\partial \mathcal{L}}{\partial V_i(\underline{w})} = \lambda_i(\underline{w}) = 0 \; i = A, B \tag{55}$$

Substituting (54) for (50) to delete $\mu(w)$ and rearranging, we can get the following.

$$\frac{\lambda_A'(w)}{\gamma} = \left[(1 - g_A)f_A^c - \frac{T_A(Z_A(w)) - T_B(Z_B(w)) - \phi_{f_A}}{G}f(\hat{\theta}(w)|w)\right]f(w) \tag{56}$$

This result can be rewritten by integrating $\lambda_A'(w)$ in (50) and using the transversality condition (55) as follows.

$$-\frac{\lambda_A(w)}{\gamma} = \int_w^{\overline{w}}\left[(1 - g_A)f_A^c - \frac{T_A(Z_A) - T_B(Z_B) - \phi_{f_A}}{G}f(\hat{\theta}(x)|x)\right]f(x)dx \tag{57}$$

On the other hand, using $\frac{\ell v''}{v'} = \frac{1}{e}$, (52) is rewritten as follows.

$$\frac{T_A'}{1 - T_A'} = -\left[1 + \frac{1}{e}\right]\frac{\lambda_A(w)}{\gamma}\frac{1}{wf_A(w)} \tag{58}$$

Finally, combining (57) and (58), we can obtain the optimal marginal income tax rates with tagging (25). Similarly, (26) is obtained by the same way.

7.3 Proof of Proposition 3

Let us start from the separable taxation with non-differentiated marginal income tax rates. By differentiating $g_B - g_A$ with respect to w, we can get the following.

$$g'_B - g'_A = \left[\frac{W''(V_B)}{\gamma} - \frac{\int_{\hat{\theta}}^{\bar{\theta}} W''(\theta G + V_A) f(\theta) d\theta}{\gamma(1 - F(\theta))} \right] \cdot V'_B \tag{59}$$

If inequality aversion is not zero, W'' is strictly increasing by the assumption. Hence, $g'_B - g'_A$ is negative for any w. By using this fact, we can derive the following inequality for any w.

$$\frac{1}{F(w)} \int_{\underline{w}}^{w} (g_B - g_A) f(w) dw > g_B - g_A > \frac{1}{1 - F(w)} \int_{w}^{\overline{w}} (g_B - g_A) f(w) dw \tag{60}$$

Therefore, equation (40) is positive, which we can conclude that dW is positive.

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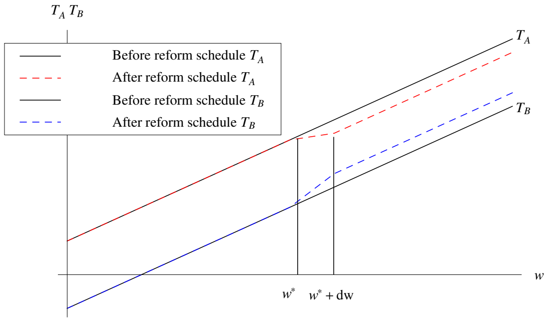


Figure 1: Negative Jointness

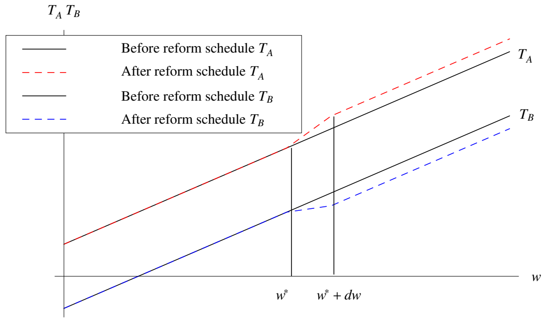
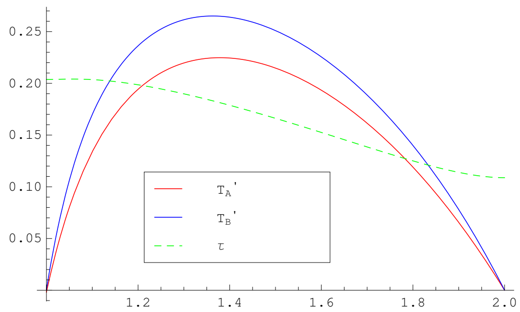
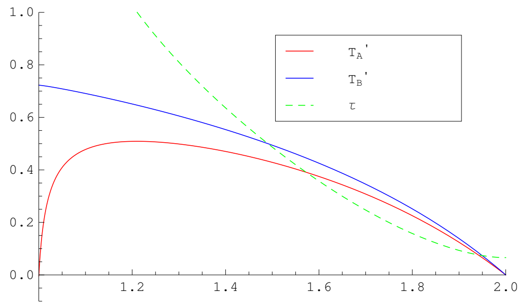


Figure 2: Positive Jointness

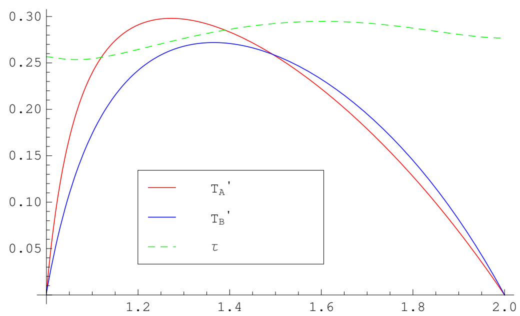


(a) Bergson-Samuelson criterion

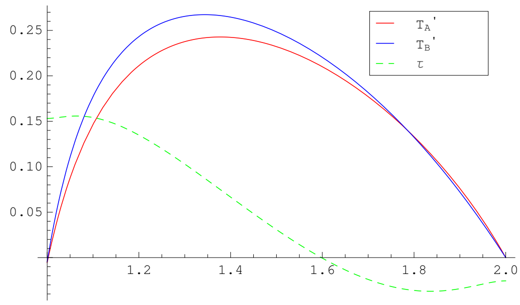


(b) Rawlsian criterion

Figure 3: The optimal differentiated marginal income tax rates under no correlation

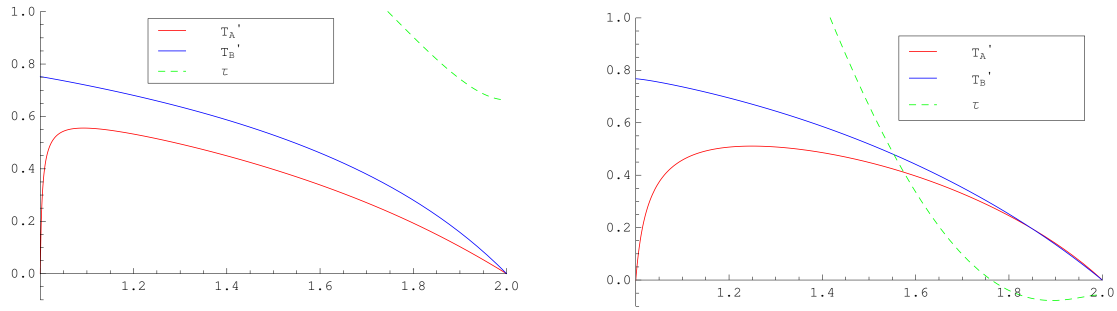


(a) Positive Correlation



(b) Negative Correlation

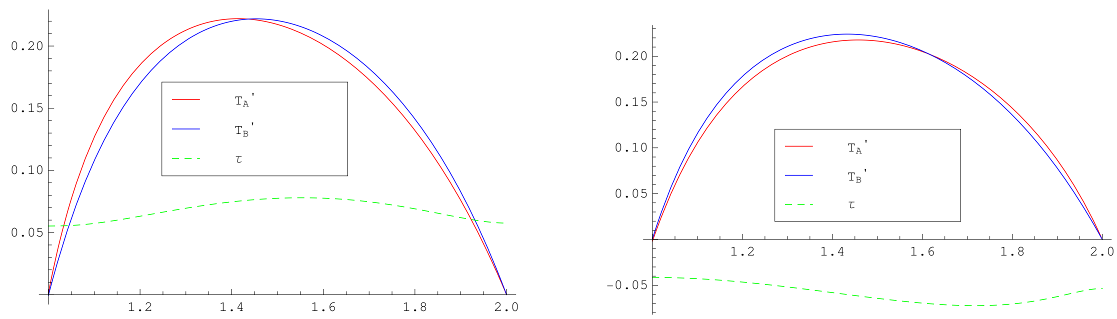
Figure 4: The optimal differentiated marginal income tax rates under Bergson-Samuelson criterion



(a) Positive Correlation

(b) Negative Correlation

Figure 5: The optimal differentiated marginal income tax rates under Rawlsian criterion



(a) Positive Correlation

(b) Negative Correlation

Figure 6: The optimal differentiated marginal income tax rates under Weighted Utilitarian